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STOCHASTIC ANALYSIS OF SEISMIC GROUND MOTIONS IN SPACE AND TIME

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SUMMARY

The spatial and temporal variability of free-field earthquake ground motion has been analysed with the aid of stochastic field model. The stochastic model developed in this paper incorporates not only the effect of the stochastic variation of the soil properties in space but also the effect of the wave propagation in the horizontal direction. On the basis of the analytical solution and a sample stochastic wave varying in space (x-direction) and time obtained from the digital simulation method, it is found that the Kanai-Tajimi spectrum of a specified point motion can be derived as a special case of the power spectrum obtained in this paper. It is also found that the stochastic spatial variability of soil properties in the horizontal direction significantly affects the spatially inchoherency of ground motion.

INTRODUCTION

Considerable attention has been recently focused on the characterization of the spatial variability as well as the temporal variability of earthquake ground motions. In this context, several empirical studies have been performed strictly from a statistical point of view (Refs.1,2,3,4). Meanwhile, it is obviously desirable to develop analytical models which reflect the underlying physics as much as possible. Such models may be used not only to interpret seismic array data possibly in terms of wave content but also as an aid to develop site-specific design parameters. In the present study, the spatial and temporal variability of earthquake ground motin has been analysed with the aid of the stochastic field model. The stochastic model developed in this study incorporates not only the effect of the spatial variability of the soil properties but also the effect of wave propagation in the horizontal direction.

STOCHASTIC GROUND RESPONSE MODEL WITH RANDOM SOIL PROPERTIES

Consider the ground layers with random soil properties resting on the rigid bedrock and subjected to earthquake ground motion as shown in Fig.1. The total soil depth is assumed to be a constant H. The input earthquake ground motion at bedrock is assumed to be a stationary random wave propagating with speed c in the x-direction and represented by

$$u_b(x, t) = u_b\left(t - \frac{x}{c}\right) \tag{1}$$

The ground displacement at the location of x and z relative to the input motion is denoted by $u_r(x,z,t)$ also shown in Fig.1. Then the total response displacement $u(x,z,t)$ can be expressed as

$$u(x, z, t) = u_b\left(t - \frac{x}{c}\right) + u_r(x, z, t) \tag{2}$$

By assuming that the shear deformation predominates in the ground layers, then the equation of motion of ground layers is governed by

$$\rho(x, z)\ddot{u}_r - \frac{\partial}{\partial z}\left[G(x, z)\frac{du_r}{dz}\right] = -\rho(x, z)\ddot{u}_b \tag{3}$$

where $\rho(x,z)$ and $G(x,z)$ are, respectively, the soil mass and the shear modulus of soil at location x and z. In this paper, it is assumed, for the first approximation, that these soil properties are the random functions only of x:

$$\rho(x, z) = \rho(z)\left[1 + f_\rho(x)\right], \quad G(x, z) = G(z)\left[1 + f_G(x)\right] \tag{4}$$

In Eq.4, the functions $\rho(z)$ and $G(z)$ are the deterministic functions of z, and $f_\rho(x)$ and $f_G(x)$ represent the stochastic fluctuations of soil properties along x-axis and have mean zero: $E[f_\rho(x)]=0$ and $E[f_G(x)]=0$. It is noted here that the soil layers are "almost homogeneous" in the sense that $E[f_\rho^2(x)]<<1$ and $E[f_G^2(x)]<<1$ in Eq.4.

By introducing the generalized displacement $u_n(x,t)$ and assuming that

$$u_r(x, z, t) = \sum_{n=1} u_n(x, t)\,\psi_n(z) \tag{5}$$

with $\psi_n(0)=1$. In Eq.5, $\psi_n(z)$ is the shape function of the n-th mode or the assumed n-th mode function satisfying the geometric boundary conditions. It is usually convenient, although not essential, to normalize the shape function in the sense of Eq.5. For the problem at hand, the geometric boundary conditions are given such that

$$u_r(x, z, t) = 0 \quad \text{at} \quad z = H \quad \text{and} \quad \frac{d}{dz}u_r(x, z, t) = 0 \quad \text{at} \quad z = 0 \tag{6}$$

The shape function may be arbitrarily assumed, provided that it satisfies the geometric boundary conditions. Obviously, however, it is more advantageous to assume a shape function that approximates the actual deformation of the ground under the specified earthquake motion at bedrock. In the present paper, the shape function is assumed, for simplicity, as

$$\psi_n(z) = \cos\left[\frac{(2n-1)\pi z}{2H}\right] \tag{7}$$

The shape function in Eq.7 corresponds the n-th mode shape of a single homogeneous infinite horizontal layer lying on rigid bedrock. Equation 7 satisfies the normalization and the geometric boundary conditions indicated by Eq.6.

Recalling that the shape function satisfying the geometric boundary conditions possesses the orthogonality such that, for $m \neq n$,

$$\int_0^H \rho(z)\,\psi_n(z)\,\psi_m(z)\,dz = 0 \tag{8}$$

$$\int_0^H G(z)\frac{d}{dz}\psi_n(z)\frac{d}{dz}\psi_m(z)\,dz = 0$$

and substituting Eq.5 into Eq.3, then multiplying it by the n-th mode shape function, and then integrating it with respect to z from zero to H, one obtains

$$\ddot{u}_n(x, t) + 2h_n(x)\omega_n(x)\dot{u}_n(x, t) + [\omega_n(x)]^2 u_n(x, t) = -\beta_n \ddot{u}_b \tag{9}$$

In deriving Eq.9, the equivalent damping ratio $h_n(x)$ has been introduced to account for, in approximation, the energy loss due , but not necessary exclusively, to the hysteretic behavior of the soil under dynamic loadings. In Eq.9, $\omega_n(x)$= ground natural circular frequency of the n-th mode and β_n = participation factor of the n-th mode. They are given as,

$$\omega_n(x) = \sqrt{\frac{[1+f_G(z)]\int_0^H G(z)\left[\frac{d}{dz}\psi_n(z)\right]^2 dz}{[1+f\rho(x)]\int_0^H \rho(z)\psi_n^2(z)\,dz}} \tag{10a}$$

$$\beta_n = \frac{\int_0^H \rho(z)\psi_n(z)\,dz}{\int_0^H \rho(z)\psi_n^2(z)\,dz} \tag{10b}$$

By using the mode shape given by Eq.7 and assuming the soil mass is constant, Eq.10b reduces to;

$$\beta_n = \frac{(-1)^{n-1}4}{(2n-1)\pi} \tag{10c}$$

Alternatively, the natural circular frequency and the equivalent damping ratio may also be expressed formally as

$$\omega_n(x) = \omega_n[1+f(x)] \qquad h_n(x) = h_n[1+h(x)] \tag{11}$$

where ω_n and h_n are the means of $\omega_n(x)$ and $h_n(x)$, and f(x) and h(x) are the homogeneous stochastic fields with zero means.

SPATIAL AND TEMPORAL VARIABILITY OF WAVE FIELDS

Expanding the impulse response function of Eq.9 into a Taylor-series around $\omega_n(x)=\omega_n$ and $h_n(x)=h_n$, neglecting the higher-order terms under the assumption of smallness of h_n, $E[f^2(x)]$ and $E[h^2(x)]$, and assuming independence among f(x), h(x) and $u_b(t-x/c)$, one can obtain the cross-spectral density function $P_{uuz}(\xi, \omega)$ between the absolute displacements u(x,z,t) and $u(x+\xi,z,t+\tau)$ as follows (Ref.5);

$$P_{uuz}(\xi, \omega) = S_{u_b u_b}(\omega)\, e^{-i\frac{\omega\xi}{c}} [A(\omega) + R_{ff}(\xi) B(\omega)] \tag{12}$$

where $i=\sqrt{-1}$, $S_{u_b u_b}(\omega)$ is the power spectral density function of ub(t-x/c), and $R_{ff}(\xi)$ is the auto-correlation function of f(x) in Eq.11. The functions A(w) and B(w) are given such that

$$A(\omega) = 1 + \omega^2 \sum_{n=1} \beta_n \psi_n(z) \left[H_n(\omega) + \overline{H}_n(\omega)\right]$$

$$+ \omega^4 \left[\sum_{n=1} \beta_n \psi_n(z) H_n(\omega)\right] \left[\sum_{n=1} \beta_n \psi_n(z) \overline{H}_n(\omega)\right] \quad (13a)$$

$$B(\omega) = \omega^4 \left[\sum_{n=1} \beta_n \psi_n(z) \omega_n H\omega_n(\omega)\right] \left[\sum_{n=1} \beta_n \psi_n(z) \omega_n \overline{H}\omega_n(\omega)\right]$$

(13b)

with,

$$H_n(\omega) = \frac{1}{(\omega_n^2 - \omega^2) + i\,2 h_n \omega_n \omega}, \quad H\omega_n(\omega) = -(2\omega_n + i\,2 h_n \omega) H_n^2(\omega)$$

(13c)

where $\overline{H}_n(\omega)$ indicates complex conjugate of $H_n(\omega)$. It is noted here that the Kanai-Tajimi spectrum of ground motion can be derived by specifying the parameters as c=infinity, n=1, $R_{ff}(\xi)=0$ in Eqs.12 and 13. The other important statistics, the coherence function $\gamma_{uuz}(\xi, \omega)$ and the power spectral density function $S_{uuz}(\kappa, \omega)$, can be derived from the following equations:

$$\gamma_{uuz}(\xi, \omega) = \frac{|P_{uuz}(\xi, \omega)|}{P_{uuz}(0, \omega)}$$

(14)

$$S_{uuz}(\kappa, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P_{uuz}(\xi, \omega)\, e^{-i\kappa\xi}\, d\xi$$

And also, the wave form u(x,z,t) can be digitally simulated by (Ref.6)

$$u(x, z, t) = \sqrt{2} \sum_{i=1}^{M} \sum_{k=1}^{N} \left[\sqrt{2 S_{uuz}(\kappa_i, \omega_k) \Delta\kappa \Delta\omega} \cos(\kappa_i x + \omega_k t + \phi_{ik}^{(1)})\right.$$

$$\left. + \sqrt{2 S_{uuz}(-\kappa_i, \omega_k) \Delta\kappa \Delta\omega} \cos(-\kappa_i x + \omega_k t + \phi_{ik}^{(2)})\right] \quad (15)$$

$$\Delta\kappa = \frac{\kappa_u}{M}, \quad \Delta\omega = \frac{\omega_u}{N}, \quad \kappa_i = i\,\Delta\kappa, \quad \omega_k = k\,\Delta\omega$$

where κ_u and ω_u are the upper values of κ and ω, and $\phi_{ik}^{(1)}$, $\phi_{ik}^{(2)}$ are independent random phases uniformly distributed between 0 and 2π.

NUMERICAL EXAMPLE

Figures 2 and 3 show the coherence function $\gamma_{uuz}(\xi, \omega)$ and the power spectral density function $S_{uuz}(\kappa, \omega)$ of ground surface displacement

$u(x,t)=u(x,o,t)$ computed by using Eqs. 13 and 14 for the following values of parameters:

$$R_{ff}(\xi)=\sigma_{ff}^2\left[1-2\left(\frac{\xi}{b}\right)^2\right]\exp\left[-\left(\frac{\xi}{b}\right)^2\right], \quad S_{u_b u_b}(\omega)=1, \tag{16}$$

where σ_{ff} =5%, b=141.42 (m), and ω_n =3(2n-1) (rad/s), n=1-10, h1=25%, h2-h10=6%, and c=1000 (m/s). In Fig.3, the peaks of power spectral density function on the line of $\omega=c\kappa$ corresponds to the wave propagation effect and the smaller peaks on the other ω-κ plane are associated with the spatial variability of soil properties in the x-direction. It is interesting to observe from Fig.2 that the coherence function of $u(x,t)$ tends to decay with increasing separation and frequency with wavy form along the frequency axis as usually observed in the statistical analyses of array data (Refs.1,2). It is a characteristic of this model that the troughs of the wavy form of the coherence function occur at the natural circular ground frequency. Figure 4 shows a sample wave form of $u(x,t)$ digitally simulated using Eq.15. It can be seen from Fig.4 that the wave propagates in the x-direction with speed of 1000 (m/s) changing the wave form due to the randomness of soil properties in the x-direction, indicating a similar trend of observations.

CONCLUSIONS

In order to characterize the spatial and temporal variability of ground motions for the seismic analysis and design of spatially extended structures, this paper presents a simple stochastic ground response model for a seismic excitation in which the soil properties are random functions of the horizontal coordinate. On the basis of the analytical solution and a digitally simulated sample stochastic wave, it is found that the Kanai-Tajimi spectrum of ground motion can be derived as a special case from the space-time power spectrum of ground motion obtained in this paper. It is also found that the stochastic variation of soil properties significantly affects the spatially incoherency of the free-field ground motions.

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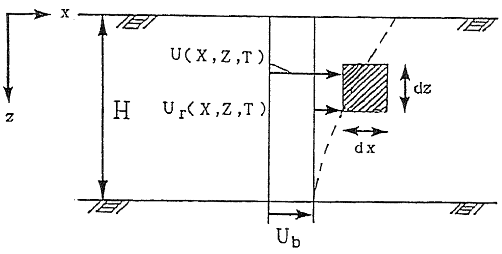

Fig. 1 Mathematical Model of Horizontally Homogeneous Ground with Random Soil Properties, and Its Notation

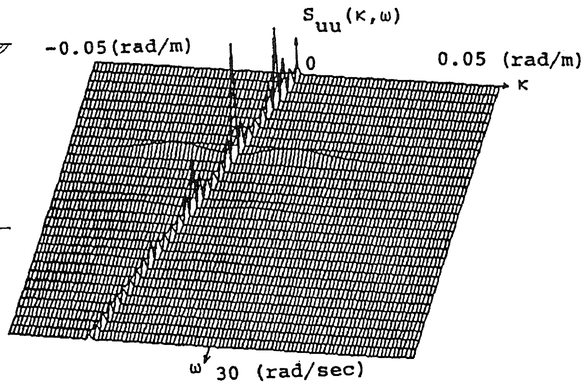

Fig. 3 Example of Space- Time Power Spectral Density Function

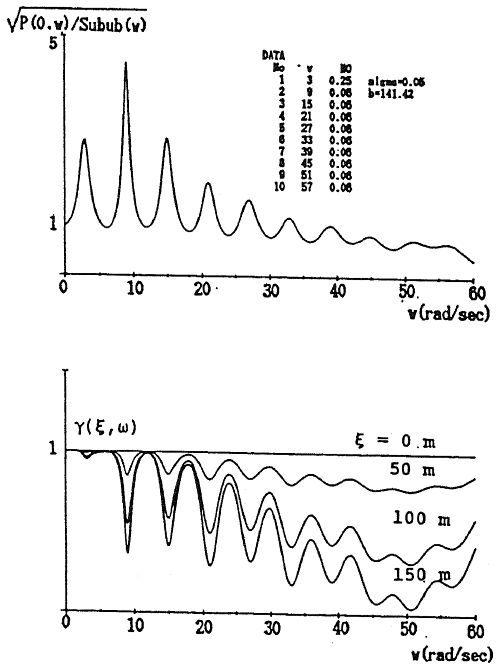

Fig. 2 Example of Transfer Function and Coherence Function

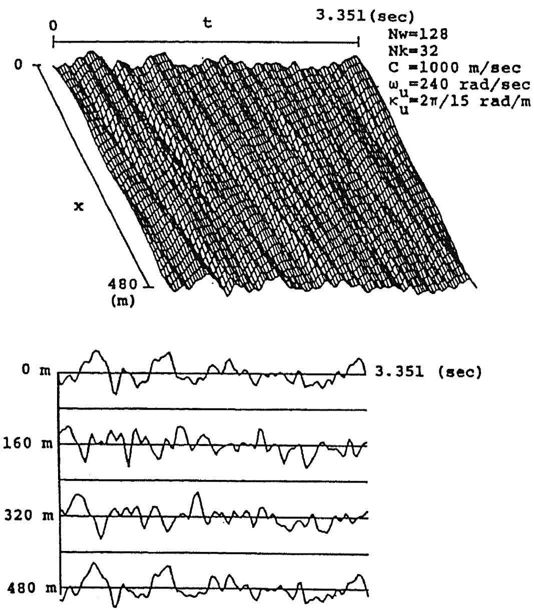

Fig. 4 Example of Digital Simulation of Stochastic Ground Motion Displacement