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EQUIVALENT LINEAR TECHNIQUE IN THE FINITE ELEMENT METHOD APPLIED TO DEFORMATION WITH VOLUME CHANGE AND TO AN AXISYMMETRIC BODY UNDER AN UNAXISYMMETRIC LOAD

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SUMMARY

Applicability of the Equivalent Linear Method in FEM is indicated with following conclusions. Calculation of rigidity and damping ratio using shear strain is reasonable when volume change and distortion exist. A Technique for FEM is proposed when an axisymmetric body is deformed by unaxisymmetric load to become partly plastic. Results from this analytical method are compared with the dynamic test results of a coal silo model.

INTRODUCTION

This paper discusses the appropriateness of determining rigidity and damping ratio from shear strain, simplification for an equivalent linear technique in cases where plasticization of an axisymmetric body occurs unevenly in the circumferential direction. The conclusions can be applied to the nonlinear dynamic behavior of a silo, or to the soil-structure interaction. Plasticization of granular contents of silos or soil often occurs prior to that of the vessel or structure. Therefore, it is important to accurately estimate the absorption of energy by plasticization. This study aimed at improving accuracy of the equivalent linear technique by stressing the reasonable conversion of complicated phenomena into a simple model, and by referring to results obtained by a more detailed method and to experimental results.

THE TREATMENT OF SYSTEMS IN WHICH FORM AND VOLUME CHANGE

Nonlinear deformation, such as of silo and of soil surrounding an embedded structure, is difficult because form and volume change simultaneously and nonlinear deformation characteristics are very complicated. Usually such deformation does not agree with the dynamic shear test or the triaxial compression test. Up to now, only shear strain (form change) was considered in deciding deformation characteristics, and the adequacy of these methods have not been well discussed yet. This paper shows that the method in which shear strain decides the deformation characteristics can give sufficient accuracy.

Assume the case in which the compressive stress (σ) varies as shown in Eq. (1) and forced shear strain γ varies with a similar function to compressive stress (Eq. (2)). From Eqs. (1) and (2), Eq.(3) can be obtained.

$$\sigma=\sigma_0+\sigma_D\cdot\sin\omega t \qquad (1)$$

$$\gamma=\gamma_D\cdot\sin\omega t \qquad (2)$$

$$\therefore \quad \sigma=\sigma_0\{1+(\gamma/\gamma_D)\cdot(\sigma_D/\sigma_0)\} \tag{3}$$

Where σ_o: Average compressive stress
σ_D: Amplitude of dynamic compressive stress
γ_D: Amplitude of shear strain

Damping ratio and decrease of rigidity when compressive stress and volume change will be studied. A modification of Iwan's model is used for this purpose. Iwan's original model(Ref. 1) shown in Fig. 1 has many basic elements in parallel, each of which has a spring and a slider in series. When the force acting is weak, elements show elastic motion due to the spring deformation, but when the force is strong, the sliders begin to move. The hysteresis curve shown in Fig. 2(a) results for a element. The modifications made to Iwan's model are that the rigidity of a spring is proportional to the root of compressive stress and that the resistance to movement of a slider is proportional to compressive stress. The skeleton curve of Iwan's model can agree with experimental results if suitable resistances of sliders are assumed. Also, the hysteresis loop agrees with Masing's Law.

When changes in compressive stress are taken into account, the hysteresis curve of a basic element is as shown in Fig. 2(b), which is determined by the following equations. When there is no sliding,

$$G_0=G_{00}\sqrt{\sigma/\sigma_0} \tag{4}$$

$$\tau=\int^{\gamma} G_0 d\gamma=\int^{\gamma} G_{00}\sqrt{\sigma/\sigma_0}\,d\gamma \tag{5}$$

The equation of the sliding line is,

$$\tau_f=\mu\cdot\sigma=\mu\cdot\sigma_0\{1+(\gamma/\gamma_D)\cdot(\sigma_D/\sigma_0)\} \tag{6}$$

Where Go: Initial rigidity (in range with no sliding)
Goo: Initial rigidity with compressive stress σ_o
τ_f: Shear stress at which slider begins to slide
μ : Friction coefficient of slider

The damping ratios $h=D/(2\pi\mu\sigma_0\gamma_D)$ are calculated for various values of the ratio σ_D/σ_o and of the ratio $\gamma_D/\gamma_{0.5}$. D is the strain energy consumed in the hysteresis shown in Fig. 3. $\gamma_{0.5}$ is the strain at which G/Goo=0.5 when $\sigma = \sigma_o$ (constant). G is the secant modulus of rigidity as shown in Fig. 3. The results shown in Table 1 and Fig. 4 are obtained. If the ratio G/Goo is calculated for the above ranges of values of σ_D/σ_o and $\gamma_D/\gamma_{0.5}$, the results in Table 2 are obtained.

It can be seen from Table 1 and Fig. 4 that except for extreme changes of compressive stress from 0 to 2.0 (i.e. $\sigma_D/\sigma_o=1.0$) or in the range of small damping, the difference of damping ratio is within 10% of its value when there is no change of compressive stress. Thus, in general the damping ratio is not substantially affected by periodic change of compressive stress. Table 2 shwos that except for the range of slight decrease of rigidity, there is no change in the ratio G/Goo from its value when there is no change of compressive stress. Moreover, the difference in the ratio G/Goo is in all cases within 10% of its value when there is no change of compressive stress. In Fig. 4, Table 1 and Table 2, it is noticeable that the effect of changing stress on the values of rigidity and damping ratio is rather small when strain is large.

Since Iwan's model can represent the various types of actual hysteresis loop, the results mentioned above show that even for systems in which both form and volume change simultaneously, it can be assumed that the relationships of G/Go to γ and of h to γ when there is no volume change can be applied.

TREATMENT OF THE CHARACTERISTIC CHANGE OF AXISYMMETRIC BODY

For equivalent linear FEM analysis on an axisymmetric body, it is necessary to obtain the distribution of strain in the circumferential direction, or the representative value of circumferential strain distribution. In this paper, Assumptions 1, 1' and 2 are used. The first is as follows. i) For horizontal, one-dimensional input motion, sufficient accuracy is obtained if only the first order term (n=1) of the expansion of displacement in the circumferential direction is used (Assumption 1). A general method to obtain rigidity and damping ratio of an axisymmetric element from the representative strain in the circumferential direction is proposed. If the maximum shear strain γ_{max} is used to calculate the representative strain, lengthy calculations are required to obtain the direction and maximum shear strain at each point. Therefore, the method of this paper is to evaluate the distribution of strain in the circumferential direction by the strain energy E_d caused by form change. E_d is given by the following equation.

$$E_d=\{(1+\nu)/6E\}\{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\}$$
$$=\{(1+\nu)/6E\}I^2 \qquad (7)$$

Where the stress invariant, $I=\{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\}^{\frac{1}{2}}$

For material tests , G/Go and h are generally related to γ_{max} . These values can also be related to the strain energy E_d because E_d can be obtained from γ_{max} . E_d is an explicit function of the stress invariant, so calculation of the maximum shear strain at each point is unnecessary.

In order to obtain the representative shear strain, the following method is used. Assume that there is horizontal excitation in one direction. The values of strain for the orthogonal coordinate system are as follows.

$$\{\varepsilon\}=[B]\{U\} \qquad (8)$$

Where, $\{\varepsilon\}^T=(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx})$

$\{U\}^T=(U_R, U_T, U_Z)$

$[B]$:Strain matrix (Ref.2, Ref.3, Ref.4)

$U_R \cos\theta$:Component of displacement in the radial (r) direction

$U_T \sin\theta$:Component of displacement in the circumferential (θ) direction

$U_Z \cos\theta$:Component of displacement in the axial (z) direction

The stress obtained by Eq. (8) can be substituted in Eq. (7), and the representative value of E_d can be compared to the results of material tests. Thus, rigidity and damping ratio for the equivalent linear method can be obtained.

When some assumptions for evaluation of strain can be made about the deformation, such as can be done for granular material in a rather rigid vessel (e.g. a silo), the calculation of stress can be simplified as follows. ii) A circle in the r-θ plane before deformation remains a circle after deformation, so $U_T = -U_R$ (Assumption 1'). iii) An r-θ plane before deformation remains so after deformation, so $\partial U_z/\partial r = U_z/r$ (Assumption 2).

Using these two assumptions,

$$\left.\begin{aligned}
&\sigma_x=\cos\theta\cdot\left\{(\lambda+2G)\cdot\frac{\partial U_R}{\partial r}+\lambda\cdot\frac{\partial U_z}{\partial z}\right\} && \tau_{xy}=G\cdot\sin\theta\cdot\frac{\partial U_R}{\partial r}\\
&\sigma_y=\lambda\cdot\cos\theta\cdot\left(\frac{\partial U_R}{\partial r}+\frac{\partial U_z}{\partial z}\right) && \tau_{yz}=0\\
&\sigma_z=\cos\theta\cdot\left\{\lambda\cdot\frac{\partial U_R}{\partial r}+(\lambda+2G)\cdot\frac{\partial U_z}{\partial z}\right\} && \tau_{zx}=G\cdot\left(\frac{\partial U_R}{\partial z}+\frac{\partial U_z}{\partial r}\right)
\end{aligned}\right\} \qquad (9)$$

Where λ : Lamé's coefficient

Using Eqs. (7) and (9), E_d can be expressed as follows.

$$E_d=\{(1+\nu)/6E\}\cdot\left[\cos^2\theta\cdot 2G^2\cdot\left(\frac{\partial U_R}{\partial r}-2\cdot\frac{\partial U_z}{\partial z}\right)^2+6G^2\cdot\left\{\left(\frac{\partial U_R}{\partial r}\right)^2+\left(\frac{\partial U_R}{\partial z}+\frac{\partial U_z}{\partial r}\right)^2\right\}\right] \qquad (10)$$

If pure shear is used in the material test, the following can be obtained.

$$E_d=(G/2)\cdot\gamma_{max}^2 \qquad (11)$$

Where E_d: Strain energy at the material test

From Eqs. (7) and (11),

$$\gamma_{max}=\sqrt{2E_d/G}=\sqrt{2\{(1+\nu)/6E\}I^2/G}=I'/\sqrt{6} \qquad (12)$$

Where I' =I/G

From Eq. (12), it can be seen that the results of the material tests can be given as a function of I'. If the root-mean-square of I' (I'rms) is used for the representative value of I', it can be given as follows from Eqs. (7) and (10).

$$
\begin{aligned}
I'_{rms} &= \left\{ \sqrt{\frac{6E \cdot E_d}{(1+\nu) \cdot G^2}} \right\}_{rms} \\
&= \sqrt{\int_0^{2\pi} \frac{6E \cdot E_d}{(1+\nu) \cdot G^2} d\theta / 2\pi} \\
&= \left[\left(\frac{\partial U_R}{\partial r} - 2 \cdot \frac{\partial U_z}{\partial z} \right)^2 + 6 \cdot \left\{ \left(\frac{\partial U_R}{\partial r} \right)^2 + \left(\frac{\partial U_R}{\partial z} + \frac{\partial U_z}{\partial r} \right)^2 \right\} \right]^{\frac{1}{2}}
\end{aligned} \quad (13)
$$

The value obtained from the above equation can be substituted in Eq. (12).

SIMULATION ANALYSIS OF THE MODEL SILO EXPERIMENT

Outline of the experiment The experiment shown in Fig. 5 was the object of the analysis. A coal silo vessel made of steel is fully loaded with coals. The model is installed on a shaking table, and given excitation in the horizontal direction by sine waves. Load cells support the cylindrical wall and the base plate, respectively, to measure base shear at each part. The coals were crushed, and screened with a 5 mm mesh. The measurement points are shown in Fig. 5.

Method of analysis The analytical model of the contents and the vessel consists of solid elements of axisymmetric FEM, as shown in Fig. 6. Concerning the physical constants for the coal, the initial rigidity of the coal (Eo and Go), Poisson's ratio, and the influence of the pressure on the initial rigidity were obtained from the elastic wave velocity. The initial rigidity is proportional to pressure to the power 0.537. The G/Go--γ and h--γ curves for the coal are obtained from the shear test. Eq. (13) was used, therefore Assumption 1' (which automatically implies Assumption 1) and Assumption 2) were assumed for simulation analysis.

Results of the analysis Fig. 7 shows the results for the resonance curves of the acceleration of the contents. When comparing the results of analysis with experimentation, the frequency range from low frequency to around the first resonance frequency is important since the base shear obtained by experiment confirmed that the force exerted on the vessel by the contents decreases above the first resonance frequency(Ref. 2). At this point, difference between the test and analysis is less than 20%, even in the case of 200 Gal excitation. Therefore, it can be stated that the hypotheses and methods presented in Sections 2 and 3 are sufficiently accurate within the scope of the simulation in this paper. In particular, the analytical method given can accurately reproduce the following characteristics observed in experiments. 1) The resonance frequency and responce magnification at resonance frequency decrease with increasing excitation. 2) The response decreases at frequencies above the first resonance and with increasing frequency. This decrease becomes less steep with increasing excitation.

CONCLUSION

1) Even for cases in which both form and volume change simultaneously, and for rather large strain as well as small strain, rigidity and damping ratio may be decided using only shear strain. Iwan's model was used to demonstrate this.

2) A method to obtain the rigidity and damping ratio for axisymmetric elements from representative strain in the circumferential direction is proposed. To obtain the representative strain, the strain energy due to deformation is used.

3) The results of simulation by the analytical method presented in this paper agree sufficiently well with the results obtained from the shaking table test of a

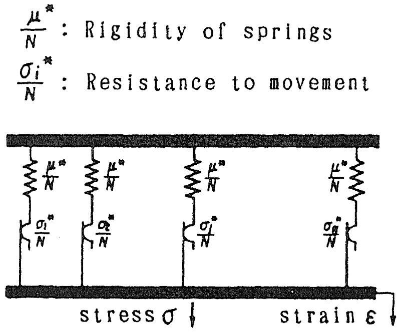

Fig. 1 Iwan's Original Model

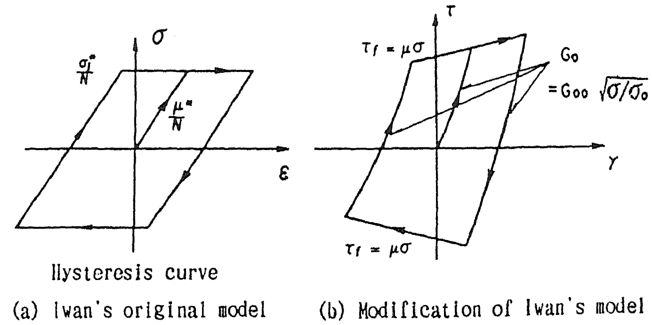

Fig. 2 Hysteresis Curve of the Basic Element

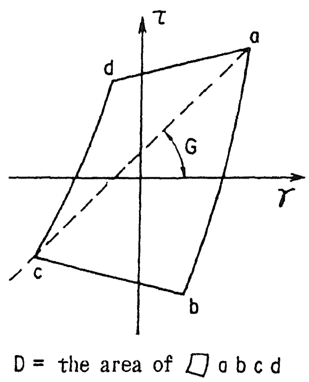

Fig. 3 Definition of G and D

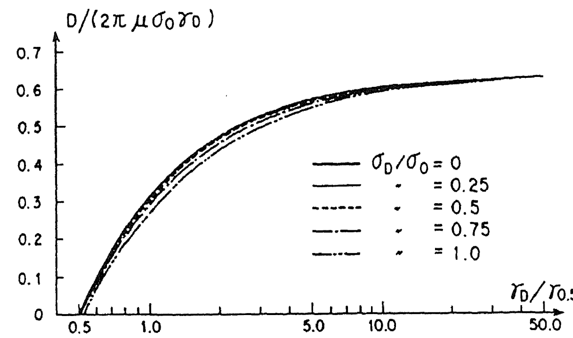

Fig. 4 Damping Ratio ($D/(2\pi\mu\sigma_0\gamma_D)$) for Various Values of Strain

Table 1 Damping Ratio ($D/(2\pi\mu\sigma_0\gamma_D)$) for Various Values of Strain

σ_D/σ_0 \ $\gamma_D/\gamma_{0.5}$	0.50	0.526	0.556	0.667	1.0	1.5	2.5	5.0	10.0	50.0
0	0.000	0.032	0.064	0.159	0.318	0.424	0.509	0.573	0.605	0.630
0.25	0.000	0.030	0.062	0.157	0.316	0.422	0.507	0.572	0.604	0.630
0.50	0.000	0.025	0.057	0.151	0.308	0.414	0.501	0.568	0.602	0.630
0.75	0.000	0.015	0.047	0.141	0.294	0.400	0.490	0.561	0.598	0.629
1.00	0.000	0.000	0.028	0.122	0.271	0.378	0.472	0.551	0.593	0.628

Table 2 Rigidity Ratio (G/Goo) for Various Values of Strain

σ_D/σ_0 \ $\gamma_D/\gamma_{0.5}$	0.50	0.526	0.556	0.667	1.0	1.5	2.5	5.0	10.0	50.0
0	1	0.95	0.9	0.75	0.5	0.333	0.2	0.1	0.05	0.01
0.25	0.997	0.95	0.9	0.75	0.5	0.333	0.2	0.1	0.05	0.01
0.50	0.989	0.95	0.9	0.75	0.5	0.333	0.2	0.1	0.05	0.01
0.75	0.973	0.95	0.9	0.75	0.5	0.333	0.2	0.1	0.05	0.01
1.00	0.943	0.943	0.9	0.75	0.5	0.333	0.2	0.1	0.05	0.01

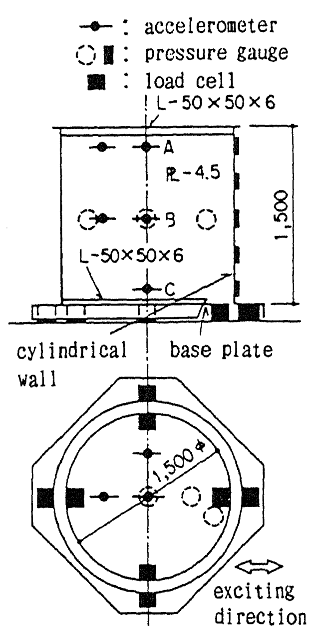

Fig. 5 Test Setup and Measurement Points

coal silo model. Therefore, from a comparison with the experiment, it can be concluded that the analytical method of this paper is adequate and may be used.

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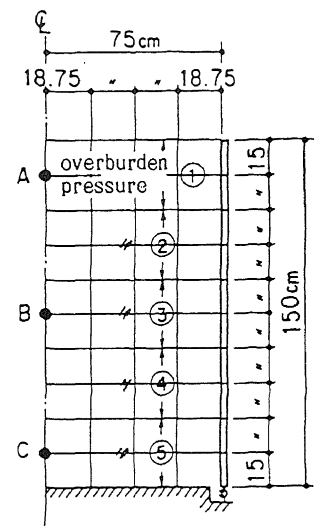

Fig. 6 Analytical Model

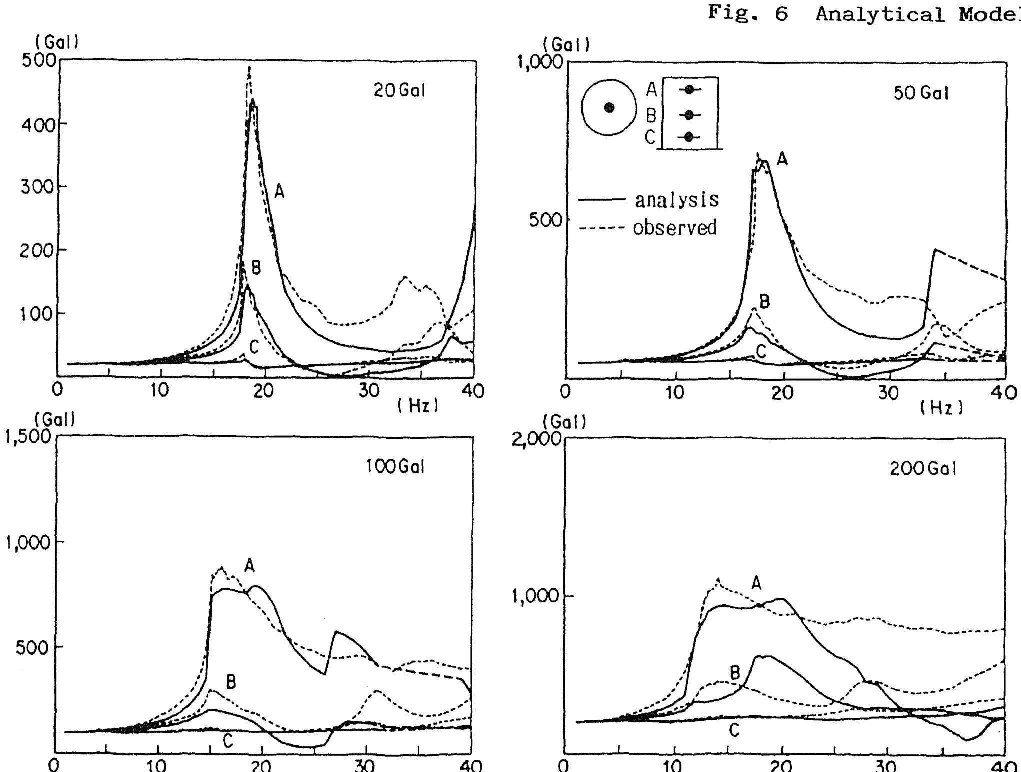

Fig. 7 Resonance Curves of the Acceleration of the Contents (Analytical and Experimental Results)