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DIGITAL SIMULATION OF SEISMIC GROUND MOTION USING STOCHASTIC WAVE THEORY

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SUMMARY

In order to describe the nature of ground motion arising from a propagating seismic wave, a stochastic wave model has been developed here and an efficient technique for digitally generating samples of such a stochastic wave is introduced as an extension of the spectral representation method. Although the stochastic wave model considered in this work is stationary and homogeneous, it is a straightforward task to extend the introduced methodology to non-stationary and/or non-homogeneous stochastic waves characterized by an evolutionary power spectrum.

INTRODUCTION

A number of stochastic models for simulation of seismic ground motion have been proposed and successfully applied to a variety of structural problems arising from seismic events (Refs. 1-17). A common limitation of all these models is that ground motion is treated as a stochastic process when its time variability is examined, or as a stochastic field when its spatial variability is considered. In the former case, the space variables are frozen, while in the latter case, time is frozen. In order to have the analysis reflect the nature of ground motion arising from a propagating seismic wave, a stochastic wave model has been developed here and an efficient technique for digitally generating samples of such a stochastic wave is introduced as an extension of the spectral representation method primarily developed by Shinozuka and his associates. The proposed model is useful for the seismic response analysis of such large-scale structures extending over a wide spatial area as water transmission and gas distribution systems and long-span bridges.

THEORY OF STATIONARY, HOMOGENEOUS STOCHASTIC WAVES

Consider the following stationary, spatially two-dimensional, homogeneous stochastic wave:

$$f_0(x_1, x_2, t) = f_0(\underline{x}, t) \tag{1}$$

with mean value zero:

$$E[f_0(\underline{x}, t)] = 0 \tag{2}$$

where $\underline{x} = [x_1\ x_2]^T$ is the vector of the space variables, t is the time variable and $E[\cdot]$ denotes the expectation. The autocorrelation function of $f_0(\underline{x}, t)$ is defined as:

$$R_{f_0 f_0}(\underline{\xi}, \tau) = E[f_0(\underline{x}, t) f_0(\underline{x} + \underline{\xi}, t + \tau)] \tag{3}$$

Since the stochastic wave is considered to be stationary and homogeneous, $R_{f_0 f_0}(\underline{\xi}, \tau)$ is symmetric with respect to the separation distance vector $\underline{\xi} = [\xi_1\ \xi_2]^T$ and the time lag τ:

$$R_{f_0 f_0}(\underline{\xi}, \tau) = R_{f_0 f_0}(-\underline{\xi}, -\tau) \tag{4}$$

Assuming that the three-fold Fourier transform of $R_{f_0 f_0}(\underline{\xi}, \tau)$ exists, the power spectral density function of $f_0(\underline{x}, t)$ is defined as:

$$S_{f_0 f_0}(\underline{\kappa}, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{f_0 f_0}(\underline{\xi}, \tau)\ \mathrm{e}^{-i\underline{\kappa} \bullet \underline{\xi}}\ \mathrm{e}^{-i\omega t}\ d\xi_1 d\xi_2 d\tau \tag{5}$$

and its inverse transform is given by:

$$R_{f_0 f_0}(\underline{\xi}, \tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{f_0 f_0}(\underline{\kappa}, \omega)\ \mathrm{e}^{i\underline{\kappa} \bullet \underline{\xi}}\ \mathrm{e}^{i\omega t}\ d\kappa_1 d\kappa_2 d\omega \tag{6}$$

The preceding two equations represent the three-dimensional version of the Wiener-Khintchine transform pair, where $\underline{\kappa} = [\kappa_1\ \kappa_2]^T$ is the wave number vector, $\underline{\kappa} \bullet \underline{\xi}$ is the inner product of $\underline{\kappa}$ and $\underline{\xi}$, and ω is the frequency. Using Eq. 4, it can be shown that the power spectral density function is real and symmetric and also non-negative:

$$S_{f_0 f_0}(\underline{\kappa}, \omega) = S_{f_0 f_0}(-\underline{\kappa}, -\omega) \tag{7}$$

$$S_{f_0 f_0}(\underline{\kappa}, \omega) \geq 0 \tag{8}$$

SIMULATION OF STATIONARY, HOMOGENEOUS STOCHASTIC WAVES

Based on the above-mentioned properties of $S_{f_0 f_0}(\underline{\kappa}, \omega)$, the stationary, spatially two-dimensional, homogeneous stochastic wave $f_0(\underline{x}, t)$ can be simulated by a stochastic wave $f(\underline{x}, t)$ in the following fashion: consider that the power spectral density function $S_{f_0 f_0}(\underline{\kappa}, \omega)$ of $f_0(\underline{x}, t)$ is of insignificant magnitude outside the region defined by:

$$-\underline{\kappa}_u \leq \underline{\kappa} \leq \underline{\kappa}_u \tag{9}$$

$$-\omega_u \leq \omega \leq \omega_u \tag{10}$$

where $\underline{\kappa}_u = [\kappa_{1u}\ \kappa_{2u}]^T$ with $\kappa_{iu} > 0$ $(i = 1, 2)$ and $\omega_u > 0$. Denote the interval vector by:

$$[\Delta\kappa_1 \quad \Delta\kappa_2 \quad \Delta\omega]^T = \left[\frac{\kappa_{1u}}{N_1} \quad \frac{\kappa_{2u}}{N_2} \quad \frac{\omega_u}{N}\right]^T \tag{11}$$

and then construct the simulated wave $f(\underline{x},t)$ by the following series, as $N_1, N_2, N \to \infty$ simultaneously:

$$f(\underline{x},t) = \sqrt{2}\sum_{k_1=1}^{N_1}\sum_{k_2=1}^{N_2}\sum_{k=1}^{N}\sum_{I_2=\pm1}\sum_{I_\omega=\pm1}\left[2S_{f_0f_0}(\kappa_{1k_1},\ I_2\cdot\kappa_{2k_2},\ I_\omega\cdot\omega_k)\ \Delta\kappa_1\Delta\kappa_2\Delta\omega\right]^{1/2}\cdot$$

$$\cdot\cos(\kappa_{1k_1}\cdot x_1 + I_2\cdot\kappa_{2k_2}\cdot x_2 + I_\omega\cdot\omega_k\cdot t + \varphi_{k_1k_2k}^{I_2I_\omega}) \qquad (12)$$

where $\varphi_{k_1k_2k}^{I_2I_\omega}$ are independent random phase angles uniformly distributed between 0 and 2π, and

$$\kappa_{1k_1} = k_1\cdot\Delta\kappa_1 \quad , \quad k_1 = 1,2,\ldots,N_1 \qquad (13)$$

$$\kappa_{2k_2} = k_2\cdot\Delta\kappa_2 \quad , \quad k_2 = 1,2,\ldots,N_2 \qquad (14)$$

$$\omega_k = k\cdot\Delta\omega \quad , \quad k = 1,2,\ldots,N \qquad (15)$$

The simulated field $f(\underline{x},t)$ is asymptotically Gaussian as $N_1, N_2, N \to \infty$ simultaneously, due to the central limit theorem.

NUMERICAL EXAMPLE

For simplicity, consider a stationary, homogeneous and non-dispersive Rayleigh wave in which case $\omega/c = \kappa = \sqrt{\kappa_1^2+\kappa_2^2}$ with c = phase velocity of Rayleigh wave. Then, $S_{f_0f_0}(\kappa_1,\kappa_2,\omega)$ degenerates into $S_{f_0f_0}(\kappa_1,\kappa_2)$. The following form of the power spectral density function $S_{f_0f_0}(\kappa_1,\kappa_2)$ is used in this numerical example:

$$S_{f_0f_0}(\kappa_1,\kappa_2) = \frac{\sigma_{yy}^2}{8\pi}\cdot b_1^3\cdot b_2\cdot\kappa_1^2\cdot\exp\left[-\left(\frac{b_1\kappa_1}{2}\right)^2-\left(\frac{b_2\kappa_2}{2}\right)^2\right] \qquad (16)$$

The corresponding autocorrelation function is given by:

$$R_{f_0f_0}(\xi_1,\xi_2) = \sigma_{yy}^2\cdot\left[1-2\left(\frac{\xi_1}{b_1}\right)^2\right]\cdot\exp\left[-\left(\frac{\xi_1}{b_1}\right)^2-\left(\frac{\xi_2}{b_2}\right)^2\right] \qquad (17)$$

The power spectrum shown in Eq. 16 is proposed by Harada and Shinozuka (Ref. 18) and is based on a wave number analysis of data from the original accelerograms recorded on January 29, 1981 (Event 5) by a SMART-1 seismograph array installed at Lotung, Taiwan. The data used represent the horizontal component of the displacement time history in the direction N13°W which is approximately the direction of the seismic source of this earthquake relative to the location of the array. The following values are used for σ_{yy}, b_1 and b_2 appearing in Eqs. 16 and 17:

$$\sigma_{yy} = 0.0124\ m \quad ; \quad b_1 = 1131\ m \quad ; \quad b_2 = 3012\ m \qquad (18)$$

The apparent lack of a frequency-wave number analysis in the estimation of the power spectrum is taken care of by the following relationship mentioned above:

$$\omega = g(\kappa_1,\kappa_2) = c\cdot\sqrt{\kappa_1^2+\kappa_2^2} \qquad (19)$$

in which the value of the phase velocity is set equal to:

$$c = 2,800 \ \frac{m}{sec} \tag{20}$$

These assumptions were made strictly for demonstration of the proposed digital simulation procedure and do not imply that the data used indeed represent a Rayleigh wave.

The simulation of a stochastic wave having the properties described above is then performed using the following expression:

$$f(x_1, x_2, t) = \sqrt{2} \sum_{k_1=1}^{N_1} \sum_{k_2=1}^{N_2} \left[2 S_{f_0 f_0}(\kappa_{1k_1},\ \kappa_{2k_2})\ \Delta\kappa_1 \Delta\kappa_2\right]^{1/2} \cdot$$

$$\cdot \left\{\cos[\kappa_{1k_1} \cdot x_1 + \kappa_{2k_2} \cdot x_2 + g(\kappa_{1k_1}, \kappa_{2k_2}) \cdot t + \varphi^{(1)}_{k_1 k_2}] + \right.$$

$$\left. + \cos[\kappa_{1k_1} \cdot x_1 - \kappa_{2k_2} \cdot x_2 + g(\kappa_{1k_1}, \kappa_{2k_2}) \cdot t + \varphi^{(2)}_{k_1 k_2}]\right\} \tag{21}$$

where:

$$\Delta\kappa_1 = \frac{\kappa_{1u}}{N_1} \quad ; \quad \Delta\kappa_2 = \frac{\kappa_{2u}}{N_2} \tag{22}$$

$$\kappa_{1k_1} = k_1 \cdot \Delta\kappa_1 \quad , \quad k_1 = 1, 2, \ldots, N_1 \tag{23}$$

$$\kappa_{2k_2} = k_2 \cdot \Delta\kappa_2 \quad , \quad k_2 = 1, 2, \ldots, N_2 \tag{24}$$

The following values are used for $N_1, N_2, \kappa_{1u}, \kappa_{2u}$:

$$N_1 = N_2 = 64 \tag{25}$$

$$\kappa_{1u} = 8.84 \cdot 10^{-3} \ \frac{rad}{m} \quad ; \quad \kappa_{2u} = 3.32 \cdot 10^{-3} \ \frac{rad}{m} \tag{26}$$

It is noted again that $\varphi^{(1)}_{k_1 k_2}$ and $\varphi^{(2)}_{k_1 k_2}$ are two sequences of independent random phase angles uniformly distributed in the range $(0, 2\pi)$.

The stochastic wave is now simulated, using Eq. 21, over a 10,000 m by 10,000 m area at 12 equispaced time instants, 0.5 *sec* apart from each other. Two of these 12 simulations are shown in Fig. 1. By studying all 12 simulations, a relatively rapid variation along the x_1-axis is clearly observed, compared to the variation along the x_2-axis. Note that the x_1-axis represents the major axis of seismic wave propagation in Event 5. From the number of peaks (4) along the x_1-axis, the apparent wave length along this axis is estimated to be around 2,500 m. Thus, the patterns observed in the simulations indicate a dominant wave with a wave length of approximately 2,500 m propagating in the negative x_1-axis direction. This is considered to be an excellent realization of the actual ground motion.

CONCLUSIONS

A stochastic wave model has been developed together with a method of generating its samples. A numerical example is presented for the case of non-dispersive Rayleigh waves. More general cases involving spatially three-dimensional seismic waves as observed on the ground surface can be treated similarly. In these general cases, however, rather than making use of a wave number domain analysis as considered here, a frequency-wave number domain analysis is necessary. Such a frequency-wave number analysis was performed by Abrahamson (Ref. 19) using data from the SMART-1 seismograph array installed at Lotung, Taiwan, considering the wave field to be stationary in certain time windows.

Although the stochastic wave model considered in this work is stationary and homogeneous, it is a straightforward task to extend the introduced methodology to non-stationary and/or non-homogeneous stochastic waves characterized by an evolutionary power spectrum.

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REFERENCES

1. Tajimi, H., "Basic Theories on Aseismic Design of Structures," *Institute of Industrial Science*, University of Tokyo, Vol. 8, No. 4, (1959).
2. Bolotin, V. V., "Statistical Theory of the Aseismic Design of Structures," *Proceedings of the 2nd World Conference on Earthquake Engineering*, Tokyo and Kyoto, Japan, Vol. II, pp. 1365-1374, (1960).
3. Cornell, C. A., "Stochastic Process Models in Structural Engineering," Technical Report No. 34, Department of Civil Engineering, Stanford University, (1960).
4. Housner, G. W. and Jennings, P. C., "Generation of Artificial Earthquakes," *Journal of Engineering Mechanics*, ASCE, Vol. 90, No. EM1, pp. 113-150, (1964).
5. Lin, Y. K., "Nonstationary Response of Continuous Structures to Random Loading," *Journal of the Acoustical Society of America*, Vol. 35, pp. 222-227, (1963).
6. Lin, Y. K., "Nonstationary Excitation and Response in Linear Systems Treated as Sequences of Random Pulses," *Journal of the Acoustical Society of America*, Vol. 38, pp. 453-460, (1965).
7. Shinozuka, M. and Sato, Y., "Simulation of Nonstationary Random Processes," *Journal of Engineering Mechanics*, ASCE, Vol. 93, No. EM1, pp. 11-40, (1967).
8. Amin, M. and Ang, A. H-S., "Nonstationary Stochastic Models of Earthquake," *Journal of Engineering Mechanics*, ASCE, Vol. 94, No. EM2, pp. 559-583, (1968).
9. Iyengar, R. N. and Iyengar, K. T. S., "A Nonstationary Random Process Model for Earthquake Accelerograms," *Bulletin of the Seismological Society of America*, Vol. 59, No. 3, pp. 1163-1188, (1969).
10. Ruiz, P. and Penzien, J., "Stochastic Seismic Response of Structures," *Journal of Engineering Mechanics*, ASCE, Vol. 97, No. EM2, pp. 441-456, (1971).
11. Shinozuka, M., "Monte Carlo Solution of Structural Dynamics," *International Journal of Computers and Structures*, Vol. 2, pp. 855-874, (1972).
12. Shinozuka, M. and Jan, C-M., "Digital Simulation of Random Processes and its Applications," *Journal of Sound and Vibration*, Vol. 25, No. 1, pp. 111-128, (1972).
13. Iyengar, R. N. and Shinozuka, M., "Effect of Self-Weight and Vertical Acceleration on the Behavior of Tall Structures During Earthquake," *Journal of Earthquake Engineering and Structural Dynamics*, Vol. 1, pp. 69-78, (1972).

14. Shinozuka, M., "Digital Simulation of Ground Acceleration," *Proceedings of the 5th World Conference on Earthquake Engineering*, Rome, Italy, pp. 2829-2838, (1973).
15. Shinozuka, M., "Digital Simulation of Random Processes in Engineering Mechanics with the Aid of FFT Technique," *Stochastic Problems in Mechanics*, (Eds. S. T. Ariaratnam and H. H. E. Leipholz), University of Waterloo Press, Waterloo, pp. 277-286, (1974).
16. Shinozuka, M., "Stochastic Fields and their Digital Simulation," *Stochastic Methods in Structural Dynamics*, (Eds. G. I. Schueller and M. Shinozuka), Martinus Nijhoff Publishers, Dordrecht, pp. 93-133, (1987).
17. Kozin, F., "Autoregressive Moving Average Models of Earthquake Records," *Journal of Probabilistic Engineering Mechanics*, Vol. 3, No. 2, (1988).
18. Harada, T. and Shinozuka, M., "Harmonic Analysis and Simulation of Homogeneous Stochastic Fields," Technical Report, Department of Civil Engineering and Engineering Mechanics, Columbia University, New York, (1986).
19. Abrahamson, N. A., "Estimation of Seismic Wave Coherency and Rupture Velocity Using the SMART-1 Strong-Motion Array Recordings," Report No. UCB/EERC-85/02, Earthquake Engineering Research Center, University of California, Berkeley, (1985).

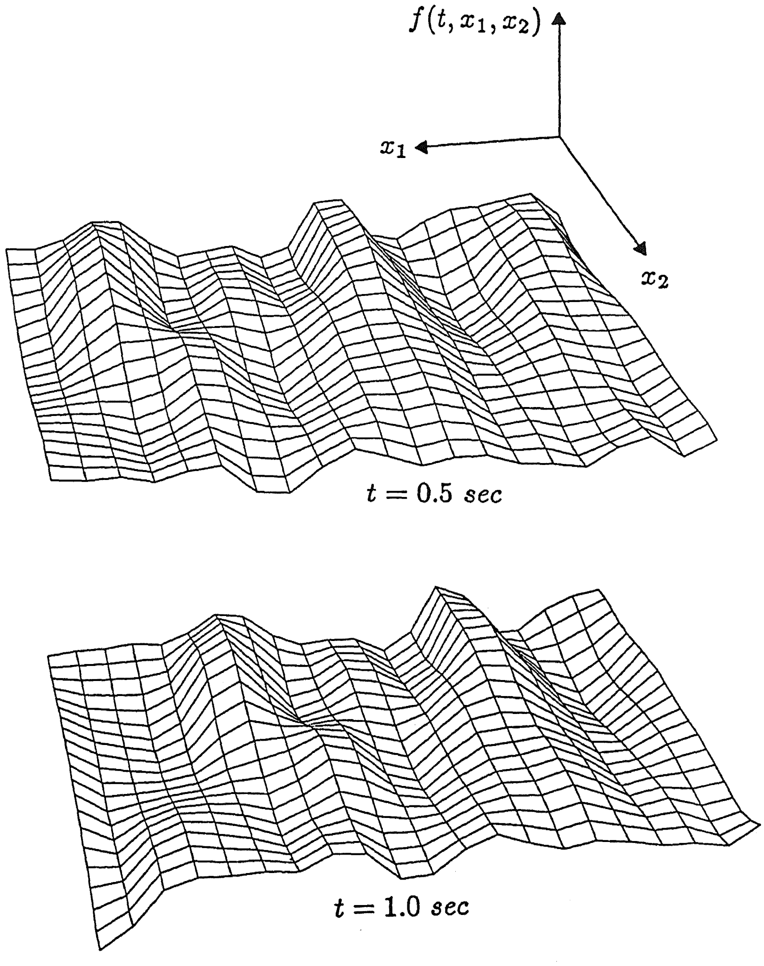

Fig. 1 Simulated Stochastic Wave at Two Time Instants.