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MODELING OF NONSTATIONARY CROSS SPECTRUM FOR MULTIVARIATE EARTHQUAKE MOTIONS BY MULTIFILTER TECHNIQUE

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SUMMARY

An effective procedure for the nonstationary cross spectrum of multivariate earthquake motions is formulated with the aid of multifilter technique. Nonstationary coherency is calculated using nonstationary cross and power spectra of the array data from SMART-1 in Taiwan. Subsequently, a coherency model is developed on the basis of an equivalent time-invariant coherency. In order to examine the validity of present model, multivariate earthquake motions are simulated by means of models of coherency and nonstationary power spectra. It has been shown by comparison with recorded data that present procedure and models are useful for an engineering purpose.

INTRODUCTION

Consideration of the spatial and the temporal variation of earthquake motions is important in the aseismic design of structures with large foundations such as dams and nuclear power plants and extended structures such as long span bridges and lifeline systems, because the earthquake motions are multivariate and multidimensional. Moreover, nonstationary characteristics of earthquake motions should be considered in the response analysis of previously mentioned important structures, especially in their inelastic responses. In such cases, an effective technique is needed for evaluating the nonstationary cross spectrum of multivariate and multidimensional earthquake motions. In previous studies for the analysis of the cross correlation of multivariate earthquake motions [Ref.1-3], the stationary random process theory has been used, without considering the nonstationarity of spectrum of earthquake motions.

The objective of this study is to formulate nonstationary cross spectrum with the aid of multifilter technique and to develop an engineering model for evaluating nonstationary cross correlation of multivariate earthquake motions.

NONSTATIONARY CROSS SPECTRUM BY MULTIFILTER TECHNIQUE

The concept of nonstationary power spectrum by multifilter technique [Ref.4] is extended to formulate a procedure for analyzing nonstationary cross spectrum of multivariate earthquake motions. Single degree-of-freedom linear oscillators are used as multifilter components. The equations of motion when subjected to random accelerations $X_j(t)$ and $X_k(t)$, respectively, are given as

$$\ddot{Y}_j(t)+2\beta\omega_o\dot{Y}_j(t)+\omega_o^2Y_j(t)=-X_j(t)$$

$$\ddot{Y}_k(t)+2\beta\omega_o\dot{Y}_k(t)+\omega_o^2Y_k(t)=-X_k(t)$$

in which β=damping factor, ω_o=natural circular frequency and $Y_j(t)$,$Y_k(t)$=random relative displacement responses of the oscillator subjected to $X_j(t)$ and $X_k(t)$, respectively. Using a stationary random vibration theory [Ref.5], cross spectrum of $Y_j(t)$ and $Y_k(t)$ is related to that of $X_j(t)$ and $X_k(t)$ as follows

$$S_{Y_jY_k}(\omega)=|H(\omega)|^2S_{jk}(\omega) \tag{2}$$

in which $S_{Y_jY_k}(\omega)$=two sided cross spectrum of $Y_j(t)$ and $Y_k(t)$, $S_{jk}(\omega)$=two sided cross spectrum of $X_j(t)$ and $X_k(t)$, and $H(\omega)$= frequency response function of a single degree-of-freedom linear oscillator.

$$H(\omega)=(\omega_o^2-\omega^2+i2\beta\omega_o\omega)^{-1} \tag{3}$$

Inverse Fourier transformation of Eq.2 leads to the cross correlation function of $Y_j(t)$ and $Y_k(t)$ represented by

$$R_{Y_jY_k}(\tau)=\int_{-\infty}^{\infty}|H(\omega)|^2S_{jk}(\omega)e^{i\omega\tau}d\omega \tag{4}$$

Above discussion is based on stationary random vibration theory. If nonstationary cross spectrum of $X_j(t)$ and $X_k(t)$, $S_{jk}(t,\omega)$, is slowly varying with time t, the nonstationary cross correlation function is represented by

$$R_{Y_jY_k}(t,\tau)=\int_{-\infty}^{\infty}|H(\omega)|^2S_{jk}(t,\omega)e^{i\omega\tau}d\omega \tag{5}$$

The complex-valued cross spectrum of Eq.5 can also be represented by

$$S_{jk}(t,\omega)=C_{jk}(t,\omega)+iQ_{jk}(t,\omega)=|S_{jk}(t,\omega)|e^{i\theta_{jk}(t,\omega)} \tag{6}$$

in which $C_{jk}(t,\omega)$=co-spectrum, $Q_{jk}(t,\omega)$=quad-spectrum and $\theta_{jk}(t,\omega)$=phase difference between the ω components of $X_j(t)$ and $X_k(t)$.

$$\theta_{jk}(t,\omega)=tan^{-1}[Q_{jk}(t,\omega)/C_{jk}(t,\omega)] \tag{7}$$

Substituting τ=0 into Eq.5 and considering that $R_{Y_jY_k}(t,0)=E[Y_j(t)Y_k(t)]$

$$E[Y_j(t)Y_k(t)]=\int_{-\infty}^{\infty}|H(\omega)|^2S_{jk}(t,\omega)d\omega \tag{8}$$

Noting that $C_{jk}(t,\omega)$ is an even function, $Q_{jk}(t,\omega)$ an odd function of ω and that $H(\omega)$ is of narrow band around the filter frequency ω_o, we have

$$E[Y_j(t)Y_k(t)]=+\pi C_{jk}(t,\omega_o)/2\beta\omega_o^3 \tag{9}$$

Subsequently, differentiating Eq.5 respecting to τ and using the same procedure as Eqs.8 and 9, following equations are deduced

$$E[\dot{Y}_j(t)Y_k(t)]=+\pi Q_{jk}(t,\omega_o)/2\beta\omega_o^2 \tag{10}$$

$$E[Y_j(t)\dot{Y}_k(t)]=-\pi Q_{jk}(t,\omega_o)/2\beta\omega_o^2 \tag{11}$$

$$E[\dot{Y}_j(t)\dot{Y}_k(t)]=+\pi C_{jk}(t,\omega_o)/2\beta\omega_o \tag{12}$$

From Eqs.9 to 12, $C_{jk}(t,\omega_o)$ and $Q_{jk}(t,\omega_o)$ are given as follows

$$C_{jk}(t,\omega_o)=\beta\omega_o^3\ E[Y_j(t)Y_k(t)+\dot{Y}_j(t)\dot{Y}_k(t)/\omega_o^2]/\pi \tag{13}$$

$$Q_{jk}(t,\omega_o)=\beta\omega_o^3\ E[\dot{Y}_j(t)Y_k(t)/\omega_o-Y_j(t)\dot{Y}_k(t)/\omega_o]/\pi \tag{14}$$

Note that,in the case of $j=k$, $Q_{jk}(t,\omega_o)$ is zero and $S_{jk}(t,\omega_o)$ equals to $C_{jk}(t,\omega_o)$, which is the nonstationary power spectrum [Ref.4]. When one sided nonstationary cross spectrum $G_{jk}(t,\omega_o)$ is considered, it can be obtained from two times of two sided cross spectrum as follows

$$G_{jk}(t,\omega_o)=C_{jk}(t,\omega_o)+iQ_{jk}(t,\omega_o)=|G_{jk}(t,\omega_o)|e^{i\theta_{jk}(t,\omega_o)} \tag{15}$$

$$C_{jk}(t,\omega_o)=2\beta\omega_o^3\ E[Y_j(t)Y_k(t)+\dot{Y}_j(t)\dot{Y}_k(t)/\omega_o^2]/\pi \tag{16}$$

$$Q_{jk}(t,\omega_o)=2\beta\omega_o^3\ E[\dot{Y}_j(t)Y_k(t)/\omega_o-Y_j(t)\dot{Y}_k(t)/\omega_o]/\pi \tag{17}$$

NONSTATIONARY COHERENCY AND ITS MODELING

<u>Nonstationary Coherency</u> Normarized cross correlation in the frequency domain can be evaluated by coherency. The nonstationary coherency $\gamma_{jk}^2(t,\omega)$ of $X_j(t)$ and $X_k(t)$ is defined by following equation

$$\gamma_{jk}^2(t,\omega)=|G_{jk}(t,\omega)|^2/G_j(t,\omega)G_k(t,\omega) \tag{18}$$

in which $G_j(t,\omega)$ and $G_k(t,\omega)$ are the nonstationary power spectra of $X_j(t)$ and $X_k(t)$, respectively. In calculating the coherency, time lag between $X_j(t)$ and $X_k(t)$ must be modified in advance [Ref.1], because, if not, the nonstationary coherency may be underestimated.

In numerical analysis, accelerograms of event 5 recorded by SMART-1 array in Taiwan(Fig.1) have been used. Fig.2 shows the nonstationary coherencies for NS components of separation distance D=200m, 1000m and 2000m, that is, for I06NS-COONS, M06NS-COONS, O06NS-COONS. The frequencies in the figure have been determined from peak frequencies of stationary cross spectra. It is found from Fig.2 that the coherencies decay with increasing separation and frequency, and that coherencies are large in the strong part of earthquake motions.

<u>Modeling of Coherency</u> The following weighted average coherency is proposed to be used as an equivalent time-invariant coherency model to replace the non-stationary coherency.

$$\bar{\gamma}_{jk}^2(\omega)=[\int_0^T \gamma_{jk}^2(t,\omega)\,|G_{jk}(t,\omega)|dt]/[\int_0^T |G_{jk}(t,\omega)|dt] \tag{19}$$

The nonstationary cross spectrum amplitude has been used as a weighting function in averaging over the time. The equivalent time-invariant coherencies are shown by dashed lines in Fig.2. Fig.3 shows the equivalent time-invariant coherencies of D=200m, 1000m and 2000m for NS components in epicentral(006-012 line) and transverse(003-009 line) directions. It can be found from Fig.3 that equivalent time-invariant coherencies in the epicentral direction are smaller than those in transverse direction. The equivalent time-invariant coherency can be represented by following model function.

$$\hat{\gamma}_{jk}^2(2\pi f)=[aD^{\alpha}f^2+1]^{-2}exp(-\beta X) \tag{20}$$

in which a,α,β=constant, f=frequency, D=separation distance, and X=projection of D to epicentral direction. Eq.20 has an advantage that it can be used in the response analysis by random vibration theory, since it is rational function of frequency. Parameters a,α,β have been determined using the array data recorded in epicentral and transverse direction (on the lines of 006-012 and 003-009). The following parameters have been determined in Eq.20.

a=0.00234; α=0.236; β=0.0000816

Fig.4 shows the model functions of equivalent time-invariant coherencies for D=200m, 1000m and 2000m in the epicentral and transverse direction. The model functions match the observed ones(Fig.3) very well.

MODELING AND SIMULATION OF MULTIVARIATE EARTHQUAKE MOTIONS

<u>Model of Nonstationary Power Spectrum</u> The following time-varying function is adopted for the model of nonstationary power spectrum $\hat{G}_j(t,\omega)$ or $\hat{G}_k(t,\omega)$[Ref.8]

$$\sqrt{\hat{G}_j(t,2\pi f)}=\begin{cases}0\ ;\ t\leqq t_s\\ \alpha_m(f)[\{t-t_s(f)\}/t_p(f)]\exp[1-\{t-t_s(f)\}/t_p(f)]\ ;\ t_s<t\end{cases} \tag{21}$$

in which $t_s(f)$, $t_p(f)$ = starting time and duration parameter, respectively, and $\alpha_m(f)$=intensity parameter which represents the peak value of $\hat{G}_j(t,2\pi f)$.

<u>Model of Nonstationary Cross Spectrum</u> Using $\hat{\gamma}_{jk}^2(\omega)$ and $\hat{G}_j(t,\omega)$ in Eqs.20 and 21 instead of $\gamma_{jk}^2(t,\omega)$, $G_j(t,\omega)$ and $G_k(t,\omega)$ in Eq.18, and then substituting them into Eq.15, model of nonstationary cross spectrum can be obtained

$$\hat{G}_{jk}(t,\omega)=\hat{\gamma}_{jk}(\omega)\sqrt{\hat{G}_j(t,\omega)\hat{G}_k(t,\omega)}\ \exp\{i\hat{\theta}_{jk}(\omega)\} \tag{22}$$

in which $\hat{\theta}_{jk}(\omega)$ is the phase difference of ω components of $X_j(t)$ and $X_k(t)$ that is determined from array data.

Simulation of Multivariate Earthquake Motions Multivariate earthquake motions are simulated using the model functions in Eqs.20 and 21, in a pairwise manner. Spatially correlated earthquake motions can be simulated by following equations, which are the extension of simulation of stationary random processes [Ref.9].

$$X_j(t)=\sum_{l=1}^{N}\sqrt{2\hat{G}_j(t,\omega_l)\Delta\omega}\cos(\omega_l t+\varphi_{jl}) \tag{23}$$

$$X_k(t)=\sum_{l=1}^{N}\sqrt{2\hat{G}_k(t,\omega_l)\Delta\omega}[\hat{\gamma}_{jk}(\omega_l)\cos(\omega_l t+\hat{\theta}_{jk}(\omega_l)+\varphi_{jl})+\sqrt{1-\hat{\gamma}_{jk}^2(\omega_l)}\cos(\omega_l t+\varphi_{kl})] \tag{24}$$

In order to illustrate the validity of the proposed method and modeling, NS components of the array data at sites in the epicentral direction (006-012) are simulated in a pairwise manner, using Eqs.23 and 24. Fig.5 shows the simulated and recorded accelerograms. It is found from the figure that the simulated accelerograms show the same intensity and nonstationarity as recorded ones. The equivalent time-invariant coherencies of I06NS-C00NS, M06NS-C00NS and O06NS-C00NS are shown in Fig.6, where the equivalent time-invariant coherencies of simulated and recorded accelerograms are compared with model function. Fig.6 shows that the simulated accelerograms reproduce the spatial correlation of multivariate earthquake motions.

CONCLUSIONS

Formulation and modeling of nonstationary cross spectrum have been investigated with the aid of multifilter technique. The major results obtained in this study can be summarized as follows.

(1)The multifilter technique has proved to be useful for the analysis of nonstationary cross spectrum of multivariate earthquake motions.

(2)It has been shown that nonstationary coherencies decay with increasing separation distance and frequency, and that the coherencies are large in the strong part of earthquake motions.

(3)The coherency has been modeled by equivalent time-invariant coherency, which is the average coherency weighted by nonstationary cross spectrum amplitude. The equivalent time-invariant coherency has been represented by the rational function of frequency, which can be used theoretical response analysis by random vibration theory.

(4)Multivariate earthquake motions have been simulated using present model functions obtained from array data of SMART-1 in Taiwan. It has been shown that simulated accelerograms reproduce the characteristics of nonstationary cross correlation of recorded ones.

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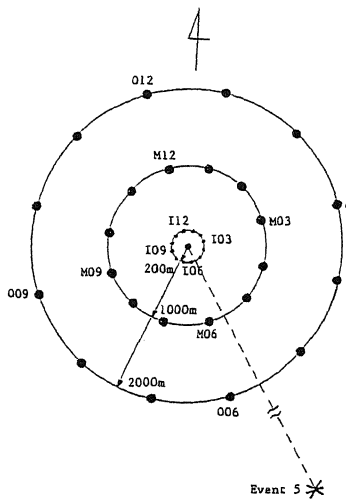


Fig.1 The SMART-1 Array
(From Bolt,et al.)

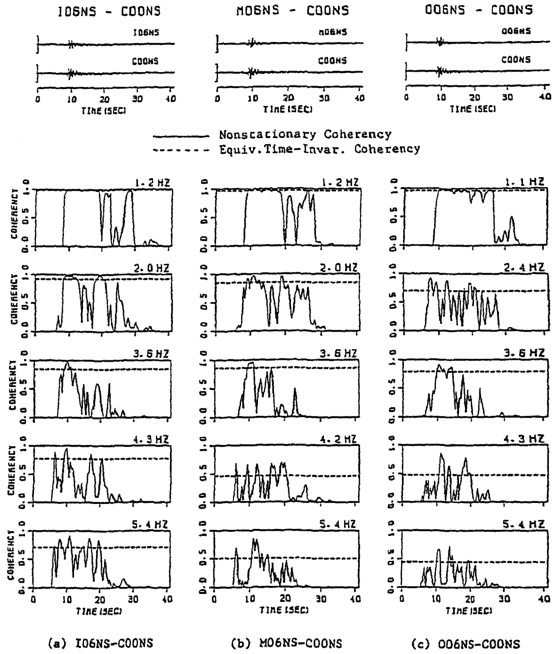


Fig.2 Nonstationary Coherencies and Equivalent Time-Invariant
Coherencies of NS Components (Epicentral Direction)

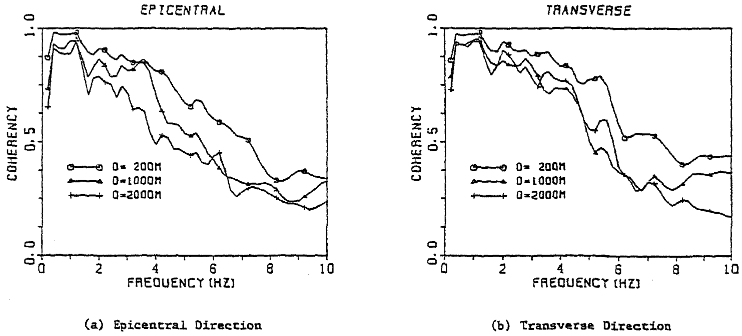
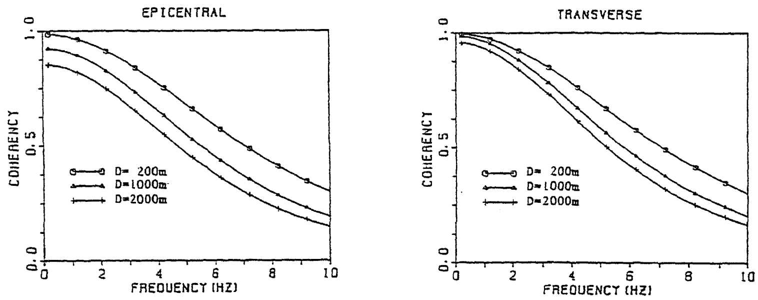


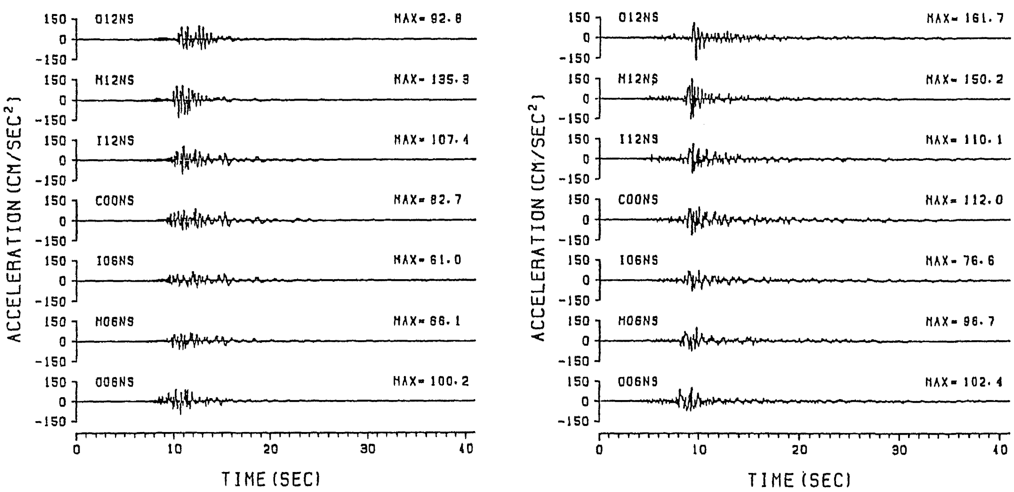
Fig.3 Equivalent Time-Invariant Coherency



(a) Epicentral Direction

(b) Transverse Direction

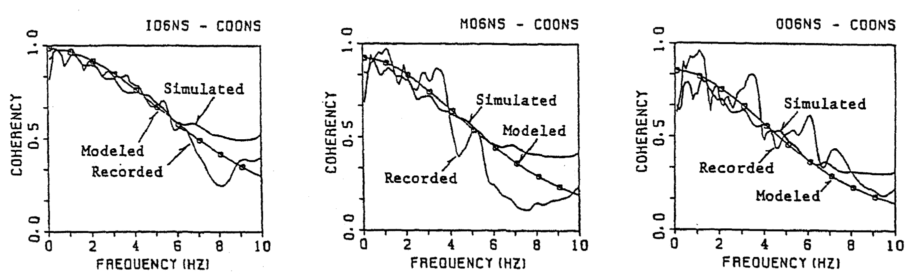
Fig.4 Model Function of Equivalent Time-Invariant Coherency



(a) Simulated

(b) Recorded

Fig.5 Time Histories of Simulated and Recorded Accelerograms
(SMART-1 Array Data ; NS Components)



(a) I06NS-COONS

(b) M06NS-COONS

(c) O06NS-COONS

Fig.6 Comparison of Equivalent Time-Invariant Coherencies of Simulated and Recorded Accelerograms with Model Functions