Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.II)

3-5-3

A SIMPLIFIED NUMERICAL ANALYSIS PROCEDURE FOR SURFACE WAVES PROPAGATING IN SEDIMENTARY LAYERS

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SUMMARY

A New finite element method is proposed to simulate ground motion in large basins. The degrees of freedom of the ground systems is drastically reduced by using the eigenfunctions for Love waves as shape functions in formulating finite element matrices. Through use of this method two types of analyses are demonstrated. One is eigen value analysis, and the other is a simulation of seismic waves which were observed on a thick sedimentary basin. Under these analyses, quantitative accuracy and applicability of this method for prospecting ground motion have been made clear.

INTRODUCTION

Not only Mexico City badly damaged in the 1985 Michoacan earthquake, but also large cities in Japan like Tokyo and Osaka are located on a large scale of sedimentary basins. Numerical prediction of earthquake motion associated with this kind of ground condition will be important for mitigating earthquake disaster in urban areas. But it still remains difficult to calculate ground motion taking the effect of a broad range of topography into account because such implementation requires considerable computer storage. In order to make it practical to simulate surface wave propagation in large areas, a simplified finite element procedure which employs a smaller number of degrees of freedom has been developed. In this paper, we will describe an outline of the procedure and demonstrate the application for the simulation of seismic waves with a main focus on Love waves.

FORMULATION OF THE FINITE ELEMENT MATRICES

Let us consider Love waves propagating in the horizontal direction of x, as shown in Fig.1. Sedimentary ground is highly idealized as a single surface layer over a rigid half-space. Movable nodes of this model are located only on the ground surface with fixed nodes at the basement. Then the finite element modeling is performed by dividing the ground into some trapezoid elements. Any arbitrary displacement in each element, shown in Fig.2, is expressed in terms of product of the nodal displacements on the surface and eigenfunctions for Love waves which satisfy the condition of the sedimentary stratigraphy.

In the above sense the displacement in the direction of y, v(x,z), can be expressed with the interpolation functions f(x,z) and the nodal displacements V

$$v(x,z) = \frac{f(x,z)}{x_j - x_i}\left(x_j - x \quad x - x_i\right)\begin{pmatrix} V_i \\ V_j \end{pmatrix} \tag{1}$$

Subscripts i and j indicate nodal points. f(x,z) satisfies the conditions of f(x,0)=1 and f(x,-H)=0, in which H is the depth of the element. This formula can be represented in the following matrix form,

$$v(x,z) = \mathbf{LV} \tag{2}$$

where

$$\mathbf{L} = \frac{f(x,z)}{x_j - x_i}\left(x_j - x \quad x - x_i\right), \quad \mathbf{V} = \begin{pmatrix} V_i \\ V_j \end{pmatrix} \tag{3}$$

Therefore a shear strain vector is given by

$$\begin{pmatrix} \gamma_{xy} \\ \gamma_{zy} \end{pmatrix} = \begin{pmatrix} \partial v/\partial x \\ \partial v/\partial z \end{pmatrix} = \frac{1}{x_j - x_i}\begin{pmatrix} -f+(x_j-x)f_x & f+(x-x_i)f_x \\ (x_j-x)f_z & (x-x_i)f_z \end{pmatrix}\begin{pmatrix} V_i \\ V_j \end{pmatrix} \tag{4}$$

or alternatively

$$\boldsymbol{\gamma} = \mathbf{BV} \tag{5}$$

where

$$\boldsymbol{\gamma} = \begin{pmatrix} \gamma_{xy} \\ \gamma_{zy} \end{pmatrix}, \quad \mathbf{B} = \frac{1}{x_j - x_i}\begin{pmatrix} -f+(x_j-x)f_x & f+(x-x_i)f_x \\ (x_j-x)f_z & (x-x_i)f_z \end{pmatrix} \tag{6}$$

Shear stress may be obtained by premultiplying GI by the $\boldsymbol{\gamma}$

$$\tau = G\mathbf{I}\boldsymbol{\gamma} = G\mathbf{BV} \tag{7}$$

in which $\mathbf{I}$ is an identity matrix and G is shear modulus. From the principle of the virtual work element stiffness matrix can be determined by

$$\mathbf{m} = \rho \int_v \mathbf{L}^T\mathbf{L}\,d(vol) \tag{8}$$

$$\mathbf{k} = G \int_v \mathbf{B}^T\mathbf{B}\,d(vol) \tag{9}$$

in which ρ is mass density and $\int_v d(vol)$ denotes volume integration through the element. The matrix $\mathbf{m}$ is the consistent matrix. In its place we have employed the lumped mass matrix, which is an approximation to the consistent matrix, in order to simplify calculations. The global stiffness and mass matrices are constructed by superposing the all element matrices. With these matrices, eigen value analysis or response analysis can be conducted in the same way as ordinary finite element methods.

ACCURACY IN EIGENVALUE ANALYSIS

Accuracy of this method was investigated through an eigenvalue analysis in comparison with an ordinary finite element method. A ground model shown in the Fig.3 was employed for this analysis. For the horizontally layered model, theoretical natural periods "T^*" and eigenvectors "u_{mn}" can be easily determined

(Ref. 1) by

$$T = \frac{4H}{Vs} \sqrt{(2m+1)^2+(2nH/L)^2} \qquad (10)$$

$$u_{mn}(x,z) = a \sin\frac{n\pi x}{L} \sin\frac{(2m+1)\pi z}{2H} \qquad (11)$$

where a is an arbitrary constant, m and n define order of vertical and horizontal mode, respectively. The finite element modeling in this method was performed by using cos $\pi z/2H$, which is the eigenfunctions for the fundamental-mode Love waves. Fig.4 shows an example of eigen vectors evaluated by each method together with distortion of ground surface at the top of each figure. As is evident from the figures, the two vectors look similar to each other. The elapsed time in calculating the first 30 modes of the ground by each method is tabulated in Table 1 with degrees of freedom of each finite element model. It can be seen from this table that the elapsed time in the computation by this method is 1/11 times as much as the one by the ordinary methods.

Fig.5 shows comparison of accuracy of the two method in the analysis. The dimensionless value, $\lambda/\Delta x$, represents a ratio of wave length to the interval between horizontally adjacent nodes. T is a natural period of the ground model calculated by each method. As seen from this figure, the accuracy decreases as the value of $\lambda/\Delta x$ becomes smaller. Although the accuracy of the proposed method is worse than that of the ordinary FEM, the value of the difference is at most 3 %. This difference is not so large considering the reduction of the elapsed time. Though, it is omitted in this paper, this method also has proved to reduce elapsed time by 1/15 in the simulation of waves induced in a model ground with accuracy almost the same as that of the ordinary FEM (Ref. 2).

SIMULATION OF GROUND MOTIONS IN THE KANTO PLAIN, JAPAN

By using this method seismic waves observed on thick sedimentary basin during the 1984 western Nagano prefecture earthquake were simulated. Fig.6 shows the map of the south part of the Kanto plain. The seismic waves were observed at the sites of ASK, NGT and YKH. On this basin, ASK is located on the bedrock and NGT and YKH are located on the thick sedimentary ground. As the epicenter of this earthquake is located about 200 km northwest to ASK, the direction of ASK-NGT-YKH line runs parallel to the radial one, approximately. Fig.7 shows a cross-section model of this area. Shear wave velocity of this model is 700m/s and the depth at NGT is 1.0km (Ref. 3). Seismic waves at NGT and YKH were simulated by applying the observed wave at ASK to the basement nodes considering shear wave velocity traveling in the bedrock(3km/s). Shape functions used in this analysis are the eigenfunctions for the fundamental mode(m=0) and the next higher mode(m=1) Love waves. Total response waves can be obtained as a sum of the result calculated by using each function.

Fig.8 shows the comparison of the observed and the calculated waveforms at NGT and YKH. Fig.9 shows Fourier spectra of the waves. The component of the longer period (about from 4 to 6 s) and shorter period (about from 1 to 2 s) in the calculated wave forms is obtained from calculation by using eigenfunction for m=0 and m=1, respectively. Despite the small number of degrees of the freedom, only 200 for this ground model, both the observed and simulated waves look rather similar regarding their predominant period, duration and amplitude.

CONCLUSION

A practical procedure is presented for numerical analysis on surface waves propagating in a large basin. As this procedure is capable of reducing the number of degrees of freedom, it seems promising for extending three-dimensional procedure.

ACKNOWLEDGMENT

The authors are grateful to H. Kobayashi and K. Seo for providing the seismograms cited here.

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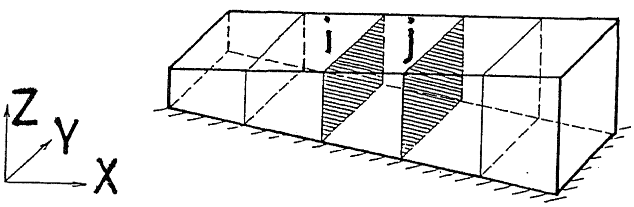

Fig.1 Finite element modeling in the proposed method.

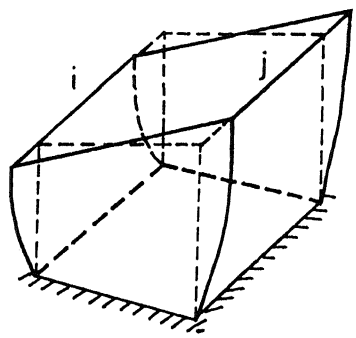

Fig.2 Trapezoid element devided by i and j sections.

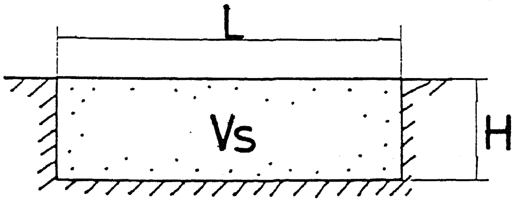

Fig.3 Ground model employed for the eigenvalue analysis.

Table 1 Comparison of CPU time elapsed in the eigenvalue analysis (by HITAC M-280H).

	CPU time	Degrees of Freedom
This method □	0.50 s	53
Ordinary FEM △	5.57 s	318
Ratio (□/△)	1/11	1/6

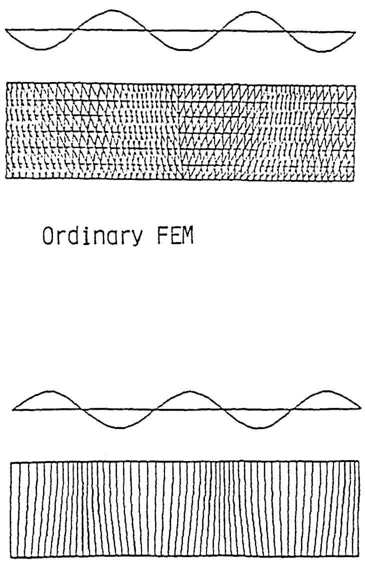

Fig.4 An example of eigen vectors. (Surface distortions are sketched at the top)

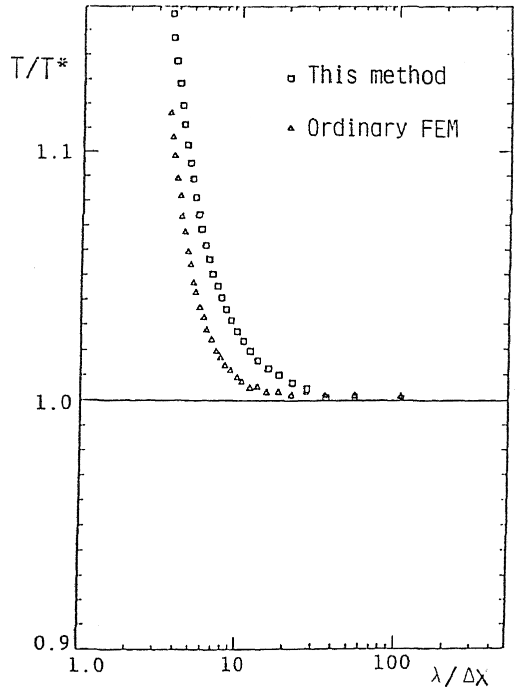

Fig.5 Comparison of accuracy in the eigenvalue analysis.

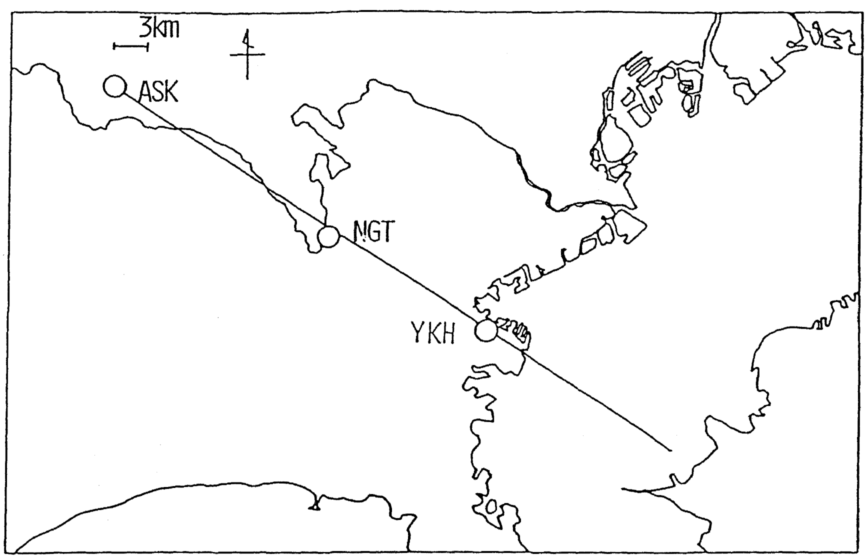

Fig.6 Map of the south part of the Kanto region.

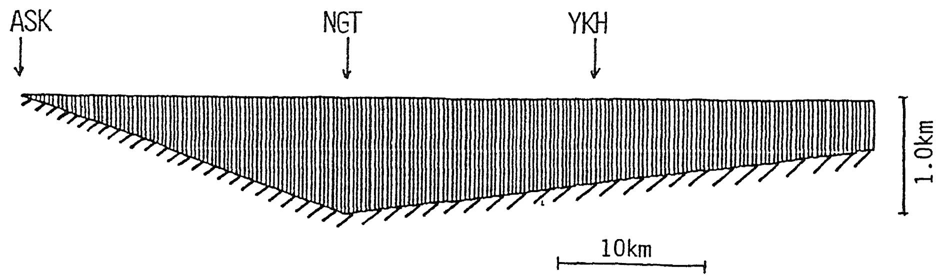

Fig.7 Cross-section of ASK-NGT-YKH line.

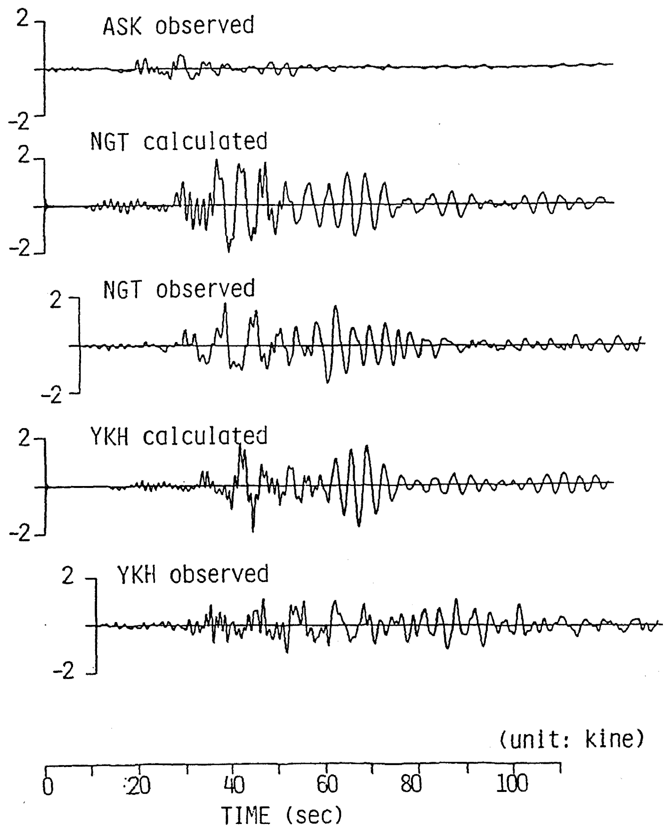

Fig.8 Observed and calculated wave forms on the ASK-NGT-YKH line.

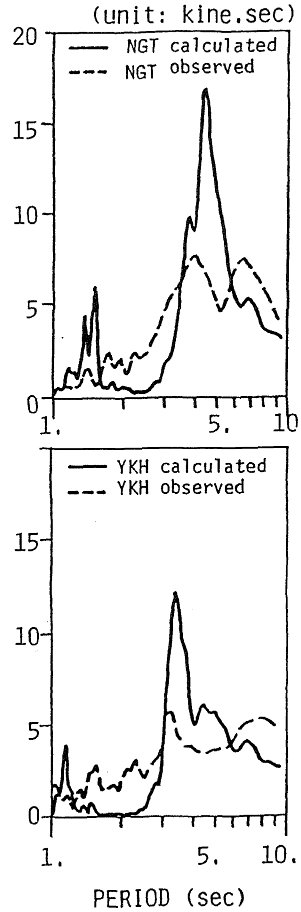

Fig.9 Fourier spectra of the calculated and observed waves.