ANALYSIS OF THEORETICAL SEISMOGRAMS BY THE THIN LAYER FINITE ELEMENT METHOD AND ITS APPLICATION TO THE 1980 IZU-HANTO-TOHO-OKI EARTHQUAKE IN JAPAN

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SUMMARY

A new method is derived to obtain theoretical seismograms for earth structures considering the layered strata and local soil conditions surrounding an observation station. The reciprocal relationship of Green's functions is employed in the formulation, and the Green's functions are obtained in the frequency domain by combining of the three-dimensional thin layer element method and the axisymmetric finite element method. The validity of this method is studied through comparison of its results for an elastic halfspace with those by the wavenumber integration method. As a numerical example, the effects of an idealized sedimentary basin surrounding an observatory is calculated. This method is further applied to the analysis of the 1980 Izu-Hanto-Toho-Oki earthquake with $M_{\text{JMA}}$ 6.7 in Japan.

INTRODUCTION

Earthquake ground motions have been studied using several theoretical kinematic fault models. The normal mode theory, the generalized rays, the discrete wave number method, the discrete wave number finite element method and the wavenumber integration method are the typical ones (Ref. 1 - 5). These layer-dependent methods are most efficient when the earth structure can be regarded as layered strata. However, some ground motions which are considered to be strongly influenced by local soil conditions have been often observed such as those in Mexico city during the Michoacan earthquake of 1985. The finite element method and finite difference method are the powerful approaches to evaluate the dynamic responses of these arbitrary earth structures (Ref. 6, 7). However, it is necessary to prevent reflecting waves at the artificial boundary within an infinite medium in order to obtain precise solutions. Therefore, these numerical methods suffer from exorbitant computational cost when the source and observer are separated or the duration of ground motions is long.

This paper presents a new method to calculate theoretical seismograms for earth structures considering the layered strata and local soil conditions surrounding an observatory. The reciprocal relationship of Green's functions is employed in the formulation, and the Green's functions are obtained in the frequency domain by combining of the three-dimensional thin layer element method by Waas (Ref. 8), Kausel (Ref. 9) and Tajimi (Ref. 10) and the axisymmetric finite element method. The results in the time domain are obtained by the FFT algorithm. Therefore, the proposed method is called the thin layer finite element (TLFE) method in this paper. The accuracy of this method is examined by comparing its results with wavenumber integral solutions for a buried point source in an elastic halfspace. As a numerical example, theoretical seismograms on the idealized sedimentary basin model are calculated in order to illustrate the effect of local soil conditions near the observation point. Finally, this method is applied to the analysis of the 1980 Izu-Hanto-Toho-Oki earthquake with $M_{\text{JMA}}$ 6.7 in Japan to examine its availability to synthesized seismograms in the period range from 1 to 5 sec.
METHOD OF ANALYSIS

Displacement field by seismic point source For linear-elastic anisotropic media, the reciprocal relationship (Ref. 11) of a Green’s function $G_{ip}(x^0:x^e)$ for the source point $x^e$ and the receiver point $x^0$ in the Cartesian coordinate system is represented as

$$G_{ip}(x^0 : x^e) = G_{ip}(x^e : x^0).$$  
(1)

Consider a steady-state problem of time dependence $\exp(i\omega t)$ with $\omega$ being a circular frequency. Then, the displacement $u(x^0, \omega)$ at observatory $x^0$ due to a double couple at the seismic source point $x^e$ buried in an elastic halfspace is expressed using Eq. (1) and the symmetry of the moment tensor as

$$u(x^0, \omega) = M(x^0, \omega) \cdot E(x^0, x^0, \omega),$$  
(2)

where $M(x^0, \omega)$ is the moment tensor and $E(x^0, x^0, \omega)$ is the strain tensor at $x^0$ due to the single force at $x^0$ (Ref. 4). The time domain response is calculated by the FFT algorithm. This concept is shown in Fig. 1.

Green’s functions The media to be used to calculate the strain tensor $E(x^0, x^e, \omega)$ are modeled as follows:

1. The media are assumed to be axisymmetric linear elastic halfspace surrounding an observatory as shown in Fig. 2.
2. The surrounding area ($r \leq r_0$) of the observatory is discretized by axisymmetric finite elements and its exterior layered strata are modeled by thin layer elements by Tajimi et al. (Ref. 10).
3. In the three-dimensional thin layer element, the linearity is assumed for the displacement distribution in depth such as in the finite element formulation, while the displacement function with respect to the horizontal direction are obtained as the exact solutions for the equation of motion in a homogenous media.
4. The source point is located in the mid depth of a thin layer element.
5. In order to express the infinity of the media, the viscous boundaries are introduced on the bottom surface of both the finite element and thin layer element regions as done by Hasegawa et al. (Ref. 12).
6. The effects of attenuating media are introduced as a frequency-independent $Q$ value using complex Lame’s constants.

The three-dimensional thin layer element method is suitable to account for the horizontal infinity of wave fields in layered media and the axisymmetric finite element method is appropriate to represent complicated media. Hence the TLFE method can accommodate both arbitrary layered strata and axisymmetric local soil conditions around the observatory with the aid of the reciprocal relationship of Green’s functions as shown in Figs. 1 and 2.

The procedure to obtain strain tensor $E(x^0, x^e, \omega)$ within the thin layer element is as follows:

(a) The displacement distribution $u(r_0)$ at the boundary ($r = r_0$) between the finite element region and the thin layer element region are obtained for the harmonic loading at the observation point $x^0$ located at the cylindrical axis in the finite element region.

(b) The displacement distribution $u(r)$ at radius $r$ within the thin layer element region can be given by the mode superposition as

$$u(r) = J(r) \cdot q,$$  
(3)

where $J(r)$ is a modal matrix including the Hankel functions depending on radius $r$, $q$ is a generalized displacement vector independent of $r$. Therefore, substituting $u(r_0)$ into Eq. (3), $q$ can be solved as

$$q = J(r_0)^{-1} \cdot u(r_0).$$  
(4)

(c) The displacement distribution $u(R)$ at the seismic source point with radius $R$ is calculated by substituting Eq. (4) and $r = R$ into Eq. (3).

(d) The strain tensor $E(x^0, x^e, \omega)$ in Eq. (2) are given by differentiating $u(R)$ analytically in the $r$-direction and numerically in the $z$-direction.

Propagating fault In order to consider a propagating fault, the fault plane is divided into rectangular subfaults. Each subfault is represented by a point source located in the center of it. The total theoretical displacements are obtained by superposing the generated displacements from each subfault considering the time lag due to the rupture propagation.
VERIFICATION OF ACCURACY

We examine the appropriate depth of the bottom boundary with viscous dashpots to obtain theoretical seismograms precisely for an uniform halfspace, which has the most rigorous condition to the infinity of the media. As a result, it becomes clear that we have to set the depth of the bottom boundary so that the following conditions be satisfied: (i) The incidence angle $\eta$ of body waves into the bottom boundary should be less than about 45° as shown in Fig. 3 (a); (ii) For Rayleigh wave mode shapes of the maximum wavelength of interest, the amplitude at the bottom boundary should be less than about 1/5 of the maximum amplitude at the surface as illustrated in Fig. 3 (b).

The layered structure in Fig. 4 is used to compare the TLFE solutions with the wavenumber integration solutions by Apsel and Luco (Ref. 5) resulting from the action of a buried double couple source. The source corresponds to a vertical strike-slip dislocation and the source time-dependence is represented by a ramp of 1 sec duration. The earth structure is modeled up to the depth of 20 km for the TLFE model. The displacement is filtered in the frequency range from 0.05 to 0.5 Hz. The three components of displacement at 15 km epicentral distance are shown in Fig. 5. The TLFE solutions agree quite well with the wavenumber integral ones. Hence the accuracy of the TLFE solutions is validated for layered strata.

THEORETICAL SEISMOGRAMS ON AN IDEALIZED SEDIMENTARY BASIN STRUCTURE

The effects of a sedimentary basin surrounding an observatory is calculated by the TLFE method. Figure 6 shows the geometry to be considered. The radius of the sedimentary basin is 30 km and its depth is 3 km. The earth structure is modeled up to the depth of 50 km. The displacement is band-pass filtered in the frequency range from 0.05 to 0.4 Hz. From the comparison of the theoretical seismograms by the models without sedimentary basin as shown in Fig. 7, it is found that the peak values of ground motion are amplified and its duration is increased due to existence of the sedimentary basin.

APPLICATION TO THE 1980 IZU-HANTO-TOHO-OKI EARTHQUAKE

Data  The 1980 Izu-Hanto-Toho-Oki earthquake ($M_{JMA}$ 6.7) occurred on 19 June, 1980, off the east coast of the Izu Peninsula, Japan. Figure 8 shows the location of the fault of this earthquake and the observation stations used in this study. We used near-field seismograms at Kawana, Ajiro and Nebukawa stations on the hard soil (Ref. 13). The recorded accelerograms are transformed into displacements by integrating in the frequency domain in order to compare those with synthetic displacements.

Structure and fault models  The detailed layered velocity structure, as listed in Table 1, is assumed based on the results of explosion seismic experiments (Refs. 14, 15). This earth structure is modeled up to the depth of 31 km so that the Moho is included. The various models with different fault parameters were assumed in previous studies (Refs. 16-19). In this study, the fault parameters, as shown in Fig. 8, are assumed based on the aforesaid distribution by Matsuurra (Ref. 20) and those commonly used in the previous investigations. In computing the synthetic seismograms, the simple model with constant rupture velocity is assumed and the fault plane is divided into 20 subfaults as shown in Fig. 9.

Synthetic seismograms  In order to fit the waveforms of synthetic seismograms to those of observed ones, the strike direction and dip angle of the fault have been changed. The synthetic and observed seismograms are band-pass filtered in the range from 0.2 to 1.0 Hz. The synthetic seismograms for the strike of N$5^\circ$W and the dip angle of 75°, which is one of the best fitted combinations, are compared with the observed seismograms in Fig. 10. The main parts of the waveforms agree well each other. The observed seismogram of the NS-component at Kawana station is excellently reproduced in the synthesis. We have estimated that the seismic moment $M_0$ is about $3.8 \times 10^{25}$ dyne-cm when the peak value of the NS-component at Kawana station is well simulated. This $M_0$ value is smaller than that of $5.0 - 7.2 \times 10^{25}$ dyne-cm used in the previous studies. This reason may be explained by the fact that the seismic moment in this study is determined for the higher frequency range than those in the previous studies. The coda parts of the waveforms cannot be simulated well. This fact suggests that the rupture process of this earthquake might be complicated (Refs. 18, 19).
CONCLUSIONS

The principal results in this study are summarized as follows:

(a) A new method to calculate three-dimensional dynamic response of an elastic halfspace to arbitrary buried seismic sources is proposed. In this method, Green's functions for the kinematic seismic source model are calculated in the frequency domain by combining the three-dimensional thin layer element method with the axisymmetric finite element method. Therefore, the proposed method is called the thin layer finite element (TLFE) method in this paper. By using the reciprocal relationship of Green's functions, this method can calculate theoretical seismograms of earth structures considering the layered strata and local soil conditions surrounding an observation station.

(b) The condition for the depth of the bottom boundary with viscous dashpots to obtain theoretical seismograms precisely is given for a uniform halfspace. The accuracy of the TLFE solutions is validated for layered strata through comparison of the theoretical seismograms with those obtained by the wavenumber integration.

(c) The effects of a sedimentary basin surrounding an observation point are found to be very important from the viewpoint of the amplitude and duration of ground motion.

(d) The main parts of the waveforms of observed seismograms at near-field of the 1980 Izu-Hanto-Toho-Oki earthquake with a moderate magnitude ($M_{L}$ 6.7) are simulated by this method. The realistic layered structure and the simple fault model with constant rupture velocity are employed in the frequency range from 0.2 to 1.0 Hz. However, the code parts of the waveforms cannot be simulated well. This fact suggests that the rupture process of this earthquake might be complicated.

ACKNOWLEDGMENTS

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REFERENCES

Fig. 1 Displacement due to a double couple using reciprocity relationship of Green's functions

Fig. 2 Modeling of elastic halfspace by TLFE method

Fig. 3 Depth of bottom boundary with viscous dashpots in order to obtain precise solutions for uniform halfspace

Fig. 4 Source-observer geometry used for comparing TLFE solutions with wavenumber integral solutions for layered halfspace

Fig. 5 Comparison of theoretical seismograms between TLFE and wavenumber integral solutions

Fig. 6 Source-observer geometry used for studying effects of sedimentary basin

Fig. 7 Comparison of theoretical seismograms between layered models with and without sedimentary basin
Fig. 8 Location of fault and hypocenter of the 1980 Izu-Hanto-Toho-Oki earthquake with $M_{\text{JMA}}$ 6.7 and strong-motion stations used in this study

Table 1 Assumed velocity structure

<table>
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<tr>
<th>DEPTH (km)</th>
<th>P-WAVE VELOCITY $V_p$ (km/s)</th>
<th>S-WAVE VELOCITY $V_s$ (km/s)</th>
<th>DENSITY (ton/m$^3$)</th>
<th>Q VALUE $Q$</th>
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<td>3.3</td>
<td>300</td>
</tr>
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</table>

* Between two successive depths marked by asterisk, velocity, density and Q value have linear gradient.

Fig. 9 Fault plane model discretized by point sources

Fig. 10 Comparison between observed displacements and synthetic ones calculated using strike of N5°W and dip angle of 75° in the frequency range from 0.2 to 1.0 Hz of the 1980 Izu-Hanto-Toho-Oki earthquake