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## STOCHASTIC ANALYSIS OF SPATIAL VARIATION OF SEISMIC WAVES

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### SUMMARY

This paper presents a method to develop a stochastic ground motion model. The parameters that control the spectral shape of ground motion is the local soil properties, corner frequency of the source as well as the travel path attenuation effects. This model is calibrated using data recorded from some events of SMART-1 array. To gain insight into the effect of local soil characteristics, the ratio of smoothed Fourier amplitude between the outcropping bedrock motion and the array response is computed. Spatial variation of seismic waves was also studied and adopted in this model. With the proposed stochastic ground motion model, the design spectrum can be constructed for any given earthquake magnitude as well as for any different site conditions.

### INTRODUCTION

It is well known that the seismic ground motion at a site is influenced by the seismic source, the travel path of the seismic waves and the local site characteristics. The SMART-1 array, located at the southern end of the essentially flat Lanyang plain, provide a valuable set of strong ground motion data for both seismological and engineering study on spatial variation of seismic waves and ground motion characteristics. In the last years, an increasing amount of effort is being devoted to the study of ground motion model (Ref. 1,2,3,4). The purpose of this paper is to quantitatively study the effects of the source, travel path, and local soil amplification on the spectral characteristics of the ground motion. A stochastic ground motion model for earthquake is developed. The spatial variation model of seismic waves, developed from array data, was adopted in the development of stochastic ground motion modelling. From which the site-dependent design spectrum can be constructed at any given earthquake magnitude and site characteristics. The ground deformation spectrum and differential response spectrum are also constructed. These two spectra provide good information for the design of long-extended lifelines.

### STOCHASTIC GROUND MOTION MODEL

Ground motion model have been developed based on physical parameters of the source and the medium (i.e., fault dimension, magnitude, shear wave velocity). Introduced by Boore (Ref. 1), the expression of the Fourier amplitude spectrum of

the ground motion is

$$R(f) = F(M, R) \cdot (2\pi f)^n S_0(f) D(f) / R \quad (1)$$

where the power  $n$  determines whether the velocity ( $n = 1$ ) or acceleration ( $n = 2$ ) is being considered. The factor  $F(M, R)$  stands for a scaling factor which is a function of earthquake magnitude  $M$ , and hypocentral distance  $R$ .  $S_0(f)$  is a source spectrum, and  $D(f)$  is a diminution factor. A commonly used stochastic model for ground movement at bedrock can be expressed in the following form (Ref. 5):

$$R(f) = \frac{f(M, R)}{R} \frac{(2\pi f)^n}{\left[1 + (\beta \frac{f}{f_2})^2\right]^{1/2} \left[1 + (\frac{f}{f_2})^2\right]} \cdot \frac{1}{\left[1 + (\frac{f}{f_m})^8\right]^{1/2}} \cdot \exp\left(\frac{-\pi f R}{Q(f) C}\right) \quad (2)$$

It is assumed herein that the regional  $Q(f)$  value is determined by averaging all of the values for the Taiwan area ( $Q(f) = 98.0 f^{1.0}$ ; for  $h < 11 \text{ km}$ ).  $C$  is the apparent phase velocity which had been estimated from the SMART-1 array site as  $C = 3.24 \text{ km/sec}$  (Event-45). A value of  $f_m = 25 \text{ Hz}$  is used in the study. An algorithm was proposed which match the peak ground acceleration (from attenuation equation) to that of the calculate peak ground acceleration from this ground motion model by random vibration theory (Ref. 7).

Figure 1 shows the Fourier amplitude spectrum (FAS) of acceleration at the outcropping bedrock (station E02) for motion in tangential direction (more like SH waves). The corner frequency  $f_2$  can be determined from the mean displacement FAS. The corresponding value of  $f_2$  for events in array site is in between  $0.67 \sim 0.9 \text{ Hz}$ . The estimated  $\beta$ -value is 0.3 (except for Event-43,  $\beta = 0$ , because of different fault mechanism). The theoretical model of ground motion is also shown in Fig. 1 for comparison. The above mentioned parameters in stochastic model can be used to predict the spectra characteristics of outcropping bedrock.

#### INFLUENCE OF LOCAL GEOLOGY ON EARTHQUAKE GROUND MOTION

It is generally accepted that a particular surface accelerogram reflects to some degree the characteristics of the near-surface soil layers at the recording site. For engineering purposes, it is often assumed that the local site effects can be represented by a simple transfer function between bedrock and the soil surface, the SMART-1 array data shows that this is not generally possible. The source mechanism, location, incident angle as well as local site characteristics must be taken into consideration in the development for an overall model for difference in ground motion. To gain insight into the effect of the depth of relatively soft subsurface depends on strong ground motion, the ratio of smoothed Fourier Spectrum between the outcropping bedrock motion and array sites is calculated for different events, as shown in Fig. 2. It is evident from the observation of these amplification ratios that no single transfer function can be postulated which is independent of the source. It will be necessary to use a fairly elaborate analytical model for the soil amplification or deduce this function from recorded data for events with similar distance and focal depth. In this study, we consider a semi-infinite medium consisting of  $N$ -parallel, homogeneous layer overlaying a half-space. The response of this layer system due to an inclined incident SH wave from the half-space at an arbitrary angle  $\theta$  was investigated. The computer code has been written to calculate the soil amplification between the free surface and the half-space outcrop. Numerical examples of a 4-layered system is given herein to illustrate the effects of anelastic attenuation ( $Q$ -value) and incident angle ( $\theta$ ) on the soil amplification.

Combining the ground motion model at the outcropping bedrock and the soil amplification developed in previous section, the model is used to reproduce ground motions of Event-45 recorded by the SMART-1 array. Figure 3 shows the comparison between the response spectrum as determined from the theoretical surface ground motion model and the data recorded from station 007, Event-45. Except for the low frequency range ( $< 0.5 Hz$ ), the theoretical ground motion model appears to do an adequate job of representing the actual surface ground motion. Based on these results, it is possible to predict the spectral characteristics of ground motion if the soil amplification function is known. The magnitude dependent response spectrum can also be constructed, as shown in Fig. 4.

### SPATIAL VARIATION OF GROUND MOTION

The observed spatial variation of the free-field motion over short a distance may have important implications for the seismic response of structures resting on multiple supports. One interesting feature of the recorded accelerations across the SMART-1 array is the considerable variation of the observed correlation between array stations.

The spectral density matrix of spatial coordinate can be reasonably well represented in the form

$$\underline{S}_a(i\bar{\omega}) = \begin{bmatrix} 1 & \gamma_{12}(i\bar{\omega}) & \gamma_{13}(i\bar{\omega}) & \cdots & \gamma_{1n_b}(i\bar{\omega}) \\ \gamma_{21}(i\bar{\omega}) & 1 & \gamma_{23}(i\bar{\omega}) & \cdots & \gamma_{2n_b}(i\bar{\omega}) \\ & & \vdots & & \\ \gamma_{n_b,1}(i\bar{\omega}) & \gamma_{n_b,2}(i\bar{\omega}) & \gamma_{n_b,3}(i\bar{\omega}) & \cdots & 1 \end{bmatrix} S_0(\bar{\omega}) \quad (3)$$

where  $S_0(\bar{\omega})$  is the ground motion model that developed previously. According to Luco and Wong (Ref. 6), the cross-spectrum between two points along a certain direction ( $x$ -axis) can be expressed in the form

$$B_{x_i, x_j}(x_i, x_j, \omega) = S_0(\omega) \cdot f_x(|x_i - x_j|, \omega) \cdot \exp\left[-i\omega\left(\frac{x_i}{C_m} - \frac{x_j}{C_m}\right)\right]$$

where  $f_x$  is the spatial coherence function for points  $x_i$  and  $x_j$  on the ground surface,  $C_m$  is the apparent phase velocity. Based on the present study of array data (Events-40, -43, and -45), a form of spatial coherence function is suggested, namely

$$f_x = \exp\left[-(a + b\omega) |x_i - x_j|^\alpha / C\right]$$

By means of a regression analysis (based on data of Events-40 and -43), it has been determined that a value of  $\alpha = 1/3$ , is appropriate. Figure 5 shows the spatial correlation of Event-45 with respect to two directional angles  $\alpha$  and  $\theta$ .

### APPLICATION OF GROUND MOTION MODEL FOR LIFELINE SEISMIC ANALYSIS

Based on the results of stochastic ground motion model as input motions, the evaluation of differential ground movement as well as the effect of differential motion on pipelines are discussed.

Ground Deformation Spectra The purpose of this section is to develop the basic relationships of relative ground displacement, on the basis of the phase delay in a long-period wave propagating between two locations and the physical ground

motion spectrum. The power spectral density function of the relative ground displacement,  $U_D(x, t)$ , is given as (Ref. 4)

$$S_{U_D}(r, \omega) = S_{U_1, U_1}(\omega) \left\{ 1 + \frac{S_{U_2, U_2}(\omega)}{S_{U_1, U_1}(\omega)} - 2 \operatorname{Re}[R(r, \omega)] \right\} \quad (4)$$

in which  $S_{U_1, U_1}(\omega)$  is the displacement power spectral density function at station 1, and  $R(r, \omega)$  is the normalized cross-spectrum. The root mean square value of relative displacement can be expressed as the square root of the zeroth order moment of the power spectrum,  $S_{U_D}(r, \omega)$ . Let  $U_D(x, t)$  be a stationary Gaussian process, the expected value of the peak response in terms of  $\operatorname{RMS}(U_D)$  is given by

$$\operatorname{Max}(U_D) = \operatorname{RMS}(U_D) \cdot p_f \quad (5)$$

in which  $p_f$  is the peak factor, which can be determined from statistics of extremes (Ref. 7). Figure 6 shows the ground deformation spectra as mentioned above. The main thrust of this study lies in that the absolute maximum relative ground displacement  $\operatorname{Max}(U_D)$  can be read from this ground deformation spectrum.

Differential Response Spectrum The differential response spectra represents the maximum relative response between pipeline segment as a function of the natural frequency of the system. The structural system model for evaluating the differential motion of pipelines subject to earthquake motion along the pipeline axis is that developed by Nelson and Weidlinger (Ref. 8) and applied by Zerva et. al. (Ref. 9). The power spectra density function of the differential displacement then developed.

$$S_{\Delta y \Delta y}(\omega) = \frac{\alpha}{\omega^4} (4\xi_0^2 \omega_0^2 \omega^2 + \omega_0^4) |H(\omega)|^2 S_{\ddot{v}_D}(\omega) \quad (6)$$

where

$$|H(\omega)|^2 = \frac{1}{(\omega_0^2 - \omega^2)^2 + 4\xi_0^2 \omega_0^2 \omega^2} \quad (7)$$

and  $S_{\ddot{v}_D}(\omega)$  is the power spectral density of the differential ground acceleration (see Eq. (4)).  $\alpha$  is the ratio between ground stiffness ( $k_g$ ) and pipe stiffness ( $k_p$ ), i.e.,

$$\alpha = \frac{k_g}{k_g + 2k_p} \quad (8)$$

Figure 7 shows the mean maximum differential displacements obtained from the differential displacement spectrum and random vibration model for pipeline separation of 0.2, 0.5, and 1.0 km (Fig. 7a) and for soil-structure relative stiffness  $\alpha$  of 0.05, 0.1, and 0.3 (Fig. 7b). The maximum differential displacement decreases with frequency, and increases with separation and  $\alpha$ -value. The shape of maximum differential displacement has significantly influenced by the shape of local soil amplification. The peaks in this spectrum are consistent with that of soil amplification function.

## CONCLUSION

This study presents the analytical equation to estimate the site-dependent response spectrum from the stochastic ground motion model. An algorithm is proposed which matches the peak bedrock acceleration calculated from the model to

that of the attenuation equation to scale the Fourier spectrum. This stochastic ground motion model is the modification of shear wave far-field source spectrum. From the study of local soil amplification, it is clear that different type of source has a strong influence on the transfer function between outcropping bedrock and soft soil. No simple soil amplification can be postulated which is independent of the value of source. It will be necessary to us a fairly elaborate model for the soil amplification function or to deduce this function from recorded data for events with similar distance and angle of incidence.

This stochastic ground motion model were used to calculate the ground deformation spectra as well as input motions to study the dynamic response of a specific pipeline system. Such a differential spectra and a differential response spectrum provide guide line for the design of lifeline system.

#### ACKNOWLEDGEMENTS

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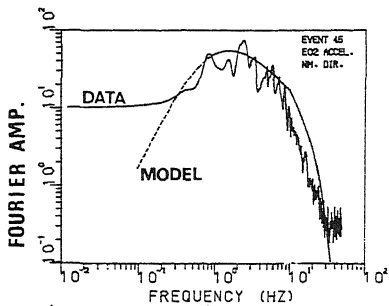


Fig.1: Comparison of Fourier amplitude spectrum.

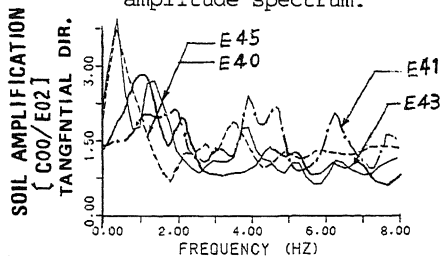


Fig.2: Calculated Soil amplification.

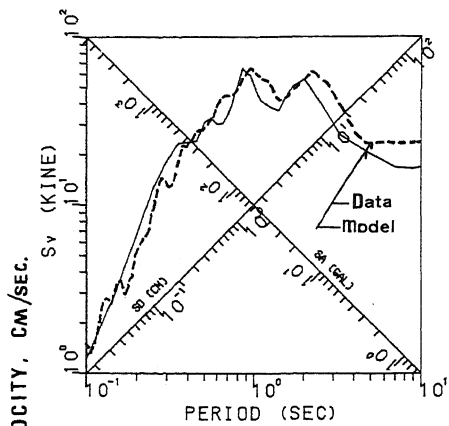


Fig.3: Comparison in spectral vel.

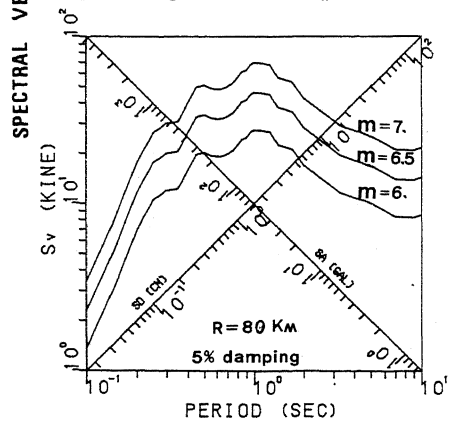


Fig.4: Estimated response spectrum with different magnitude.

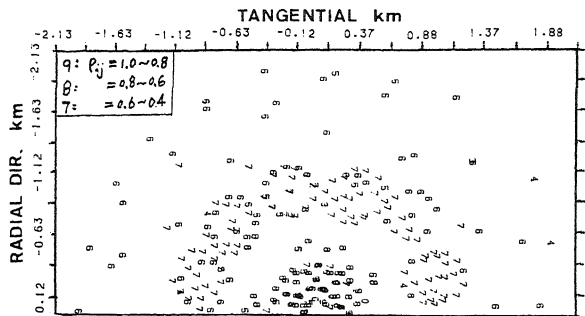


Fig.5: Two Dimensional spatial correlation function of Event-45.

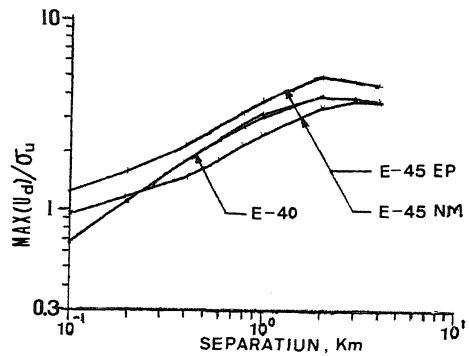


Fig.6: Ground deformation spectrum.

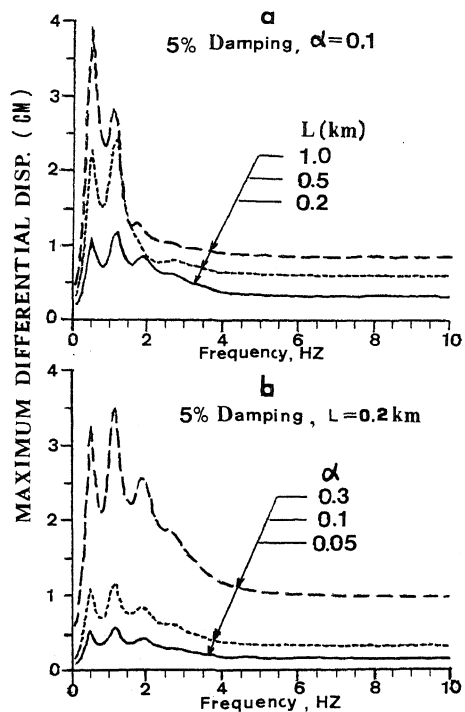


Fig.7: Differential displacement response spectrum.