DIGITAL SIMULATION OF SEISMIC GROUND MOTION USING STOCHASTIC WAVE THEORY

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SUMMARY

In order to describe the nature of ground motion arising from a propagating seismic wave, a stochastic wave model has been developed here and an efficient technique for digitally generating samples of such a stochastic wave is introduced as an extension of the spectral representation method. Although the stochastic wave model considered in this work is stationary and homogeneous, it is a straightforward task to extend the introduced methodology to non-stationary and/or non-homogeneous stochastic waves characterized by an evolutionary power spectrum.

INTRODUCTION

A number of stochastic models for simulation of seismic ground motion have been proposed and successfully applied to a variety of structural problems arising from seismic events (Refs. 1-17). A common limitation of all these models is that ground motion is treated as a stochastic process when its time variability is examined, or as a stochastic field when its spatial variability is considered. In the former case, the space variables are frozen, while in the latter case, time is frozen. In order to have the analysis reflect the nature of ground motion arising from a propagating seismic wave, a stochastic wave model has been developed here and an efficient technique for digitally generating samples of such a stochastic wave is introduced as an extension of the spectral representation method primarily developed by Shinozuka and his associates. The proposed model is useful for the seismic response analysis of such large-scale structures extending over a wide spatial area as water transmission and gas distribution systems and long-span bridges.

THEORY OF STATIONARY, HOMOGENEOUS STOCHASTIC WAVES

Consider the following stationary, spatially two-dimensional, homogeneous stochastic
wave:

\[ f_0(x_1, x_2, t) = f_0(\xi, t) \quad (1) \]

with mean value zero:

\[ E[f_0(\xi, t)] = 0 \quad (2) \]

where \( \xi = [x_1 \ x_2]^T \) is the vector of the space variables, \( t \) is the time variable and \( E[\cdot] \) denotes the expectation. The autocorrelation function of \( f_0(\xi, t) \) is defined as:

\[ R_{f_0, f_0}(\xi, \tau) = E[f_0(\xi, t)f_0(\xi + \xi, t + \tau)] \quad (3) \]

Since the stochastic wave is considered to be stationary and homogeneous, \( R_{f_0, f_0}(\xi, \tau) \) is symmetric with respect to the separation distance vector \( \xi = [\xi_1 \ \xi_2]^T \) and the time lag \( \tau \):

\[ R_{f_0, f_0}(\xi, \tau) = R_{f_0, f_0}(-\xi, -\tau) \quad (4) \]

Assuming that the three-fold Fourier transform of \( R_{f_0, f_0}(\xi, \tau) \) exists, the power spectral density function of \( f_0(\xi, t) \) is defined as:

\[ S_{f_0, f_0}(\kappa, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{f_0, f_0}(\xi, \tau) \ e^{-i\kappa \cdot \xi} \ e^{-i\omega t} \ d\xi_1 d\xi_2 d\tau \quad (5) \]

and its inverse transform is given by:

\[ R_{f_0, f_0}(\xi, \tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{f_0, f_0}(\kappa, \omega) \ e^{i\kappa \cdot \xi} \ e^{i\omega t} \ d\kappa_1 d\kappa_2 d\omega \quad (6) \]

The preceding two equations represent the three-dimensional version of the Wiener-Khinchine transform pair, where \( \kappa = [\kappa_1 \ \kappa_2]^T \) is the wave number vector, \( \kappa \cdot \xi \) is the inner product of \( \kappa \) and \( \xi \), and \( \omega \) is the frequency. Using Eq. 4, it can be shown that the power spectral density function is real and symmetric and also non-negative:

\[ S_{f_0, f_0}(\kappa, \omega) = S_{f_0, f_0}(-\kappa, -\omega) \quad (7) \]

\[ S_{f_0, f_0}(\kappa, \omega) \geq 0 \quad (8) \]

SIMULATION OF STATIONARY, HOMOGENEOUS STOCHASTIC WAVES

Based on the above-mentioned properties of \( S_{f_0, f_0}(\kappa, \omega) \), the stationary, spatially two-dimensional, homogeneous stochastic wave \( f_0(\xi, t) \) can be simulated by a stochastic wave \( f(\xi, t) \) in the following fashion: consider that the power spectral density function \( S_{f_0, f_0}(\kappa, \omega) \) of \( f_0(\xi, t) \) is of insignificant magnitude outside the region defined by:

\[ -\kappa_u \leq \kappa \leq \kappa_u \quad (9) \]

\[ -\omega_u \leq \omega \leq \omega_u \quad (10) \]

where \( \kappa_u = [\kappa_{1u} \ \kappa_{2u}]^T \) with \( \kappa_{iu} > 0 \) \((i = 1, 2)\) and \( \omega_u > 0 \). Denote the interval vector by:

\[ [\Delta \kappa_1 \ \Delta \kappa_2 \ \Delta \omega]^T = \left[ \begin{array}{c} \kappa_{1u} \\ \kappa_{2u} \\ \omega_u \\ N_1 \\ N_2 \\ N \end{array} \right]^T \quad (11) \]
and then construct the simulated wave $f(\mathbf{z}, t)$ by the following series, as $N_1, N_2, N \rightarrow \infty$ simultaneously:

$$f(\mathbf{z}, t) = \sqrt{2} \sum_{k_1=1}^{N_1} \sum_{k_2=1}^{N_2} \sum_{k=1}^{N} \sum_{I_3=\pm 1}^{L_2=\pm 1} \sum_{I_4=\pm 1}[2S_{f_0 f_0}(\kappa_{1 k_1}, I_2 \cdot \kappa_{2 k_2}, I_0 \cdot \omega_k) \Delta \kappa_1 \Delta \kappa_2 \Delta \omega]^{1/2} \cdot \cos(\kappa_{1 k_1} \cdot x_1 + I_2 \cdot \kappa_{2 k_2} \cdot x_2 + I_0 \cdot \omega_k \cdot t + \varphi_{k_1 k_2 k})$$

(12)

where $\varphi_{k_1 k_2 k}$ are independent random phase angles uniformly distributed between 0 and $2\pi$, and

$$\kappa_{1 k_1} = k_1 \cdot \Delta \kappa_1, \quad k_1 = 1, 2, \ldots, N_1$$

(13)

$$\kappa_{2 k_2} = k_2 \cdot \Delta \kappa_2, \quad k_2 = 1, 2, \ldots, N_2$$

(14)

$$\omega_k = k \cdot \Delta \omega, \quad k = 1, 2, \ldots, N$$

(15)

The simulated field $f(\mathbf{z}, t)$ is asymptotically Gaussian as $N_1, N_2, N \rightarrow \infty$ simultaneously, due to the central limit theorem.

**NUMERICAL EXAMPLE**

For simplicity, consider a stationary, homogeneous and non-dispersive Rayleigh wave in which case $\omega/c = \kappa = \sqrt{\kappa_1^2 + \kappa_2^2}$ with $c = \text{phase velocity of Rayleigh wave}$. Then, $S_{f_0 f_0}(\kappa_1, \kappa_2, \omega)$ degenerates into $S_{f_0 f_0}(\kappa_1, \kappa_2)$. The following form of the power spectral density function $S_{f_0 f_0}(\kappa_1, \kappa_2)$ is used in this numerical example:

$$S_{f_0 f_0}(\kappa_1, \kappa_2) = \frac{\sigma_{yy}^2}{8 \pi} \cdot b_1^3 \cdot b_2 \cdot \kappa_1^2 \cdot \exp \left[ -\left( \frac{b_1 \kappa_1}{2} \right)^2 - \left( \frac{b_2 \kappa_2}{2} \right)^2 \right]$$

(16)

The corresponding autocorrelation function is given by:

$$R_{f_0 f_0}(\xi_1, \xi_2) = \sigma_{yy}^2 \cdot \left[ 1 - 2 \left( \frac{\xi_1}{b_1} \right)^2 \right] \cdot \exp \left[ -\left( \frac{\xi_1}{b_1} \right)^2 - \left( \frac{\xi_2}{b_2} \right)^2 \right]$$

(17)

The power spectrum shown in Eq. 16 is proposed by Harada and Shinozuka (Ref. 18) and is based on a wave number analysis of data from the original accelerograms recorded on January 29, 1981 (Event 5) by a SMART-1 seismograph array installed at Lotung, Taiwan. The data used represent the horizontal component of the displacement time history in the direction N139°W which is approximately the direction of the seismic source of this earthquake relative to the location of the array. The following values are used for $\sigma_{yy}$, $b_1$ and $b_2$ appearing in Eqs. 16 and 17:

$$\sigma_{yy} = 0.0124 \text{ m} \quad b_1 = 1131 \text{ m} \quad b_2 = 3012 \text{ m}$$

(18)

The apparent lack of a frequency-wave number analysis in the estimation of the power spectrum is taken care of by the following relationship mentioned above:

$$\omega = g(\kappa_1, \kappa_2) = c \cdot \sqrt{\kappa_1^2 + \kappa_2^2}$$

(19)
in which the value of the phase velocity is set equal to:

$$c = 2,800 \frac{m}{sec}$$  \hspace{1cm} (20)

These assumptions were made strictly for demonstration of the proposed digital simulation procedure and do not imply that the data used indeed represent a Rayleigh wave.

The simulation of a stochastic wave having the properties described above is then performed using the following expression:

$$f(x_1, x_2, t) = \sqrt{2} \sum_{k_1=1}^{N_1} \sum_{k_2=1}^{N_2} 2S_{f_0 f_0}(\kappa_{1k_1}, \kappa_{2k_2}) \Delta \kappa_1 \Delta \kappa_2 \frac{1}{2} \cdot \left\{ \cos[\kappa_{1k_1} \cdot x_1 + \kappa_{2k_2} \cdot x_2 + g(\kappa_{1k_1}, \kappa_{2k_2}) \cdot t + \varphi^{(1)}_{k_1 k_2}] + \\
+ \cos[\kappa_{1k_1} \cdot x_1 - \kappa_{2k_2} \cdot x_2 + g(\kappa_{1k_1}, \kappa_{2k_2}) \cdot t + \varphi^{(2)}_{k_1 k_2}] \right\}$$  \hspace{1cm} (21)

where:

$$\Delta \kappa_1 = \frac{\kappa_{1u}}{N_1} ; \quad \Delta \kappa_2 = \frac{\kappa_{2u}}{N_2}$$  \hspace{1cm} (22)

$$\kappa_{1k_1} = k_1 \cdot \Delta \kappa_1 , \quad k_1 = 1, 2, \ldots, N_1$$  \hspace{1cm} (23)

$$\kappa_{2k_2} = k_2 \cdot \Delta \kappa_2 , \quad k_2 = 1, 2, \ldots, N_2$$  \hspace{1cm} (24)

The following values are used for \(N_1, N_2, \kappa_{1u}, \kappa_{2u}\):

$$N_1 = N_2 = 64$$  \hspace{1cm} (25)

$$\kappa_{1u} = 8.84 \cdot 10^{-3} \frac{rad}{m} ; \quad \kappa_{2u} = 3.32 \cdot 10^{-3} \frac{rad}{m}$$  \hspace{1cm} (26)

It is noted again that \(\varphi^{(1)}_{k_1 k_2}\) and \(\varphi^{(2)}_{k_1 k_2}\) are two sequences of independent random phase angles uniformly distributed in the range \((0, 2\pi)\).

The stochastic wave is now simulated, using Eq. 21, over a 10,000 m by 10,000 m area at 12 equispaced time instants, 0.5 sec apart from each other. Two of these 12 simulations are shown in Fig. 1. By studying all 12 simulations, a relatively rapid variation along the \(x_1\)-axis is clearly observed, compared to the variation along the \(x_2\)-axis. Note that the \(x_1\)-axis represents the major axis of seismic wave propagation in Event 5. From the number of peaks (4) along the \(x_1\)-axis, the apparent wave length along this axis is estimated to be around 2,500 m. Thus, the patterns observed in the simulations indicate a dominant wave with a wave length of approximately 2,500 m propagating in the negative \(x_1\)-axis direction. This is considered to be an excellent realization of the actual ground motion.

CONCLUSIONS

A stochastic wave model has been developed together with a method of generating its samples. A numerical example is presented for the case of non-dispersive Rayleigh waves. More general cases involving spatially three-dimensional seismic waves as observed on the ground surface can be treated similarly. In these general cases, however, rather than making use of a
wave number domain analysis as considered here, a frequency-wave number domain analysis is necessary. Such a frequency-wave number analysis was performed by Abrahamson (Ref. 19) using data from the SMART-1 seismograph array installed at Lotung, Taiwan, considering the wave field to be stationary in certain time windows.

Although the stochastic wave model considered in this work is stationary and homogeneous, it is a straightforward task to extend the introduced methodology to non-stationary and/or non-homogeneous stochastic waves characterized by an evolutionary power spectrum.

ACKNOWLEDGMENTS

This work was supported by Contract No. NCEER 87-3008 under the auspices of the National Center for Earthquake Engineering Research under NSF Grant No. ECE-86-07591.

REFERENCES


![Graph](image)

**Fig. 1** Simulated Stochastic Wave at Two Time Instants.