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3-7-1 ESTIMATION OF THE EVOLUTIONARY PROCESS OF STRONGLY NONSTATIONARY EARTHQUAKE RECORDS

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SUMMARY

A procedure is introduced to estimate Priestley's evolutionary process from a single earthquake record. The procedure is applicable for strongly non-stationary ground motion and does not rely on the assumption of slowly varying statistics of the stochastic process. The suggested procedure is based on two statistical tests, namely the run-test for smoothing the spectral estimates and a recently developed stationarity test to find the evolution in time. The suggested procedure reduces the randomness of the estimated evolutionary process significantly.

INTRODUCTION

It is widely known that earthquakes are unpredictable in a deterministic sense. For this reason, extensive studies has been made to model the ground motion as a stochastic process and to determine the corresponding stochastic response.

When nonlinear (hysteretic) structural response subjected to extreme earthquake loading is considered, basically two stochastic models for the ground motion have been found to be adequate, namely Priestley's evolutionary spectrum [1,2] and the random pulse train model proposed independly by Lin [3] and Cornell [4]. Both are capable to model the nonstationarity of ground motion with respect to its intensity and frequency content. Lately, a strong tendency is found to incorporate the rapidly increasing knowledge from seismology into predictive stochastic models for the ground motion in terms of physical parameters like seismic moment, apparent stress drop, directivity of slip motion, duration of rupture, depths and epicentral distance, local tectonic and special soil conditions at the site [5,6,7,8]. These stochastic models in development need to be scaled and verified on available data, i.e. by an evolutionary process obtained from data only. The estimation of the governing parameters for the evolutionary process, however, is by no means a simple task. The difficulties arise from two sources. One is due to insufficient statistical information, since the estimation procedure must be applied to one single record available. Another source comes from the impossibility to evaluate the evolutionary process with a high resolution in the time domain as well as in the frequency domain (uncertainty principle [1]). Mainly two estimation procedures are presently used based on work by Priestly [2] and the multifilter technique introduced by Kameda [9] which seems more suitable for engineering purposes. Both methods estimate the evolutionary process by passing the recorded time history through a filter and inferring from its output the evolutionary input. Transient effects are neglected, assuming the parameter of the process to be slowly varying with respect to time and frequency. These approaches are *satisfactory for nonlinear response analysis if the ground motions meets the* assumed conditions. For many earthquake records in the near field, however, where often short strongly peaked intensities occur, the above assumptions are hardly justified. Another shortcoming is the unavoidable randomness of the estimated evolutionary process due to the uncertainty principle. Thus, a procedure based on less restrictive assumptions and less randomly varying estimates is desirable for the purpose of fitting predictive models with available data.The approach suggested herein aimes at a reduction of the randomness of the estimated parameters and on less restrictive assumptions.

PARTITION INTO SUBPROCESSES

Priestley's evolutionary process has the well known representation

$$a(t) = \int_{-\infty}^{+\infty} A(t,\omega)\, e^{i\omega t}\, dX(\omega) \tag{1}$$

where $A(t,\omega)$ is a deterministic function of the circular frequency ω and time t. $X(\omega)$ in eq.(1) represents a stationary process with orthogonal increments. The modulating function $A(t,\omega)$ allows the assignment of a distinct time dependent modulating function $A(t,\omega)$ to an arbitrary narrow frequency band of the ground motion. Clearly, given a single earthquake record such an unique assignment can not be possible, since only quite limited statistical information can be extracted from one single record, especially when short strongly non-stationary earthquake induced ground motion in the near field is recorded. However, a high resolution with respect to both time and frequency, is not required for structural engineering purposes. Experience even shows that for stochastic response analysis of a linear system it is sufficient to model earthquake as uniformly modulated stationary process, i.e. $A(t,\omega) = A(t)$. The response of a strongly nonlinear hysteretic structure, however, is strongly influenced by the low-frequency content of the ground motion which might start delayed and last longer than the higher frequency components. Thus, it is natural to regard the ground motion $a(t)$ as sum of several (2-3) subprocesses $a_j(t)$

$$a(t) = \sum_{j=1}^{M} a_j(t) \quad ; \quad 2 \le M \le 3 \tag{2}$$

where one subprocess represents the low-frequency motion, one the frequency range significant for the linear response analysis and one for the high frequency range. Let the Fourier transform of the ground acceleration $a(t)$ be

$$A(\omega) = \sum_{j=1}^{M} A_j(\omega) \tag{3}$$

$$\text{where} \qquad A(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} a(t)\, e^{-i\omega t}\, dt \quad \text{and} \quad A_j(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} a_j(t)\, e^{-i\omega t}\, dt$$

In contrast to a stationary process, all amplitudes $A(\omega)$ might be correlated with each other. Although the correlation between different amplitudes $A(\omega_1)$ and $A(\omega_2)$ is not known, it can be assumed heuristically that the correlation decreases with increasing $|\omega_1 - \omega_2|$. Studies of earthquake records indicate an increasing correlation between amplitudes for higher frequencies. It also can be reasoned that the correlation between the amplitudes $A(\omega_K)$ determines the time varying intensity of the process. Let the Fourier transform $A_j(\omega)$ of the subprocess $a_j(t)$ be determined by applying filters $W_j(\omega)$ on $A(\omega)$.

$$A_j(\omega) = A(\omega) \cdot W_j(\omega) \qquad \text{where} \quad \sum_{j=1}^{M} W_j(\omega) = 1 \tag{4}$$

The filters $W_j(\omega)$ should be selected such that the correlation between two distinct amplitudes $A(\omega)$ is essentially preserved in the subprocesses. An example for selecting the filters is given in Fig.1. The choice of continuous trapezoidal or triangular filters $W_j(\omega)$ with the height equal one maintains possible correlation in neighboring amplitudes fairly well, but ignores correlation between amplitudes of frequencies within and outside the filter. Since the correlation in $A_j(\omega)$ is preserved best when $W_j(\omega)$ is 1, the vertex of the filter is placed benefically at local maxima of $|A(\omega)|^2$.

PARAMETER ESTIMATION FOR THE SUBPROCESSES

A Fourier transform of the amplitudes $A_j(\omega)$ results in subprocesses $a_j(t)$ in the time domain. They are considered as single realization of a uniformly modulated stochastic process written as,

$$a_j(t) = f_j(t)\ x_j(t) \quad ; \quad x_j(t) = \int_{-\infty}^{\infty} e^{-i\omega t}\, dX_j(\omega) \tag{5}$$

where $dX_j(\omega)$ are orthogonal increments. The amplitudes $dX_j(\omega)$ of the stationary process x_j are related with its spectral density function $S_j(\omega)$ (see e.g.[6]). In order to determine the subprocess $a_j(t)$ in stochastic terms, its spectral density function $S_j(\omega)$ and the modulating function $f_j(t)$ need to be estimated from a single realization $a_j(t)$.

SPECTRAL DENSITY FUNCTION

The spectral density function is usually evaluated from the raw- or sample spectrum

$$S_{j,T}(\omega) = \frac{1}{T}\,|A_j(\omega)|^2 \tag{6}$$

where it is actually implied that the process $a_j(t)$ is a realization of a stationary process with a duration T. It can be shown, however, that the error induced by this assumption is negligible compared with indispensible smoothing of the raw spectrum. The spectrum must be smoothed because the raw spectrum is not a reliable estimate of the spectrum $S_j(\omega)$ no matter how large the duration T or which data window has been used. This is clearly seen from the following inequality:

$$\mathrm{Var}\{S_{j,T}(\omega)\} \geq E^2\{S_{j,T}(\omega)\} \tag{7}$$

In order to obtain a more reliable estimate $S_{j,w}(\omega)$ for the spectral density function $S_j(\omega)$, the raw spectrum is convoluted with a spectral window W_s

$$S_j(\omega) \approx S_{j,W}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{j,T}(\omega-\alpha)\ W_s(\alpha)\ d\alpha \tag{8}$$

where $W_s(\alpha)$ is a real even function normalized such that a constant raw spectrum remains unchanged. Its inverse is called lag window $w(\tau)$. The smoothing results in a considerable decrease of the variance of the estimated spectral density. The estimated spectral density is strongly effected by the choice of the spectral window where its band width B is the crucial parameter. Is the band width B selected too large, peaks and troughs of physical origin will be missed, or for too small B, the estimated power spectrum shows peaks and troughs at random.

A statistical test procedure is suggested to select a suitable band width B. Consider the series of random variables

$$U_k = \frac{S_{j,T}(\omega_k)}{S_j(\omega_k)} \quad \text{and} \quad U_k' = \frac{S_{j,T}(\omega_k)}{S_{j,W}(\omega_k)} \tag{9}$$

It can be shown that the random variables U_k are identically distributed and approximately independent of each other (exact if $a_j(t)$ is a stationary process) and nearly exponentially distributed. Since $S_{j,w}(\omega_k)$ estimates the unknown spectral density $S_j(\omega_k)$, the series of random variables is tested if the components are identically distributed and independent. This hypothesis is tested by the run test (e.g.[10]) with a certain level of significance. In a first step, the band width is selected quite high. Then the band width B is successively decreased until the hypothesis is accepted by the run test.

UNIFORMLY MODULATING FUNCTION

The estimation of the modulating function $f_j(t)$ gives rise to the fundamental question how to distinguish, on the basis of a single realization of a stochastic process with a finite (short) duration T, between a stationary and nonstationary process. The problem might also be stated in the following way: Some moving average estimates $\sigma^2_{aa}(t) = f^2(t)\ \sigma^2_{xx}$ of a realization of stochastic process exhibit "natural" fluctuations also in case the process is stationary. To what degree can these fluctuations be assigned to "natural" fluctuations of a stationary process or to the modulating function f(t)?

Clearly, the above stated problem cannot be solved without a criterion for stationarity. The usual approach of evaluating moving average estimates (including filtering) relies on the assumption of a function f(t) varying slowly with respect to time. Considering the estimates for the variance $\sigma_{xx}(k\cdot\Delta t)$, where Δt represents a time step, as random variables, the application of the non-parametric run test has been suggested [11]. However, the run test is only applicable for cases where correlations between neighboring random variables are negligibly small.

For testing stationarity, further essential progress can be achieved, if the random behavior of all estimates can be characterized and utilized in the testing procedure. Such a procedure has been recently presented in [12] which transforms the estimate of a stationary process into independent standard normal variables. Thus, testing stationarity is converted into a statistical test. These transformed random variables can be associated with a set of "perfectly" distributed random variables representing the underlying stationary process. The latter can be transformed into the original space by inverse transformations. A comparison of the estimates for the given stochastic process with the estimates for the stationary process allows the final estimation of the modulating functions $f_j(t)$.

EVOLUTIONARY PROCESS

The orthogonal processes $X_j(\omega)$, j=1,2,M are assumed to have the same orientation in the complex plane. Then the orthogonal processes $X_j(\omega)$ are fully correlated with each other for equal frequency ω and the following relation holds

$$E\ [\ dX_j(\omega)\ dX_k^*(\omega)\ ] = \{S_j(\omega)S_k(\omega)\}^{1/2}\ d\omega \qquad \text{where } dX(\omega) = \sum_{j=1}^{M} dX_j(\omega) \tag{10}$$

The assumption of full correlation guarantees that the spectrum $S(\omega)$ is not influenced by the filters used in eq.(5). As a consequence of full correlation the power spectrum $S(\omega)$ associated with the orthogonal process $X(\omega)$ in eq.(1) and the modulating function $A(t,\omega)$ can be calculated as follows:

$$S(\omega) = \{\sum_{j=1}^{M} [S_j(\omega)]^{1/2}\ \}^2 \quad \text{and} \quad A(t,\omega) = (\ \sum_{j=1}^{M} f_j(t)\sqrt{S_j(\omega)}\ )/\ \sum_{j=1}^{M}\sqrt{S_j(\omega)} \tag{11}$$

Alternatively, the stationary process X(t) in eq.(1) might be assumed white noise with unit intensity I. In this case, the modulating function $A'(t,\omega)$ reads:

$$A'(t) = \frac{1}{2\pi}\sum_{j=1}^{M} f_j(t)\sqrt{S_j(\omega)} \tag{12}$$

NUMERICAL EXAMPLE

A Friuli earthquake recorded at 10 May 1976 at the Maiano station in Italy is selected as a typical European earthquake record. Its accelerogram is shown in Fig. 1a where the recorded motion lasts only 10 sec. The amplitudes $|A(\omega)|$ of the Fourier spectrum are shown in Fig. 1b. The associated filter $Wj(\omega)$ (see eq.(5)) can be seen from Fig. 1c.

The accelerograms of the subprocesses are shown in Fig.2 at the bottom where the original record is plotted again for the purpose of comparison at the top. An identical scale is used for all four plots. By evaluating the parameters of the uniformly moduated subprocesses $a_j(t)$, their power spectra $S_j(\omega)$ and

S(ω) has been estimated first using the suggested procedure. Subsequently, the modulating functions have been determined. The estimated power spectra as well as the modulating function are shown in Fig. 3b and 3a, respectively. Finally, the modulating function determined by eq.(19) is seen from Fig.3c.

CONCLUSIONS

The following conclusion can be drawn:

1. A procedure for estimating from a single earthquake record the parameters defining an evolutionary process has been shown.
2. The procedure is applicable and reliable also for short strongly nonstationary ground motion .
3. The procedure does not rely on the assumption of slowly varying with respect to time.
4. The procedure is capable to account for random fluctuations of the estimates by applying statistical test procedures.

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REFERENCES

1. Priestley, M.B. "Evolutionary spectra and nonstationary processes," Journal of the Royal Statistical Society, Ser.B,27, 204-237, (1965).
2. Priestley, M.B. "Power spectral analysis of nonstationary random processes," Journal of Sound and Vibration, 6, 86-97, (1967).
3. Lin, Y.K."Application of nonstationary shot noise in the study of system response to a class of nonstationary excitations," Journal of Applied Mechanics, 30, 555, (1963).
4. Cornell, C.A."Stochastic process models in structural engineering," Department of Civil Engineering Technical Report No. 34, Stanford University, Stanford California (1964).
5. Schuëller, G.I., Scherer, R.J. "Artifical acceleration time histories based on nonstationary stochastic source-model," European Earthquake Engineering, Vol.1, 34-40, (1988).
6. Lin, Y.K., Yong, Y. "Evolutionary Kanai-Tajimi type earthquake models," Lectures Notes in Engineering, Vol.32, Lin, Y.K., Minai, R. (Eds.) Stochastic approaches in earthquake engineering, U.S.-Japan Joint Seminar, Boca Raton, Florida, USA, 174-203, (May 6-7 1987).
7. Faravelli, L. "Source-to-site seismic models in structural dynamics," Proc. of the Third Intern. Conference on Recent Advances in Structural Dynamics, Vol II, 1021-1032, (18-22 July 1988).
8. Scherer R.J.:"Source and Site Effects - Preliminary Results based on the Friuli Earthquake Sequence 1976", in "Strong Ground Motion Seismologie" ed. by Erdik M.O., Toköz M.N., 545-558, D. Reidel Publ. Comp., (1987).
9. Kameda, H."Evolutionary spectra of seismogram by multifilter,"Jour. Eng. Mech. Div., ASCE, Vol. 101, No.EM6, 787-801,(Dec.1975).
10. Dixon W.J., Massey, F.J. "Introduction to statistical analysis," McGraw-Hill, Inc., Intern. Student Edition (1983).
11. Scherer, R.J., Schuëller, G.I. "Die Genauigkeit und Leistungsfähigkeit der Spektralmethode," Berichte zur Zuverlässigkeitstheorie der Bauwerke, Sonderforschungsbereich 96, Heft 60, Tech. Universität München (1981).
12. Pradlwarter, H.J."Estimation of modulating functions of earthquake records," in Lecture Notes in Engineering 31:Stochastic Structural Mechanics, Lin, Y.K., Schuëller, G.I. (Eds.), Springer Verlag, Berlin, Heidelberg, New York, 311-326, (1987).

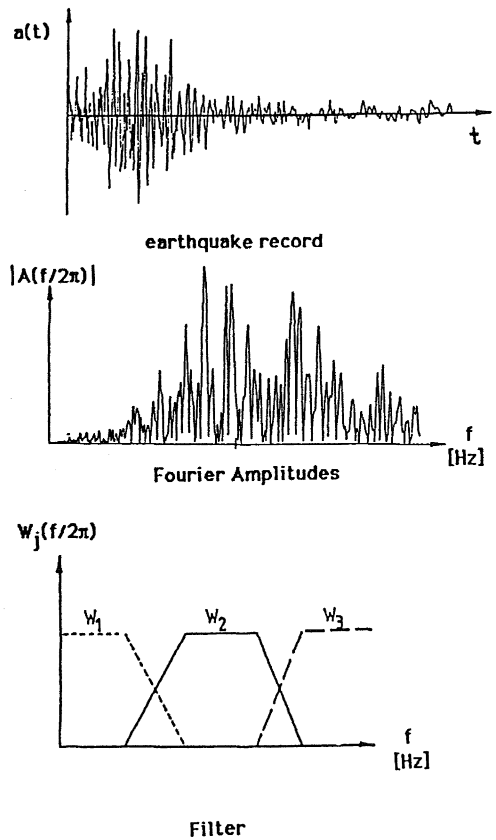

Fig.1a-c: Selection of filter

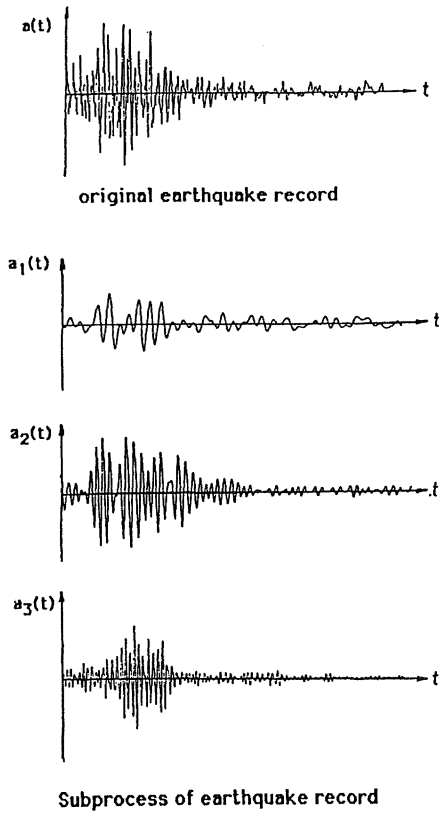

Fig.2a-b: Subprocesses of earthquake record

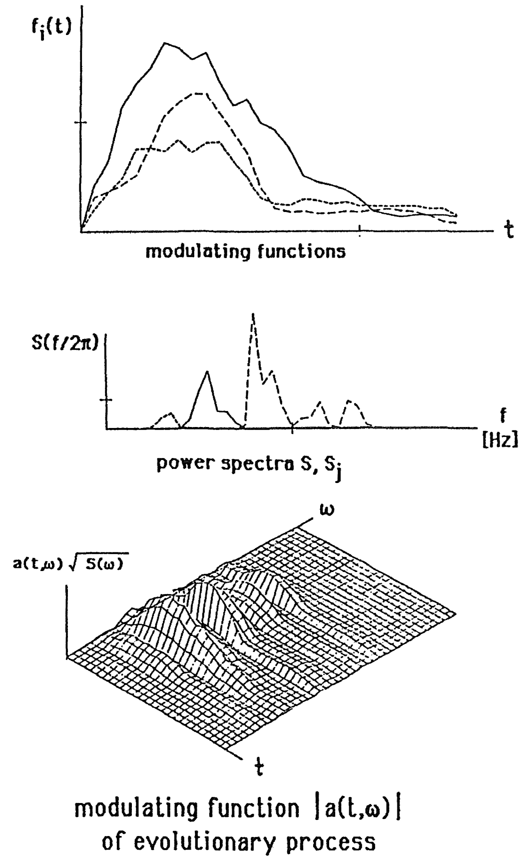

Fig.3a-c: Estimation of the evolutionary process