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AUTOREGRESSIVE MODEL OF MULTIPLE POINTS EARTHQUAKE GROUND MOTIONS

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Summary

An input stochastic model properly representing mutually correlated earthquake ground motions is inevitably required in probabilistic response analysis of large scale line-like or network-like structures such as suspension bridges and buried pipelines. Hoshiya[1] proposed an effective method of obtaining response covariances in a recursive form for a multiple-degree of freedom linear structural system subjected to multiple support nonstationary seismic excitations. But this method requires a pair of input ground acceleration and velocity at each nodal point. This paper proposes a simulation method to obtain a pair of input acceleration and velocity at each node.

Introduction

An autoregressive random process model was used to investigate a simulation method[2] of spatially and temporally variative ground motion when characteristics of motion are prescribed by a cross spectral density function matrix of frequency and space vector between spatial points. In the simulation, surface ground which two dimensionally spreads in relatively small region of approximately 0.2 square kilometers is assumed to be homogeneous and nonisotropic with respect to the direction of wave propagation. The formulation of a cross spectral density function matrix was based on the model suggested by Harichandran and Vanmarcke[3], in their study on array data in Lotung, Taiwan.

Stochastic Representation of Earthquake Ground Motion Harichandran and Vanmarcke[3], proposed the following cross spectral density function model of space and time random field, assuming the homogeneity of surface layers of ground and the nonisotropy with respect to the direction of wave propagation, and regarding the earthquake motion time series as a stationary process with zero mean.

$$S(\vec{x}_a,f) = S(f)\gamma(\vec{x}_a,f)$$
$$\gamma(\vec{x}_a,f) = |\gamma(\vec{x}_a,f)|\exp(-i2\pi fd)$$
$$|\gamma(\vec{x}_a,f)| = A\cdot\exp\left\{\frac{-2|\vec{x}_a|}{\alpha\theta(f)}(1-A+\alpha A)\right\} + (1-A)\exp\left\{\frac{-2|\vec{x}_a|}{\theta(f)}(1-A+\alpha A)\right\} \tag{1}$$
$$\theta(f) = k\left\{1+\left(\frac{f}{f_a}\right)^b\right\}^{-1/2}$$
$$d = \frac{\vec{c}\cdot\vec{x}_a}{|\vec{c}|^2}$$

where $S(\vec{x}_0,f)$ is a cross spectral density function for two spatial points whose separation is given by a distance separation vector $\vec{x}_0$. $\theta(f)$ is called frequency dependent correlation distance, which shows the degree of correlation between earthquake ground motions as a frequency-dependent spatial scale of fluctuation. The coherence $|\gamma(\vec{x}_0,f)|$ decays exponentially with increase of the ratio of magnitude of the distance separation vector and the frequency dependent correlation distance. Parameter d is a lag-shift parameter which describes the phase difference of motions at different spatial points. The nonisotropy is dependent upon the directions of both the propagation velocity vector $\vec{c}$ and the distance separation vector $\vec{x}_0$.

Autoregressive Model of Wave Propagation Propagating earthquake motions may be represented by the following correlated stationary autoregressive random processes with zero mean.

$$_su_i(k) = \sum_{p=1}^{m}\sum_{j=1}^{M} b_{ip}(j)\,_su_p(k-j) + {}_s\varepsilon_i(k) \qquad i=1,2,\cdots,m \tag{2}$$

in which m = number of spatial points, i = a locational index of a spatial point, k = an index of discrete time t in $t=(k-1)\Delta t$, Δt=equal time interval of the given time series, $b_{ip}(j)$= a deterministic function which is governed by the frequency characteristics and $_s\varepsilon_i(k)$ = an error function which is a white noise with zero mean. The coefficients $b_{ip}(j)$ are to be determined so that the time series $_su_i(k)$ may possess the prescribed correlation function matrix, or equivalently the cross spectral density matrix consisting of eq(1),which describes the characteristics of propagating earthquake motions.

Appling the least mean square error criterion to eq(2), the coefficients may be determined by solving the following equations.

Let

$$R_{pq}(\ell) = R(\vec{pq},\ell\cdot\Delta t) = \int_{-\infty}^{\infty} S(\vec{pq},f)e^{i2\pi f\ell\Delta t}df \tag{3}$$

and

$$\begin{bmatrix} R_{11} & R_{21} & \cdots & R_{m1} \\ R_{12} & R_{22} & \cdots & R_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ R_{1m} & R_{2m} & \cdots & R_{mm} \end{bmatrix} \begin{bmatrix} B_{n1} \\ B_{n2} \\ \vdots \\ B_{nm} \end{bmatrix} = \begin{bmatrix} Q_{1n} \\ Q_{2n} \\ \vdots \\ Q_{mn} \end{bmatrix} \qquad (n=1,2,\cdots,m) \tag{4}$$

where R_{pq} and Q_{qn} consist of components of $R_{pq}(\ell)$, and

$$B_{np} = [\, b_{np}(1), b_{np}(2), \cdots, b_{np}(M)\,]^T \tag{5}$$

It is noted that eq.(4) is a m-th order simultaneous matrix equation and can be solved algebraically for the coefficients B_{np} for n=1,2,•••,m.

Then, the error function ${}_s\varepsilon_i(k)$ in eq.(1) may be generated by

$$\begin{bmatrix} {}_s\varepsilon_1(k) \\ {}_s\varepsilon_2(k) \\ \vdots \\ {}_s\varepsilon_n(k) \end{bmatrix} = \begin{bmatrix} C_{11} & & & \\ C_{21} & C_{22} & & 0 \\ \vdots & \vdots & \ddots & \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix} \begin{bmatrix} \xi_1(k) \\ \xi_2(k) \\ \vdots \\ \xi_n(k) \end{bmatrix} = C\,\xi(k) \tag{6}$$

$$\sigma = C\,C^T$$

$$\sigma_{iq} = E[\,{}_s\varepsilon_i(k)\,{}_s\varepsilon_q(k)\,] = R_{iq}(0) - \sum_{P=1}^{m}\sum_{j=1}^{M} b_{qp}(j)R_{pi}(j) \tag{7}$$

In the above equation (6), $\xi_i(k)$ is mutually independent random trains with zero mean and their variance is unity. It is pointed out that a pair of input ground acceleration and velocity should be simulated based on common random variables so that the compatibility might be maintained mathematically for the relationship between accelaration and velocity.

For a prescribed $S(\overrightarrow{pq},f)$ or $R(\overrightarrow{pq},\ell\Delta t)$, the coefficients $b_{ip}(j)$ and the error terms ${}_s\varepsilon_i(k)$ are evaluated respectively by eq.(4) and eq.(6). Then, a set of sample time histories are simulated from eq.(2). A nonstationary process model may be easily expressed with an envelope function g(t) as follows.

$$\begin{aligned} {}_nu_i(k) &= g(k)\cdot {}_su_i(k) \\ &= \sum_{P=1}^{m}\sum_{j=1}^{M} b_{ip}(j,k)\,{}_nu_p(k-j) + {}_n\varepsilon_i(k) \qquad i=1,2,\cdots,m \end{aligned} \tag{8}$$

in which g(k)=an envelope function which represents an evolutionary amplitude trend by

$$g(t) = \{\frac{t-d}{T_p}\}\exp\{\frac{1-(t-d)}{T_p}\} \tag{9}$$

in which Td=the time lapse for the maximum amplitude and d=the lag shift parameter which is specified by eq. 1.

Example A pair of nonstationary input ground accelaration and velocity at each point was simulated for a region in Fig.1, where data in Table 1 were used. As the power spectral density function at a representative point of the homogeneous field, the following equation was used.

$$_aS(f) = (2\pi f)^2{}_vS(f) = (2\pi f)^4{}_DS(f) = \frac{64}{6\pi f_g^5} f^4 \cdot \exp\{-\frac{4|f|}{f_g}\} \tag{10}$$

Some of the results are shown in Figs. 2 and 3. The auto and cross correlation functions of simulated waves were compared to the prescribed auto and cross correlation functions in Figs. 4 and 5.

The proposed simulation method can describe the characteristics of spatially and temporally variative earthquake ground motions. Furthermore, the autoregressive model of this study may be properly adapted as the input earthquake ground motions in a recursive covariance matrix equation proposed by Hoshiya[1].

References

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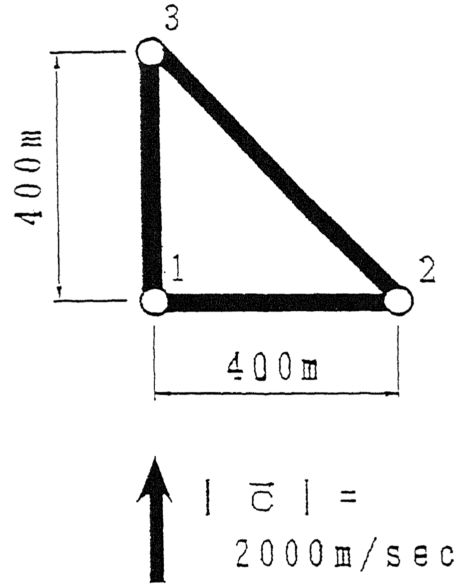

Fig. 1 Homogeneous Field

Table-1

Input Data

Input Data	Values
f_g in eq.(9)	2.0Hz
A	0.736
α	0.147
k in eq.(1)	5210m
f_a	1.09Hz
b	2.78
$\lvert \bar{c} \rvert$	2000m/s
m	6
M in eq.(2)	40
Δt	0.10sec
T_D in eq.(9)	10.0sec

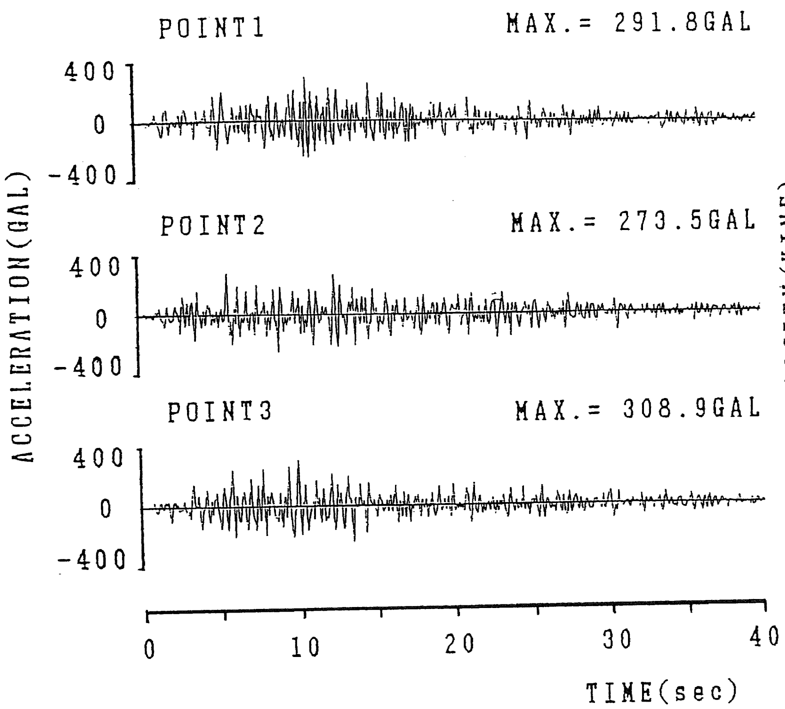

Fig. 2 Nonstationary Acceleration Time Histories

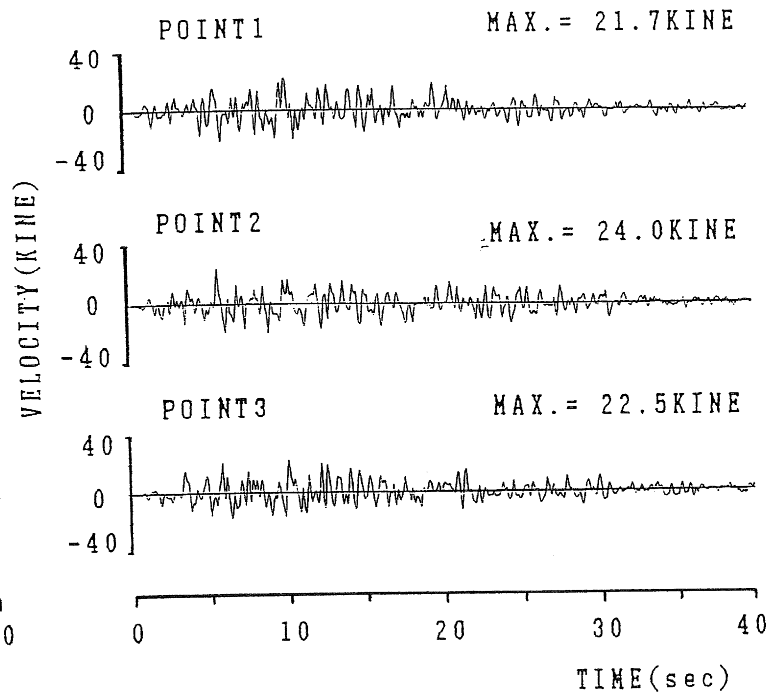

Fig. 3 Nonstationary Velocity Time Histories

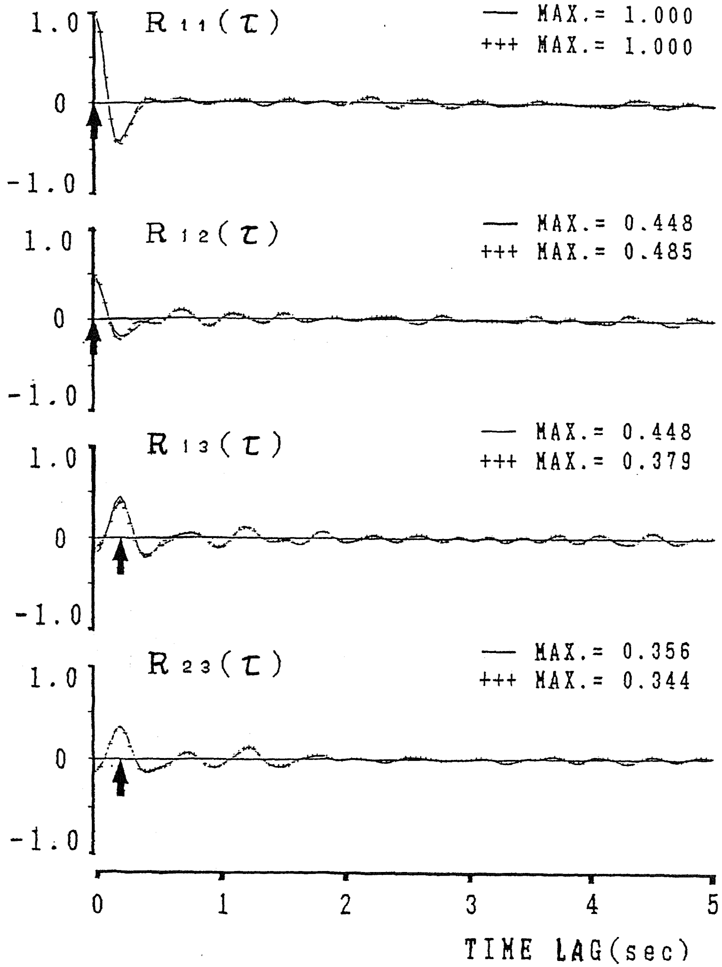

Fig. 4 Cross Correlation Functions of Nonstationary Accelerations

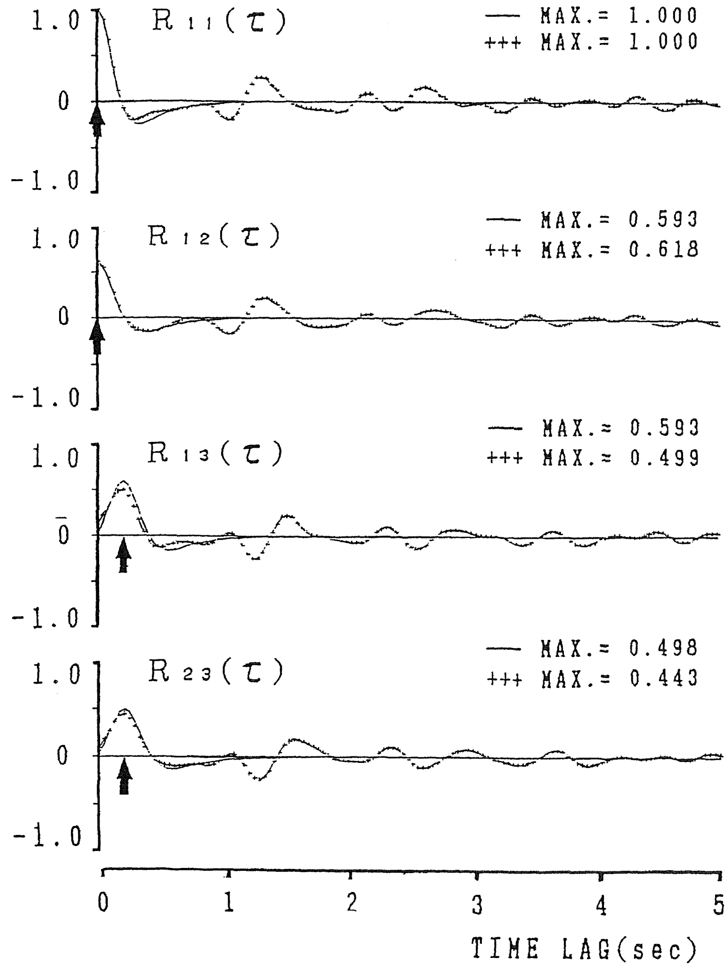

Fig. 5 Cross Correlation Functions of Nonstationary Velocities