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STUDIES ON SEPARATION AND SYNTHESIS OF NONSTATIONARY SEISMIC WAVES USING FFT TECHNIQUE BASED ON HYPERFUNCTION THEORY

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SUMMARY

The objectives of this paper are to present an FFT technique symmetrical in time and frequency domains, which can evaluate Hilbert transforms, complex envelopes, and generating functions in hyperfunction theory, and to show ideas to separate seismic waves into several meaningful components and to synthesize nonstationary seismic waves with definite characters, by means of applying the FFT technique to the homomorphic deconvolution problems.

INTRODUCTION

For earthquake engineering, it will be quite important to grasp the effectiveness of FFT(fast Fourier transform) technique considering its applicability. In general, many researchers may consider that FFT can only give the Fourier series expansions of periodic time functions, and the idea prevents FFT from being applied to the theoretical problems such as calculating Hilbert transforms and evaluating the Fourier integrals of Delta function $\delta(t)$ and step function $U(t)$. However, provided that functions to be considered are extended to distributions, the applicability of FFT will be improved, and the Hilbert transforms and complex envelopes of such functions will be able to be easily evaluated by FFT(see Ref.1, for example).

In Ref.1, by the idea of complex envelope, the phase properties of seismic waves are discussed not only in frequency domain but in time domain. That is, the symmetrical treatment of FFT is performed in the paper. Of course, such symmetrical properties of FFT analyses are not familiar to the field of earthquake engineering. However, the symmetry is not perfect, since, in frequency domain, there are not any real causal functions comparative to seismic waves in time domain.

In this paper, we will make sure the symmetrical properties of FFT analyses of seismic waves by using the hyperfunction theory which examines the nature of hyperfunctions through their generating functions. Furthermore, we develope the symmetrical properties of FFT to homomorphic deconvolution problems of seismic waves such as calculating their minimum-phase-shift(MPS in short) and all-pass(AP in short). Namely, we introduce the MPS and AP in time domain. Based on these developed theories, we will show the method to separate and synthesize nonstationary seismic waves whose amplitude and phase characteristics in both domains are controlled.

In Fig.1, for a real causal function x(t), the flowchart to determine hyperfunctions x(t) and X(ω) and their generating functions f(t) and F(ω) is shown. From the figure, we can make sure the symmetrical nature of the FFT technique in this diagram by considering the relations between the hyperfunctions and generating functions in both domains and the existence of the real causal frequency function X(ω) comparable to x(t). It is important to note that these functions are determined by means of simple operations such as FFT, IFFT, and U( )Re[ ], and that it is not necessary to calculate the Hilbert transforms directly.

In Fig.2, theoretical examples are shown for the delta function δ(t) and the step function U(t). The generating function of δ(t) is the inverse Fourier transform of 4U(ω) and the generating function of U(t) is 1. In the FFT analyses, though the values of singularity functions such as δ(t) and 2δ(t)-2/πit are meaningless in time domain, we can determine the values by IFFT from their Fourier transforms determined in advance.

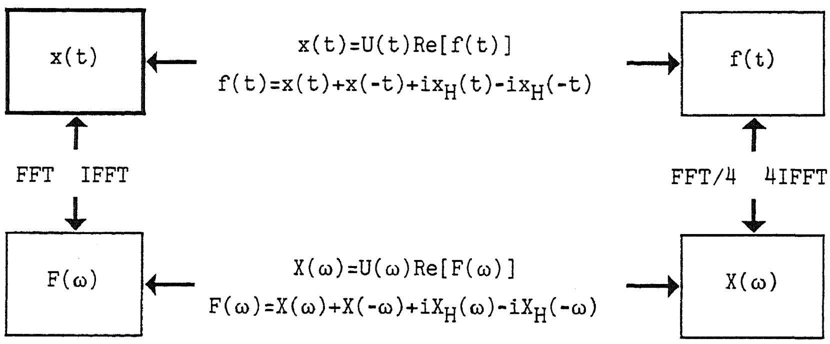

Fig.1 Method to determine the generating functions, f(t) and F(ω), and the hyperfunctions, x(t) and X(ω), from a real causal function x(t). Where, U(t) and U(ω) are step functions, Re[F(ω)] means the real part of F(ω), the suffix H indicates Hilbert transform. IFFT means inverse Fourier transform by FFT, 4IFFT and FFT/4 mean IFFT of 4X(ω) and FFT of f(t)/4, respectively. The generating functions f(t) and F(ω)are complex valued and the hyperfunctions x(t) and X(ω) are real causal.
Since the relation $-T_N \leqq t \leqq T_N$ for f(t) is required, the duration of FFT must be greater than $2T_N$, where T_N is the duration of x(t).

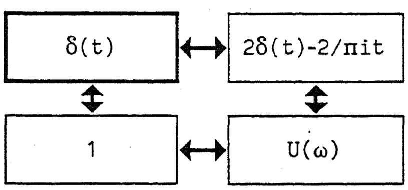

Fig.2(1) hyperfunctions and generating functions obtained from delta function δ(t)

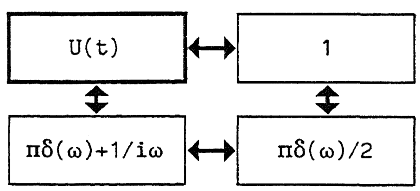

Fig.2(2) hyperfunctions and generating functions obtained from step function U(t)

SEPARATION OF SEISMIC WAVE BY MPS AND AP

In Fig.3, the flowchart to calculate MPS and AP is shown. The problem to determine MPS and AP is one concerning the homomorphic deconvolution(see Refs.2,3) based on information about the amplitude of complex valued functions. Accordingly, the generating functions, $F(\omega)$ and $f(t)$, play an important role. It is noteworthy that MPS and AP can be determined in time and frequency domains.

By these deconvolutions, several complex valued functions can be obtained from one seismic wave $x(t)$ as shown in Fig.4. These functions will give us useful information on the analyses of seismic waves.

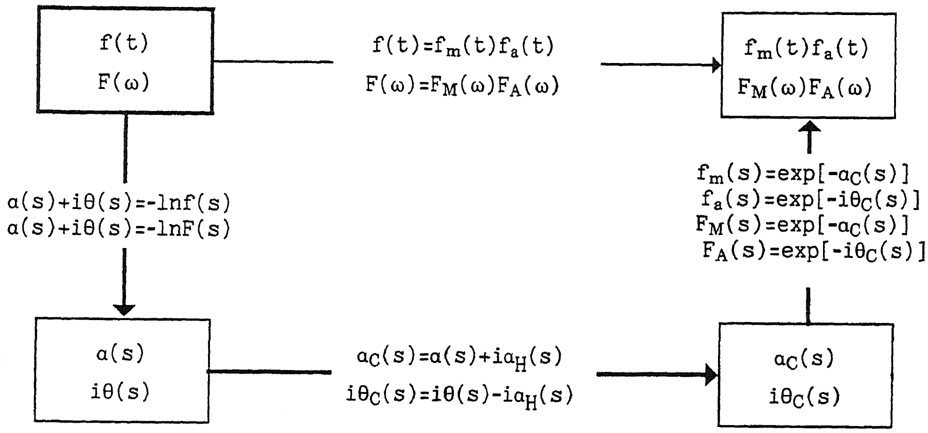

Fig.3 Method to determine minimum-phase-shift(MPS)s, $f_m(t)$ and $F_M(\omega)$, and all-pass(AP)s, $f_a(t)$ and $F_A(\omega)$, from generating functions, $f(t)$ and $F(\omega)$. The function $\alpha_C(s)$ is the complex envelope of $\alpha(s)$ which can be calculated by FFT. Obviously, for MPSs, $|f_m(t)|=|f(t)|$ and $|F_M(\omega)|=|F(\omega)|$ hold. Furthermore, for APs, $|F_A(\omega)|=1$ and $|f_a(t)|=1$ hold.

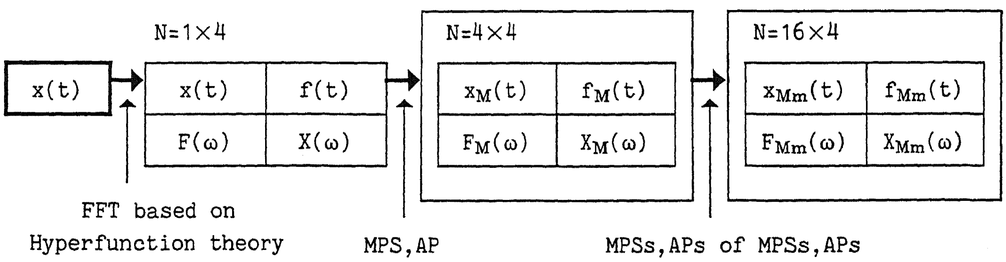

Fig.4 Functions obtained from a seismic wave $x(t)$ by the FFT technique in this paper. N means the total number of generating functions and hyperfunctions. The relations such as $F_{MM}(\omega)=F_M(\omega)$, $F_{MA}(\omega)=1$, $F_{AM}(\omega)=1$, $F_{AA}(\omega)=F_A(\omega)$, etc., can easily be derived from among these functions.

In Figs.6,7,8, we show examples of these functions for a real seismic wave. The generating functions and hyperfunctions(GH functions in short) of the seismic wave are shown in Fig.6. The GH functions obtained from the MPS of the generating functions calculated in Fig.6 are shown in Figs.7,8.

SYNTHESIS OF SEISMIC WAVES

It is possible, based on the separated functions shown in Fig.4, to synthesize several seismic waves whose amplitude and phase characteristics are controlled. In Fig.5, the idea of the syntheses is illustrated.

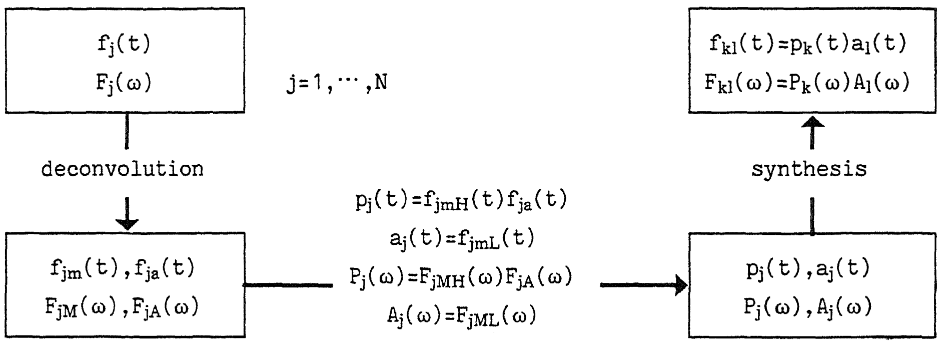

Fig.5 Method to synthesize artificial seismic waves. The homomorphic deconvolutins such as $f_{jm}(t)=f_{jmL}(t)f_{jmH}(t)$ and $F_{jM}(\omega)=F_{jML}(\omega)F_{jMH}(\omega)$ are introduced. In which, $f_{jmL}(t)$ and $F_{jML}(\omega)$ are determined from the long-period properties of $X_jm(\omega)$ and $x_{jM}(t)$, respectively.

In Fig.5, first, GH functions of N seismic waves are calculated, and then, the "amplitudes", $a_j(t)$ and $A_j(\omega)$, and the "phases", $p_j(t)$ and $P_j(\omega)$, are determined by the idea of homomorphic deconvolution. If $F_{lM}(\omega)=F_{lML}(\omega)$, then synthesized $F_{kl}(\omega)$ has the same Fourier amplitude with $F_l(\omega)$. And if $F_{lM}(\omega)\neq F_{lML}(\omega)$, then synthesized $F_{kl}(\omega)$ has the similar Fourier amplitude character with $F_l(\omega)$. The same is true for $f_{kl}(t)$ by changing Fourier amplitude for time envelope. In Fig.9, examples of synthesized seismic waves based on the method presented in Fig.5 are shown.

CONCLUSIONS

The FFT technique based on the hyperfunction theory is very clear and will provides us with a lot of potential in the analyses of seismic waves. In particular, the symmetrical nature in time and frequency domains will be effectively applicable to the problems to extract the physical meanings of seismic waves and to synthesize nonstationary seismic waves.

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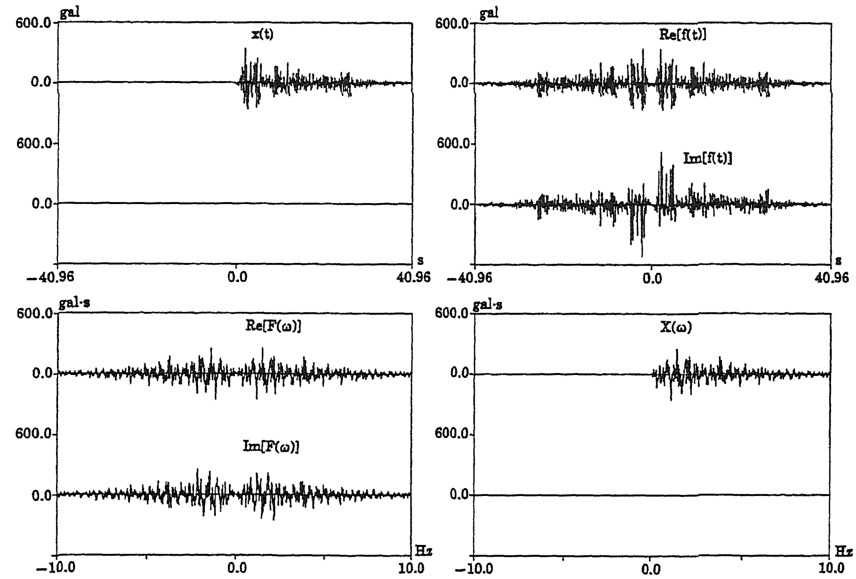

Fig.6 GH functions of a real seismic wave(NS component of 1940 Elcentro) In this FFT analysis, the duration $2T_N$=81.92(Δt=0.01) is considered. It is evident from the figure that, in the time domain, there is a complex valued function f(t) which satisfies $f(-t)=f^*(t)$ and $x(t)=U(t)Re[f(t)]$. We can find clear symmetrical relations between time and frequency domains.

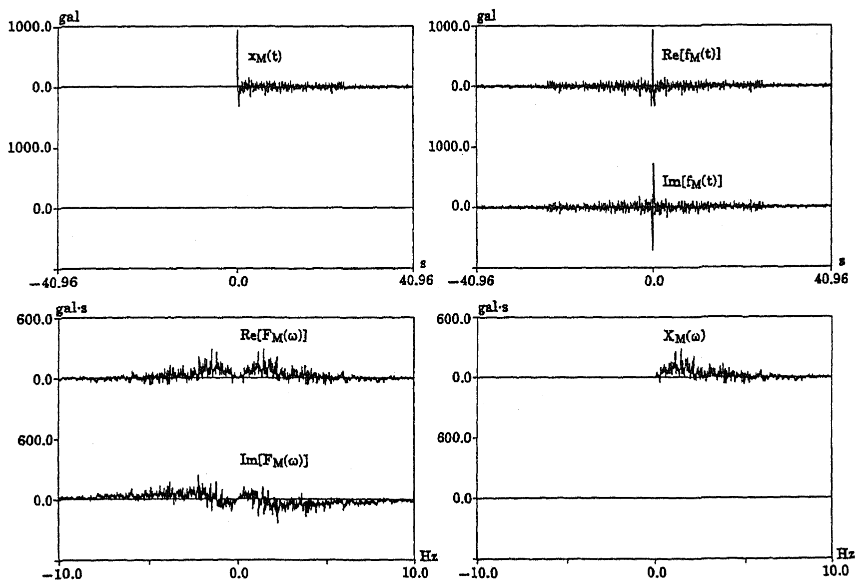

Fig.7 GH functions of the MPS $F_M(\omega)$ which is determined from the generating function $F(\omega)$ in Fig.6. $x_M(t)$ is the causal time function which has a positive peak value near t=0. Accordingly, $Re[F_M(\omega)]$ tends to a positive function.

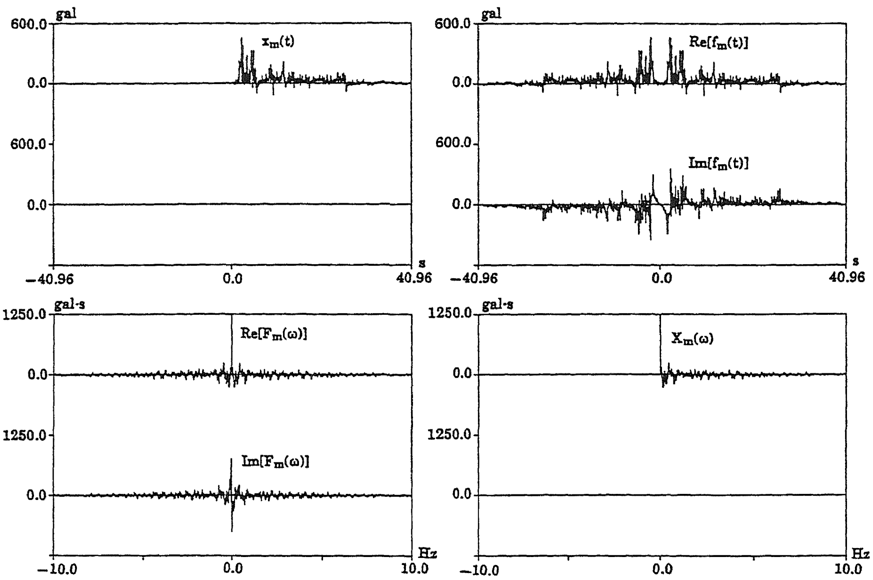

Fig.8 GH functions of the MPS $f_m(t)$ which is determined from the generating function f(t) in Fig.6. These GH functions have quite similar nature to the functions shown in Fig.7, provided we change time for frequency. That is, $X_m(\omega)$ is the causal frequency function which has a positive peak value near ω=0, and $Re[f_m(t)]$ tends to a positive function.

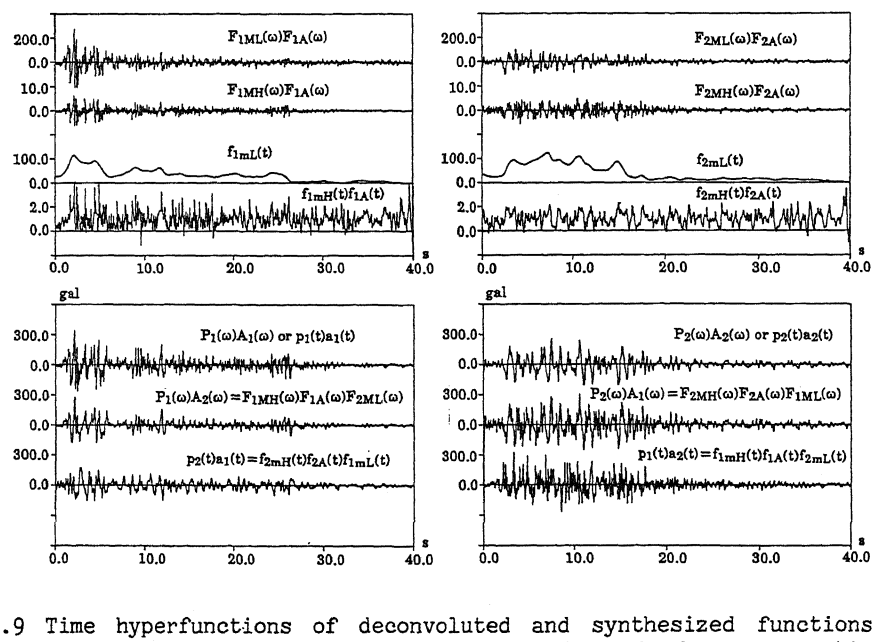

Fig.9 Time hyperfunctions of deconvoluted and synthesized functions for two seismic waves. By selecting the amplitude and phase properties of complex valued functions, we can synthesize several seismic waves with intentional properties.