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HAZARD-CONSISTENT MAGNITUDE AND DISTANCE FOR EXTENDED SEISMIC RISK ANALYSIS

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SUMMARY

A method is developed to deal with multi-variate seismic hazard problems for extended seismic risk analysis. For the purpose of such an extension, the use of "hazard-consistent magnitude" and "hazard-consistent distance" is proposed. They are defined as conditional mean values given that the ground motion intensity exceeds the level determined from a specified annual probability of exceedance. Other ground motion parameters are determined as such corresponding to the hazard-consistent magnitude and the hazard-consistent distance. Liquefaction assessment is presented as a typical engineering application.

INTRODUCTION

Probabilistic seismic hazard analysis is widely applied in seismic load estimation involving various sources of uncertainty. Conventional methods deal with a single ground motion intensity parameter, such as peak ground acceleration. It is often required in engineering application to estimate not only intensity parameter, but also other ground motion parameters, such as predominant period, ground motion duration, etc. Its exact probabilistic treatment will involve the joint probability distribution of all parameters. However, this way is very complicated mathematically and it is difficult to obtain the data of the joint probability distribution of all parameters. In this study, a method is proposed for extending seismic hazard analysis that can deal with multiple ground motion parameters simultaneously with a simple and rigorous application of probability concept. After the ground motion intensity is determined corresponding to the risk parameter, such as the annual probability of exceedance, other ground motion parameters are estimated by using "hazard-consistent magnitude" and "hazard-consistent distance" proposed in this study. They are defined as conditional mean values given that the ground motion intensity exceeds the level determined from the annual probability of exceedance. In this manner, one can obtain all parameters to characterize a site-dependent ground motion on the basis of a single risk parameter.

DEFINITION OF HAZARD-CONSISTENT MAGNITUDE AND DISTANCE

Determination of Intensity Parameter First the ground motion intensity parameter is determined by using conventional methods of seismic hazard analysis (Ref.1, 2). The seismic region surrounding the site is divided into several small areas (so called "source-area"). In each source-area, parameters characterizing its seismic activities, such as the occurrence rate, b-value of the recurrence rule, upper bound of magnitude, etc. are assumed to be uniform. Earthquakes are assumed to occur randomly, and independently according to the Poisson process in each source-area.

Let p_0 denote the annual probability of exceedance for a ground motion intensity parameter Y to exceed a value y at the site ; i.e.,

$$p_0 = 1 - exp(-\sum_k w_k) \tag{1}$$

where w_k is the annual occurrence rate of the event that the ground motion intensity Y exceeds the value of $y(p_0)$ during any of earthquakes occurring in the source-area k, which is represented by ;

$$w_k = \lambda_k \cdot s_k \cdot \sum_i \sum_j P(Y \geqq y(p_0) | m_i, \delta_j) \cdot P_k(m_i) \cdot P_k(\delta_j) \tag{2}$$

in which λ_k is the annual average number of earthquakes per unit area in the source-area k, and s_k is the total area of the source-area k. The summation conventions are used instead of integrals by discretizing the earthquake magnitude and the source-area, for which serial numbers i and j are used, respectively. $P_k(m_i)$ and $P_k(\delta_j)$ are probability mass functions of magnitude M and epicentral distance Δ of earthquakes occurring in the source-area k, respectively. $P(Y \geqq y(p_0) | m_i, \delta_j)$ is the conditional probability of $Y \geqq y(p_0)$ given that an earthquake occurs, whose magnitude is m_i and epicentral distance is δ_j. This probability depends on the ground motion attenuation and the attenuation uncertainty.

The relation between p_0 and $y(p_0)$, called a hazard curve, is illustrated in Fig. 1 (a) → (b).

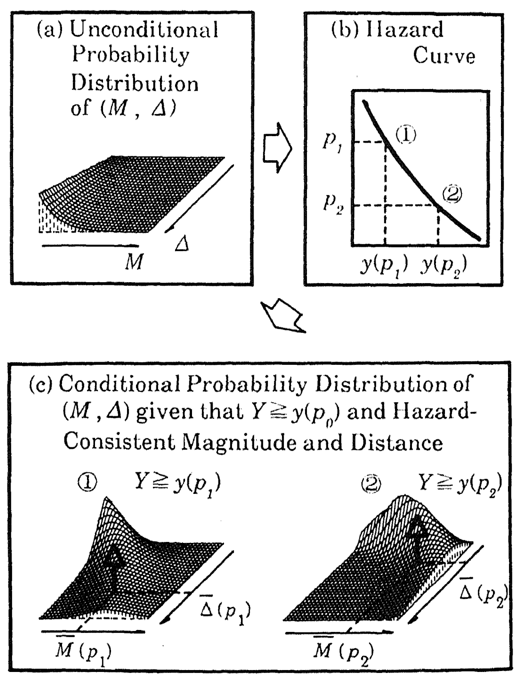

Fig. 1 Definition of Hazard-consistent Magnitude and Distance

<u>Definition of Hazard-consistent Magnitude and Distance</u> After determination of $y(p_0)$, the following discussion is carried out under the condition that the ground motion intensity parameter Y exceeds the value of $y(p_0)$. Conditional probability distributions of magnitude M and epicentral distance Δ given that $Y \geqq y(p_0)$ depend on the risk parameter p_0 as shown in Fig. 1 (c). The proposal of this study is the determination of other ground motion parameters by use of the conditional probability distributions of magnitude and epicentral distance calculated by an extended seismic hazard analysis technique. "Hazard-consistent magnitude" $\bar{M}(p_0)$ is defined as the mean value of magnitude from this conditional probability distribution. "Hazard-consistent distance" $\bar{\Delta}(p_0)$ is defined likewise.

The formulation of hazard-consistent magnitude $\bar{M}(p_0)$ and hazard-consistent distance $\bar{\Delta}(p_0)$ is described below. Conditional mean values of magnitude $E_k(M | Y \geqq y(p_0))$ and epicentral distance $E_k(\Delta | Y \geqq y(p_0))$ given that $Y \geqq y(p_0)$ under the condition that an earthquake occurring in the source-area k, are given as ;

$$E_k(M \mid Y \geqq y(p_0)) = \sum_i \sum_j m_i \cdot P_k(m_i, \delta_j \mid Y \geqq y(p_0)) \tag{3}$$

$$E_k(\Delta \mid Y \geqq y(p_0)) = \sum_i \sum_j \delta_j \cdot P_k(m_i, \delta_j \mid Y \geqq y(p_0)) \tag{4}$$

where $P_k(m_i, \delta_j \mid Y \geqq y(p_0))$ is the conditional probability mass function of the magnitude M and the epicentral distance Δ given that Y exceeds $y(p_0)$ under an earthquake occurring in the source-area k, which is calculated from ;

$$P_k(m_i, \delta_j \mid Y \geqq y(p_0)) = \frac{P(Y \geqq y(p_0) \mid m_i, \delta_j) \cdot P_k(m_i) \cdot P_k(\delta_j)}{P_k(Y \geqq y(p_0))}$$

$$= \frac{P(Y \geqq y(p_0) \mid m_i, \delta_j) \cdot P_k(m_i) \cdot P_k(\delta_j)}{\sum_i \sum_j P(Y \geqq y(p_0) \mid m_i, \delta_j) \cdot P_k(m_i) \cdot P_k(\delta_j)} \tag{5}$$

For earthquakes occurring only in the source-area k, the conditional mean values of magnitude and epicentral distance are obtained from Eqs. (3) ~ (5). On this basis, the hazard-consistent magnitude $\overline{M}(p_0)$ and the hazard-consistent distance $\overline{\Delta}(p_0)$ considering all source-areas are defined as ;

$$\overline{M}(p_0) = E(M \mid Y \geqq y(p_0)) = \left\{ \sum_k E_k(M \mid Y \geqq y(p_0)) \cdot w_k \right\} / \left\{ \sum_k w_k \right\}$$

$$= \frac{\sum_k \lambda_k \cdot s_k \cdot \sum_i \sum_j m_i \cdot P(Y \geqq y(p_0) \mid m_i, \delta_j) \cdot P_k(m_i) \cdot P_k(\delta_j)}{\sum_k \lambda_k \cdot s_k \cdot \sum_i \sum_j P(Y \geqq y(p_0) \mid m_i, \delta_j) \cdot P_k(m_i) \cdot P_k(\delta_j)} \tag{6}$$

$$\overline{\Delta}(p_0) = E(\Delta \mid Y \geqq y(p_0)) = \left\{ \sum_k E_k(\Delta \mid Y \geqq y(p_0)) \cdot w_k \right\} / \left\{ \sum_k w_k \right\}$$

$$= \frac{\sum_k \lambda_k \cdot s_k \cdot \sum_i \sum_j \delta_j \cdot P(Y \geqq y(p_0) \mid m_i, \delta_j) \cdot P_k(m_i) \cdot P_k(\delta_j)}{\sum_k \lambda_k \cdot s_k \cdot \sum_i \sum_j P(Y \geqq y(p_0) \mid m_i, \delta_j) \cdot P_k(m_i) \cdot P_k(\delta_j)} \tag{7}$$

The conditional variances and the conditional covariance of magnitude and epicentral distance given that $Y \geqq y(p_0)$ are determined by means of the similar formulation of $\overline{M}(p_0)$ and $\overline{\Delta}(p_0)$.

RISK-BASED DETERMINATION OF GROUND MOTION PARAMETERS

Seismic hazard analysis can be extended to simultaneous determination of multiple ground motion parameters as a function of a single risk parameter p_0 by using the above concepts. The method of extended seismic hazard analysis consists of the following 2 steps.

(1) Determination of the ground motion intensity parameter $y(p_0)$ corresponding to the annual probability of exceedance p_0 by using hazard curves.

(2) Estimation of other ground motion parameters as conditional mean values given that $Y \geqq y(p_0)$.

If any ground motion parameter X, such as predominant period, ground motion duration, etc. is related to the magnitude M and the epicentral distance Δ in the form of $X=g(M,\Delta)$, the conditional mean value of X given that $Y\geqq y(p_0)$ is represented by (direct method) ;

$$\bar{X}(p_0)=E(X|Y\geqq y(p_0))=\frac{\sum_k \lambda_k\cdot s_k\cdot\sum_i\sum_j g(m_i,\delta_j)\cdot P(Y\geqq y(p_0)|m_i,\delta_j)\cdot P_k(m_i)\cdot P_k(\delta_j)}{\sum_k\lambda_k\cdot s_k\cdot\sum_i\sum_j P(Y\geqq y(p_0)|m_i,\delta_j)\cdot P_k(m_i)\cdot P_k(\delta_j)} \qquad (8)$$

An approximate value for the conditional mean $\bar{X}(p_0)$ can be easily obtained by use of the hazard-consistent magnitude $\bar{M}(p_0)$ and the hazard-consistent distance $\bar{\Delta}(p_0)$. The first-order approximation is given as follows.

$$\bar{X}(p_0)\approx g(\bar{M}(p_0),\bar{\Delta}(p_0)) \qquad (9)$$

The second-order approximation of $\bar{X}(p_0)$ can be calculated by use of the conditional variances and the conditional covariance of magnitude and epicentral distance given that $Y\geqq y(p_0)$, in addition to the hazard-consistent magnitude and the hazard-consistent distance (Ref.3).

NUMERICAL EXAMPLES

Liquefaction assessment of sand deposits is described as a typical example application of the extended seismic hazard analysis. Three sites, Tokyo, Osaka and Sendai in Japan, are analyzed. The peak ground acceleration on the ground surface is used as the intensity parameter. Various conditions, such as the source-area models for each site, the historical earthquake data, and upper and lower bounds of magnitude in each source-area are based on Ref.4. Attenuation equation for the peak ground acceleration for soft soils (type 4) in Specifications for highway bridges (Ref.5) is used. Attenuation uncertainty is assumed as 0.5 of logarithmic standard deviation.

Fig.2 shows the hazard curve of $y(p_0)$, the hazard-consistent magnitude $\bar{M}(p_0)$ and the hazard-consistent distance $\bar{\Delta}(p_0)$ for Tokyo, Osaka and Sendai. The hazard-consistent magnitude tends to increase with decrease in p_0, whereas the hazard-consistent distance decrease as p_0 decreases. Note also there are characteristic differences in their curves for individual sites reflecting their seismic environments. This aspect is observed clearly in Fig.3, which shows the conditional probability distributions of the magnitude M and the epicentral distance Δ for three sites, when p_0 corresponds to 10^{-1} and 10^{-2}. These distributions which are the base of the hazard-consistent magnitude and the hazard-consistent distance, characterize the site-dependent seismic environment. For example, in Tokyo, the frequency of earthquake

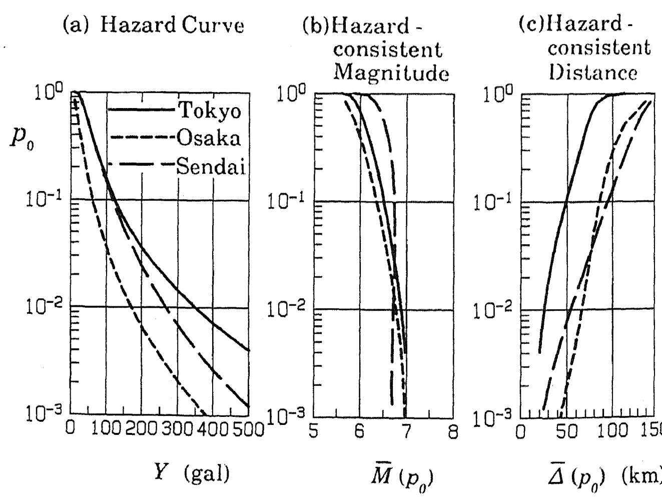

Fig. 2 Hazard Curve, Hazard-consistent Magnitude and Hazard-consistent Distance

occurrence at a close distance is relatively high, then the value of hazard-consistent distance is smaller than other sites with same p_0.

Liquefaction assessment of sand deposits generally requires estimation of ground motion intensity and ground motion duration. In Tokimatsu and Yoshimi's method (Ref.6) widely used in Japan and also employed in this study, the peak ground acceleration and the earthquake magnitude are used as corresponding parameters. Therefore, the hazard-consistent magnitude $\overline{M}(p_0)$ proposed in this study, in addition to the peak ground acceleration $y(p_0)$ are conveniently used.

Fig.4 shows the SPT N-value profile and the underground water level of model soil deposits. All sand deposits are assumed to have a unit weight of 1.9tf/m^3 and a fines content of 5% or less. Distribution of the liquefaction safety factor F_L for a period of 100 years was calculated under the seismic environment in Tokyo, Osaka and Sendai.

Table 1 shows the peak ground acceleration for the annual probability of exceedance of 10^{-2}, and the corresponding hazard-consistent magnitude in the three sites. According to Table 1, the peak ground acceleration becomes larger in the order of Tokyo, Sendai and Osaka. On the other hand, the hazard-consistent magnitude becomes larger in the order of Tokyo, Osaka and Sendai, thus reflecting local seismic environments.

Fig.5 shows the distribution of F_L. According to this figure, the F_L values become smaller in the order of Tokyo, Sendai and Osaka, which corresponds to the order of the peak ground acceleration. In this example, the 100 year period is considered. The results of liquefaction assessment depend

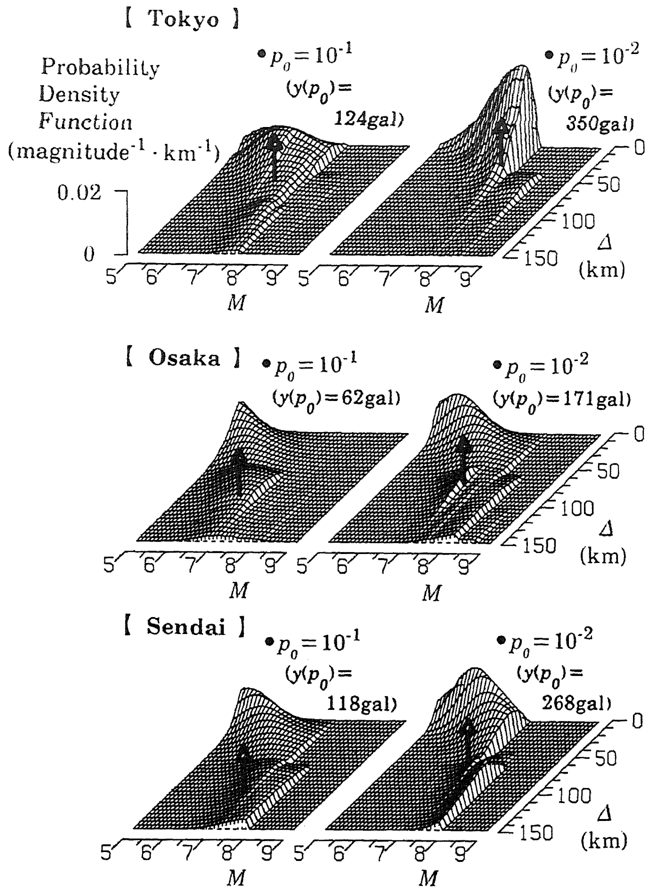

Fig. 3 Conditional Probability Distributions of Magnitude and Distance

Table 1 Peak Ground Acceleration and Hazard-consistent Magnitude for 100-years

Site	Tokyo	Osaka	Sendai
Peak Ground Acceleration (gal)	350	171	268
Hazard-Consistent Magnitude	6.88	6.78	6.72

on the considered period, because of the change of the expected seismic loads. Application of the extended seismic hazard analysis described in this paper allows the determination of the seismic load in the form reflecting such differences in the seismic environment and the considered period.

There are many engineering applications of the extended seismic hazard analysis besides of above example, such as the simulation of the earthquake ground motions (Ref.7).

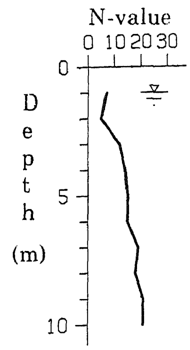

Fig. 4 Soil Profile

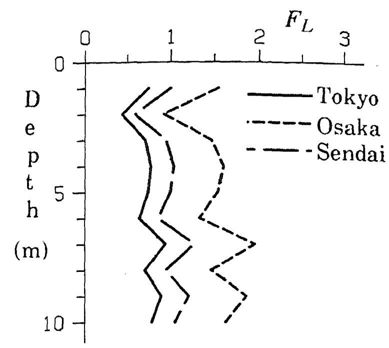

Fig. 5 Distribution of Liquefaction Safety Factor

CONCLUSIONS

This study has proposed a new method of simultaneous determination of multiple ground motion parameters in the context of the probabilistic seismic hazard analysis. The concepts are simple and useful to determine the seismic load for various engineering problem, such as, liquefaction assessment described above, earthquake ground motion simulation for dynamic response analysis, etc. The main results may be summarized as follows.

(1) The conditional mean values of magnitude and epicentral distance under the condition that the ground motion intensity Y exceeds the value of $y(p_0)$ were respectively defined as the "hazard-consistent magnitude" and "hazard-consistent distance".
(2) A method for the simultaneous determination of multiple earthquake ground motion parameters as a function of a single risk parameter such as the annual probability of exceedance was proposed by extending the conventional seismic hazard analysis. The determination of other ground motion parameters can be easily calculated by use of the hazard-consistent magnitude and the hazard-consistent distance.
(3) The above method was applied to the liquefaction assessment as a example application in engineering problem.

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