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## PREDICTION OF STRONG GROUND MOTIONS BY A SIMPLIFIED SYNTHESIS METHOD OF ACCELEROGRAMS

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### SUMMARY

This paper deals with a simplified method for synthesizing strong-motion accelerogram from small earthquake records by use of the faulting source model. The method is derived by extending the spectra relation satisfied in a point source model to ones in the faulting source model. The important terms employed in the method are the scaling law of source spectra, the number of sub-faults and so on. They were obtained statistically from the past strong-motion accelerograms observed in Japan. The method was applied to several representative earthquakes in Japan, and it is shown that the synthesized accelerograms by the method agree relatively well with the observed ones in the points of amplitude, duration and spectra characteristics.

### INTRODUCTION

Hartzell proposed a semi-empirical method, in which records of small earthquakes such as foreshock or aftershock are regarded as a kind of Green's function, for synthesizing records during a large earthquake(Ref.1). This method is quite skillful because ground motions can be evaluated by use of faulting models without estimating the complicated interference due to the propagation media of waves. Motivated by Hartzell's study in which displacement motions over several seconds were only dealt with, many researchers have applied the method to acceleration motions(Ref.2). Regrettably, however, the technique has not succeeded satisfactorily in predicting acceleration motions. The greatest reason is that the faulting parameters intimately associated with acceleration motions have not been made clear, and therefore the effective technique for superposing small earthquake records of acceleration has not become valid.

As for the inhomogeneous characteristics of source fault possibly related with acceleration motions, the author made clear the average aspect of the inhomogeneity over earthquake fault, and derived a simplified scaling law of source spectra with the aid of statistical analyses for strong-motion accelerograms obtained in Japan(Ref.5). Based on the results, the present paper deals with a *method of predicting strong-motion accelerograms from small earthquake records.*

### METHOD FOR SYNTHESIZING SPECTRA OF A LARGE EARTHQUAKE BY USE OF THE FAULTING MODEL

We now consider a main shock  $m$  and its related small shock  $e$  as shown in Fig.1. Since the shocks  $m$  and  $e$  generally occur almost in the same region, the

frequency response functions  $G_m(\omega)$  and  $G_e(\omega)$  for both shocks are equal to each other, assuming that the local soil layers have little material non-linearity. Accordingly we can obtain the following expression(Ref.3):

$$F_m(\omega) = F_e(\omega) \cdot \frac{R_m(\omega) \cdot S_m(\omega)}{R_e(\omega) \cdot S_e(\omega)}, \quad (1)$$

where  $F(\omega)$ : Fourier transform of the ground motions,  $R(\omega)$ : frequency response function due to the propagation media of waves which will be called "propagation spectra" hereafter,  $S(\omega)$ : source spectrum, and  $m$  and  $e$  indicate the main shock and its related small shock, respectively.

Eq.(1) is a spectra relation satisfied in a point source model. A large earthquake, on the other hand, has been known to consist of fault, so we need to extend the equation to the faulting source model. As shown in Fig.2, we here suppose that the mainshock has a rectangular fault and is composed of sub-faults of  $N$  in all which are referred to as "elementary earthquakes" in this paper. Denoting the source spectrum and propagation spectrum due to the  $n$ -th elementary earthquake as  $S_n(\omega)$  and  $R_n(\omega)$ , respectively and assuming the linearity of both spectra, the term  $S_m(\omega) \cdot R_m(\omega)$  in Eq.(1) is expressed as(Ref.3)

$$R_m(\omega) \cdot S_m(\omega) = \sum_{n=1}^N R_n(\omega) \cdot S_n(\omega) \quad (2)$$

Here, we set the origin of the Cartesian co-ordinates at the initiating point of rupture which is  $r_0$  away from the observation site, as illustrated in Fig.2. Furthermore, the rupture of fault is assumed to propagate from the origin in all directions with uniform velocity  $v$ . The spectra  $S_n(\omega)$  and  $R_n(\omega)$  related to each elementary  $n$ -th earthquake over the fault may be different in each element. Particularly the phase components of  $S_n(\omega)$  and  $R_n(\omega)$  may change remarkably from element to element because faulting of each element occurs successively propagating over the entire fault plane. The amplitude components of them, on the other hand, would be less dependent on element compared with the phase components. On the assumption that the variations of their amplitude components are little, accordingly,  $S_n(\omega)$  and  $R_n(\omega)$  are given by (Ref.3)

$$S_n(\omega) = |S_0(\omega)| \cdot e^{-i\phi_n(\omega)} \cdot e^{-i\omega\sqrt{x_n^2 + y_n^2 + z_n^2}/v}, \quad (3)$$

$$R_n(\omega) = |R_0(\omega, r_n)| \cdot e^{-i\psi_n(\omega)} \cdot e^{-i\omega\frac{r_n - r_0}{\beta}}, \quad (4)$$

where  $|S_0(\omega)|$ : source amplitude spectrum averaged for all elements,  $\phi_n(\omega)$ : phase angle except the propagation effect of rupture,  $(x_n, y_n, z_n)$ : co-ordinates of the center of the  $n$ -th element,  $|R_0(\omega, r_n)|$ : representative amplitude spectrum of propagation spectrum,  $\psi_n(\omega)$ : phase angle peculiar to  $R_n(\omega)$ ,  $\beta$ : propagation velocity of earthquake wave,  $r_n$ : distance between the center of the  $n$ -th element and the observation site, and  $\omega$ : circular frequency.

Similarly, the small earthquake spectra  $R_e(\omega)$  and  $S_e(\omega)$  in Eq.(1) can be expressed as follows separating their amplitude and phase:

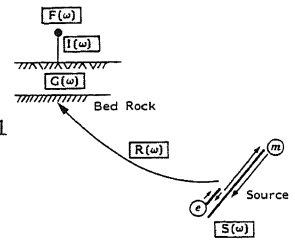


Fig.1 Schematic profile for earthquake ground motions

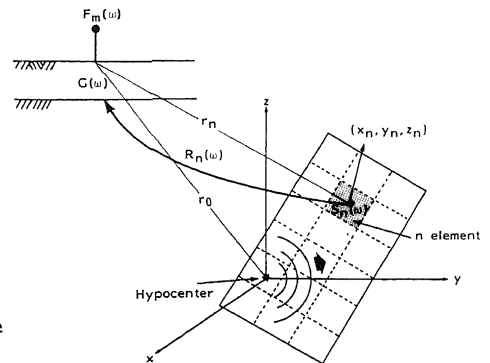


Fig.2 Earthquake ground motions due to the fault model

$$R_e(\omega) = |R_e(\omega)| \cdot e^{-i\eta_e(\omega)} \quad (5) \quad S_e(\omega) = |S_e(\omega)| \cdot e^{-i\xi_e(\omega)} \quad (6)$$

Although  $S_n(\omega)$  varies its phase components from element to element as implied in Eq.(3), the variation may have a incoherent character and may be independent from a statistical standpoint. In addition to such incoherency of elementary earthquake spectrum  $S_n(\omega)$ , the resultant source spectrum  $S_m(\omega)$  of the overall fault is given as linear summation of  $S_n(\omega)$  on the condition that the whole length of the fault is much smaller than the epicentral distance. Thus the expected value of amplitude for the resultant source spectrum  $S_m(\omega)$  may be given, by considering the root mean square of  $S_n(\omega)$  for each element, as follows(Ref.3).

$$E[|S_m(\omega)|] = \sqrt{N} |S_e(\omega)| \quad (7)$$

where  $E[ ]$  means the expected value and  $| \cdot |$  absolute value.

From Eqs.(1) ~ (7), we get the following expression, assuming that the terms  $E[|S_m(\omega)|] / |S_e(\omega)|$  and  $|R_0(\omega)| / |R_e(\omega)|$  are scaled with earthquake magnitude  $M$ , focal depth  $D$ , epicentral distance  $r$  of the main shock  $m$  and its related small shock  $e$ (Ref.3):

$$F_m(\omega) = F_e(\omega) \frac{1}{\sqrt{N}} \sum_{n=1}^N \frac{|R(\omega, r_n)| \cdot |S(\omega, M_m, D_m)|}{|R(\omega, r_e)| \cdot |S(\omega, M_e, D_e)|} \cdot e^{-i\omega \left( \frac{r_n - r_e}{v} + \frac{\sqrt{z_n^2 + z_n^2 + z_n^2}}{v} \right)} \cdot e^{-i\phi_n(\omega)} \quad (8)$$

where the subscripts  $m$  and  $e$  mean, respectively, mainshock and its related small shock, and  $\phi_n(\omega) = \phi_m(\omega) + \phi_n(\omega) - \eta_e(\omega) - \xi_e(\omega)$ .

#### SCALING LAW OF SOURCE SPECTRA AND PROPAGATION SPECTRA, AND FAULTING PATTERN OF MAINSHOCK BY USE OF STATISTICAL ANALYSES

In order to make Eq.(8) possible to evaluate, we need to grasp the scaling law of source spectrum  $S(\omega)$  and propagation spectrum  $R(\omega, r)$ , and the total number  $N$  of sub-faults to be counted for the mainshock. The author attempted to clarify these parameters statistically using a multiple regression analyses of strong-motion spectra as well as referring to a theoretical faulting model. In the analyses, the spectra of 228 strong-motion accelerograms obtained in Japan were calculated and they were dealt with statistically with the aid of a special multiple regression technique by which the spectra peculiar to source and the spectra due to propagation media of waves were simply scaled in terms of earthquake magnitude, focal depth and source-to-observation distance(Refs.4, 5). After incorporating the statistical spectra with the specific barrier model of Papageorgiou and Aki(Ref.6), moreover, the general faulting pattern of inhomogeneous fault was made clear. The results showed that the total number of sub-faults, namely, the total number of cracks over the whole fault is relatively stable regardless of earthquake magnitude  $M$ , ranging from 10 to 20 between  $M=6$  and  $M=8$ . The mean value of the total cracks number estimated statistically is nearly equal to 16. Therefore, the number  $N$  of the elementary earthquakes for the mainshock would be set simply to be 16 irrespective of earthquake magnitude  $M$  in the following. The spectra scaling laws, meanwhile, were derived from the statistical analyses of strong-motion spectra as follows(Ref.3):

$$\frac{|S(\omega, M_m, D_m)|}{|S(\omega, M_e, D_e)|} = 10^{a(\omega)(M_m^2 - M_e^2) + b(\omega)(M_m - M_e)} \cdot 10^{c(\omega)(D_m - D_e)} \quad (9)$$

$$\frac{|R(\omega, r_n)|}{|R(\omega, r_e)|} = \left( \frac{r_n + 30}{r_e + 30} \right)^{d(\omega)} \quad (10)$$

where  $M$ ,  $D$  and  $r$  mean earthquake magnitude, focal depth and source-to-station distance, respectively, the subscripts  $m$ ,  $e$  and  $n$  indicate the mainshock, its related small shock and the number of element of the mainshock fault each, and  $a(\omega)$ ,  $b(\omega)$ ,  $c(\omega)$  and  $d(\omega)$  are the multiple regression coefficients obtained from the statistical analyses.

The statistical analyses of the strong-motion spectra were fully described in

Refs.4 and 5. Anyway, we can easily estimate the spectra ratios in Eqs.(9) and (10) by employing the statistical coefficients provided that familiar earthquake factors such as magnitude, focal depth and source-to-station distance are available. An example of the spectra ratios is showed, respectively, in Figs.3 and 4.

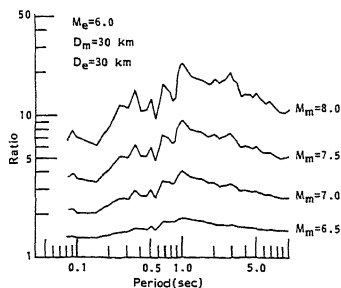


Fig.3 Example of source spectra ratio

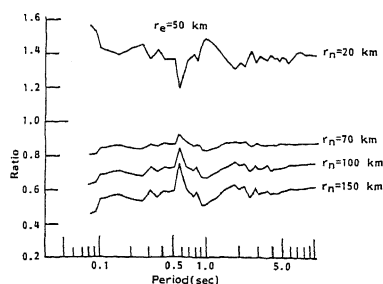


Fig.4 Example of propagation spectra ratio

On the other hand,  $\phi_n(\omega)$  in Eq.(8) is determined by the composite effects due to the phase components of elementary earthquake spectrum and the small shock spectrum. It may vary at random from element to element reflecting the complicated faulting mechanism, and so it would be relevant that we regard it as a stochastic variable rather than deterministic one. The detailed stochastic manner, however, has not been known yet. For this reason, it was varied in this study as the uniform pseudo-random number of  $0 \sim 2\pi$  for each element and moreover was dealt with to be constant irrespective of  $\omega$  to avoid its interference to the propagation manner of fault rupture.

Now that the principal terms in Eq.(8) have been given as mentioned above, we can synthesize the mainshock spectra  $F_m(\omega)$  in accordance to the equation after evaluating  $F_e(\omega)$  which is obtained as Fourier transform of the accelerogram due to the pertinent small shock. Finally, the inverse Fourier transform for  $F_m(\omega)$  leads to the objective accelerogram of the mainshock.

#### PREDICTION EXAMPLE OF STRONG-MOTION ACCELEROGRAM FROM SMALL EARTHQUAKE RECORD

We applied the present method to several representative earthquake occurred in Japan. These are the 1968 Tokachi-Oki earthquake ( $M=7.9$ ), the 1973 Nemuro-Hantooki earthquake ( $M=7.4$ ), the 1978 Miyagiken-Oki earthquake ( $M=7.4$ ) and the 1983 Nihonkai-Chubu earthquake ( $M=7.7$ ) (Refs. 3 and 7). In this paper, we show only the results of the 1983 Nihonkai-Chubu earthquake of 26 May, 1983 on account of space consideration. The earthquake occurred in the Japan sea off Akita and Aomori prefectures located in northern Japan and caused severe damage to both prefectures. Fig.5 indicates the origins of the mainshock and representative aftershocks  $e_1$  and  $e_2$  of the earthquake together with the principal sites which observed the accelerograms due to these shocks. The mainshock fault which was deduced from the distribution of aftershocks is also illustrated in Fig.5. The earthquake factors of the main-shock and the aftershocks are listed on Table 1 and the fault parameters for the mainshock are summarized

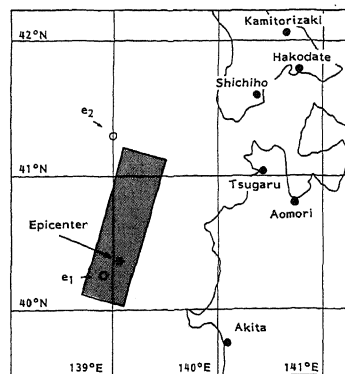


Fig.5 The map showing the 1983 Nihonkai-chubu earthquake

Table 1 The earthquake factors of the 1983 Nihonkai-chubu earthquake

	Date	Magnitude	Depth (km)	Epicentral Distance (km)	
				(Akita)	(Others)
Mainshock m	May 26, 1983	7.7	14.0	(Akita)	107.0
				(Aomori)	156.0
				(Hakodate)	211.0
				(Tsugaru)	137.0
				(Shichino)	177.0
(Kamitorizaki)	236.0				
Aftershock e1	June 9, 1983	6.1	23.0	(Akita)	113.0
Aftershock e2	June 21, 1983	7.1	6.0	(Aomori)	160.0
				(Hakodate)	195.0
				(Tsugaru)	129.0
				(Shichino)	123.0
				(Kamitorizaki)	169.0

Table 2 The fault parameters of the 1983 Nihonkai-chubu earthquake

Fault Length	125 km
Fault Width	35 km
Center of Fault	40.61°N 139.11°E
Dip Angle	20°
Dip Direction	N105°E
Division of Fault	16 (4x4)
Velocity of Fault Rupture	2.5 (km/sec)
Velocity of Earthquake wave	3.5 (km/sec)

in Table 2. Fig.6 shows the accelerogram obtained at Akita site during the aftershock e<sub>1</sub>. By employing it as the small earthquake record pertinent for synthesis, the accelerogram due to the mainshock was synthesized as indicated in Fig.7.

In Fig.7 the acceleration record observed at Akita site during the mainshock is also drawn for comparison. It is found in Fig.7 that the synthesized accelerogram is consistent comparatively well with the observed one in the points of amplitude and duration. Fig.8 shows the comparison between the response spectra calculated from the synthesized and observed accelerograms. We can see from Fig.8 that both of the synthesized and observed agree relatively well in spectral characteristics, too.

Similar results to Fig.7 are shown in Fig.9 for the other observation sites. It should be noted in Fig.9 that the accelerograms due to the aftershock e<sub>2</sub> were used as the small earthquake records. We can also find in Fig.9 that there is a relatively good consistence between the synthesized accelerograms and observed ones.

#### CONCLUDING REMARKS

In this study we have derived a simplified semi-empirical method for predicting strong-motion accelerograms from small earthquake records. The parameters that we mainly need in this method are earthquake magnitude, focal depth and source-to-station distance of a mainshock and its related small shock, which are quite familiar with engineers, and the fault characteristics of the mainshock. Even if the present method requires such simple source parameters, it was shown that the accelerograms predicted by the method are consistent relatively well with the observed ones in the points of amplitude, duration and spectral characteristics. Also the method was confirmed to have a versatility when selecting small earthquake records although the results were not shown in this paper (Refs.3 and 7). It would be thus

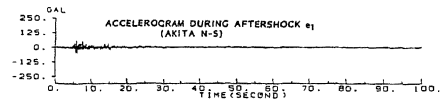


Fig.6 The accelerogram at Akita site during the aftershock e<sub>1</sub>

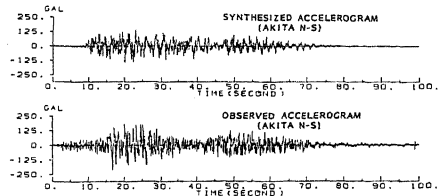


Fig.7 Comparison between the synthesized accelerogram and observed one at Akita site during the mainshock

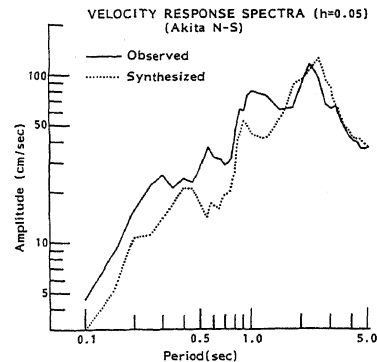


Fig.8 Comparison of the spectra calculated from the accelerograms at Akita site

concluded that the method presented here is a simplified and effective tool for predicting strong-motion accelerograms.

#### ACKNOWLEDGEMENTS

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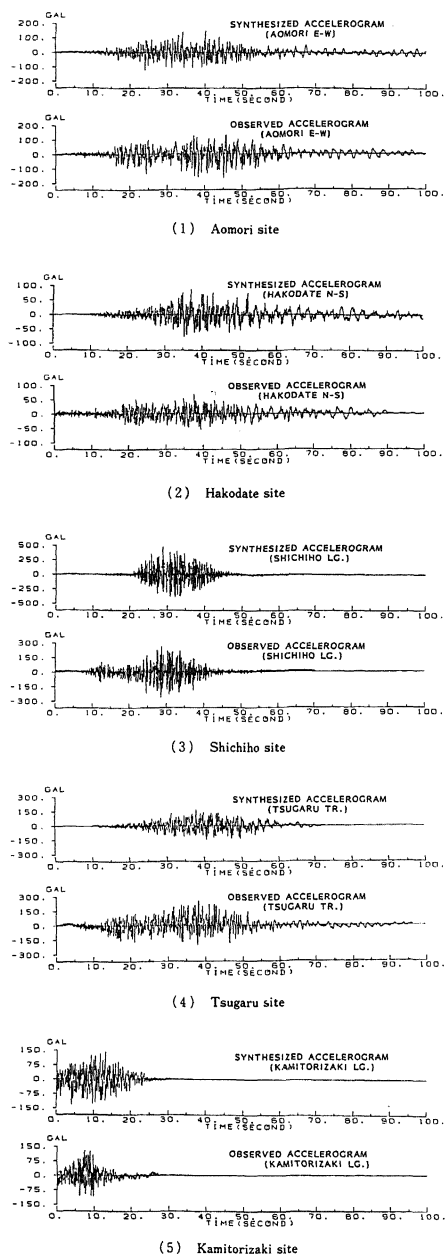


Fig.9 Comparison between the synthesized accelerograms and observed ones during the mainshock