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# ABSORBING BOUNDARIES FOR WAVE PROPAGATION PROBLEMS (USING KOSLOFF'S METHOD IN ABSORBING REGION)

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## SUMMARY

Reflection from boundary of numerical grids have always presented the difficulties in applying discrete method to simulate physical phenomena. In this paper, absorbing boundary condition , which is proposed by Kosloff, are applied to the numerical simulations of two-dimensional SH wave propagation problem by finite difference method. The effectiveness of this absorbing boundary condition is evaluated by comparing the wave propagation theory.

## INTRODUCTION

In the numerical simulation of wave propagation problems by finite element and finite difference method, it is necessary to eliminate the boundary events which are generated by the boundary of the numerical grids. These events arise because the numerical mesh covers a finite region of space. A number of methods have been proposed for constructing absorbing boundaries.

Lysmer et al.(Ref. 1) proposed a " viscous boundary ", which absorbed scattering waves effectively. Lysmer et al.(Ref. 2) proposed a " transmitting boundary ", which is intended to absorb body waves and surface waves on the lateral infinite boundary. These absorbing boundary condition are dependent on the frequency. Smith (Ref. 3), Cundall et al.(Ref. 4) and Kunar et al.(Ref. 5) proposed a " nonreflecting boundary " , which can be achieved by averaging the solution of two problems, one involving fixed(Dirichlet) boundary condition and one involving free(Neumann) boundary conditions. This absorbing boundary is independent on the frequency.

On the other hand, Kosloff et al.(Ref. 6) proposed the method, which absorbed radiating wave from the interior region to outward on the absorbing region around about a interior region. This method is applied to the Schrodinger equation and to the acoustic equation in one and two dimension by the Foureir method. But it is not clear how to apply this type of boundary condition to discrete method such as the finite difference method. Therefore, in this study Kosloff's method is applied to the simulation of two-dimensional SH wave propagation problems by finite difference method.

## METHODS

### Absorbing Boundary Condition for Acoustic and Elastic Wave Equations

For simplicity, one-dimensional wave propagation problem is considered. Wave motion in the interior region is given by equation (1)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

where C is phase velocity and u is present displacement.
In order to derive the equation in the absorbing region, equation (1) is rewritten as two coupled first-order differential equations in time.

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ V \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{\partial^2}{\partial x^2} & 0 \end{bmatrix}\begin{bmatrix} u \\ V \end{bmatrix} \tag{2}$$

The first equation in (2) expressed the relation $V = \partial u/\partial t$, whereas the second equation is identical to (1). The absorbing boundary condition is achieved by replacing (2) with the system

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ V \end{bmatrix} = \begin{bmatrix} -\gamma & 1 \\ \frac{\partial^2}{\partial x^2} & -\gamma \end{bmatrix}\begin{bmatrix} u \\ V \end{bmatrix} \tag{3}$$

where $\gamma$ is the potential, which play the same role of the optical potential in the Schrodinger equation. This potential, which is shown in Fig-1 , is expressed in the equation (4)

$$\gamma = \frac{U_0}{\cosh^2(\alpha x)} \tag{4}$$

where, Uo is constant, $\alpha$ is a decay factor and X denotes the distance from the boundary. In constructing numerical schemes, it sometimes may be more convenient to work with a single second-order equation. This equation is obtained from (3) after elimination of the variable V, giving

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - 2\gamma \frac{\partial u}{\partial t} - \gamma^2 u \tag{5}$$

Equation (5) can be integrated in time with a suitable stable time differencing scheme. And in case of the two-dimensional problem, wave equation is given by the equation (6)

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - 2\gamma \frac{\partial u}{\partial t} - \gamma^2 u \tag{6}$$

For the elastic problem, the elastic wave equations in absorbing region can be derived by the same way to the acoustic case. Final wave equations form can be expressed

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2} + (\alpha^2 - \beta^2)\frac{\partial^2 w}{\partial x \partial z} + \beta^2 \frac{\partial^2 u}{\partial z^2} - 2\gamma \frac{\partial u}{\partial t} - \gamma^2 u \tag{7}$$

$$\frac{\partial^2 w}{\partial t^2} = \beta^2 \frac{\partial^2 w}{\partial x^2} + (\alpha^2 - \beta^2)\frac{\partial^2 u}{\partial x \partial z} + \alpha^2 \frac{\partial^2 w}{\partial z^2} - 2\gamma \frac{\partial w}{\partial t} - \gamma^2 w \tag{8}$$

where, $\alpha$ and $\beta$ are velocity of compressional and shear waves, respectively.

<u>Derivation of Reflection and Transmission Coefficients for The One-Dimensional Acoustic Wave Equation</u> Consider one-dimensional wave propagation in region $-\infty < X < \infty$ . According to Kosloff (Ref.6), X axis is divided into three regions which is shown in Fig-2. A plane wave (1*exp(-ikx)) in the region X<a (Region-1) generates a reflected wave (R*exp(ikx)) at the boundary at X=a. The reflected coefficient R is generally complex. In the region X>b (Region-3) only the transmitted wave (T*exp(-ikx)) propagates toward infinity with T denoting the generalized transmission coefficient. In the reign a<X<b (Region-2) transmitted wave (C*exp(-ikx)) at the boundary at X=a and reflected wave (D*exp(ikx)) at the boundary at X=b are generated. It is considered that the reflection wave and transmission wave propagating from the region a<X<b are absorbed in this region

by way of introducing absorbing coefficients. Absorbing coefficients differs form zero only in the region a<X<b. Accordingly the region a<X<b is divided into small region : a=X0<X1<X2<----<Xm=b. Applying the condition for continuity of displacement and continuity of traction at the each interface, the equation, which relates the amplitude coefficient in the (m)th region to the corresponding coefficients in the (m+1)th region, can be derived.

$$\begin{bmatrix} C_{m+1} \\ D_{m+1} \end{bmatrix} = [F_m] \begin{bmatrix} C_m \\ D_m \end{bmatrix} \tag{9}$$

where [Fm] is propagator matrix and is referred to Ref. 4.
A successive application of (9) will yield a connection between the coefficients of at any two regions. The boundary condition at X=a and X=b are given by

$$\begin{bmatrix} 1 \\ R \end{bmatrix} \qquad \begin{bmatrix} Te^{\frac{-i\omega b}{c}} \\ 0 \end{bmatrix} \tag{10}$$

respectively. Putting these condition into (9) and rearranging, we obtain

$$\begin{bmatrix} Te^{\frac{-i\omega b}{c}} \\ 0 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} 1 \\ R \end{bmatrix} \tag{11}$$

The matrix elements are formed for the product of the individual matrices is (9) in each region. The generalized reflection and transmission coefficients can be obtained for (11) giving

$$R = -\frac{T_{21}}{T_{22}} \quad , \qquad T = \frac{T_{11}T_{22} - T_{12}T_{21}}{T_{22}} e^{\frac{i\omega b}{c}} \tag{12}$$

The absorbing effects can be evaluated by the magnitude of the reflection coefficient R and transmission coefficient T of the absorbing region. When these coefficients are kept small, the propagation wave does not reflect from the absorbing boundary into the interior region and also they does not transmit to the exterior region. Fig-3 shows the reflection and transmission coefficients as a function of wavenumber. The parameters are C= 100 (m/sec.),number of partitions m=20. Fig-3a is a example which absorbing parameters are not adequate. In this case, the transmission coefficient is fairly large. In Fig-3b, these coefficients are small except at the extreme of small wave number. In this way the undetermined parameters, Uo and $\alpha$ , affect the values of the magnitude R and T. Accordingly it is necessary to estimate the optimal values of Uo and $\alpha$ , which make small the coefficients R and T as possible. As these parameters can not be evaluated analytically, they have to be evaluated numerically by way of trial and error.

## APPLICATION AND RESULTS

It is considered that propagation waves from the interior region into the exterior region such as reflected wave, which is generated by the reflected of incident wave on the free surface of half space medium, is downward, and as scattering wave, which is caused by artificially or irregular topography in a half space, is radiating to outward, are all absorbed in the absorbing region. The wave motion can be obtained by integrating the equations (5) and (6) in regard to time. For the purpose of solving these equations, finite difference method is applied in this applications.

Response of Half Space Medium Fig-4 show a geometry of analytical half space model,which removed irregularity. Continuous lines indicate to the interior region. The absorbing region is illustrated by assemblage of little open circle. The interior region is 80m x 40m and the absorbing region is 160m x 80m. Mesh size of each region is dx=dz=2m and phase velocity is C=100 (m/sec.). The absorbing region is surrounding by the numerical mesh and parameters in the equation (4) are given by Uo = 140 (1/sec.) and $\alpha$ = 0.083 (1/m). The period of input wave is 0.18 sec. Fig-6 shows the result of the simulation ,which the impulse displacement is given at the point O in Fig-4. Fig-7 shows the result of the simulation, which SH wave propagate from bottom vertically. The bold lines in Fig-6a and Fig-7a show time history of response at the point P in the soil and at center point O on the free surface of the model, respectively . The thin lines in Fig-7a is indicate response at the same point calculated by wave propagation theory. In each case, a little reverberation remains just after a main shock. Its reverberation depends on the discreteness of space and time. And also a small long period vibration remains the latter part of the time history. This vibration is occurred becouse the reflection coefficient R can not be perfectly zero in the absorbing region. But,these reverberation is able to be negligible.

Response of Bank and Surrounding Soil Fig-5 shows a analytical model, which have bank and surrounding soil topography. The property of soil is uniform and analytical region size and mesh size are same to the previous model in Fig-4 except the bank. The bank size is 10m x 12m. The period of incident wave is 0.18 sec. and is propagating vertically from bottom. And the parameters for the calculation are C=100 (m/sec.). ,Uo =140 (1/sec.) and $\alpha$ =0.083 (1/m). In this model, it is considered that scattering wave are generated by the presence of the bank in addition to reflection wave on the surface. Fig-8 shows the results of the simulation computed and Fig-8a shows time history of response at a center of bank.

## CONCLUSIONS

Kosloff's method is applied to the simulation of the one or two-dimensional SH wave propagation problems. This absorbing parameters, which has been established to one-dimensional wave propagation problem,are well applied to eliminate the two-dimensional wave propagation problem. As the absorbing region is added around interior region, total analytical region become fairly large. But if the absorbing parameters in the absorbing region can be estimated before the analysis, the boundary condition on the outside absorbing region can be given by the fixed(Dirichlet) boundary condition and special boundary technique is unnecessary.

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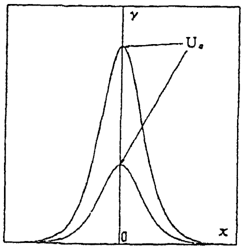


Fig-1. The shape of function Y

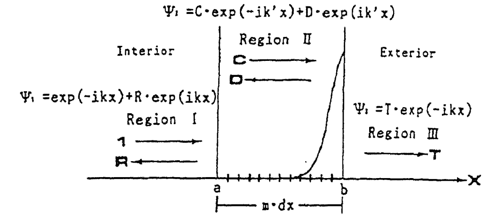


Fig-2. One-dimensional model

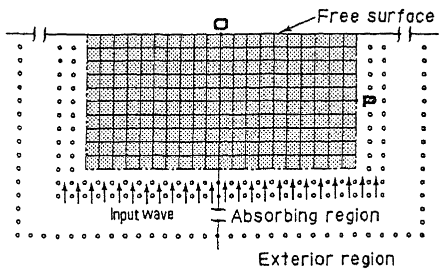


Fig-4. Two-dimensional half space model

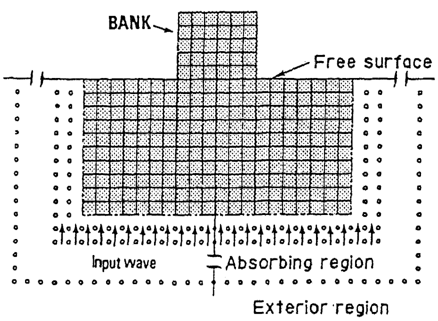


Fig-5. Bank and surrounding soil topography

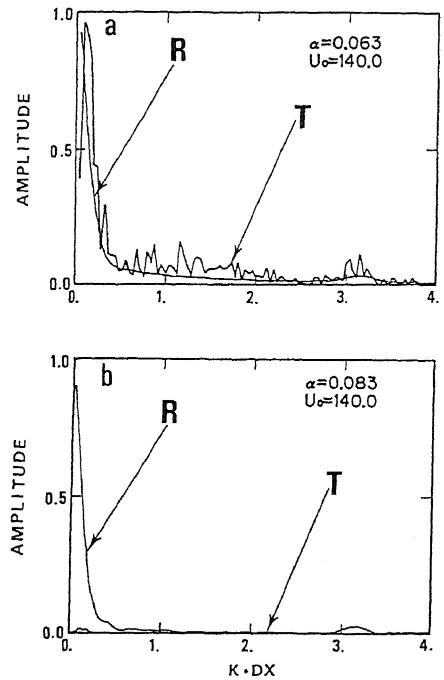


Fig-3. Reflection and Transmission coefficients absorbing region as a function of wavenumber
(a) $U_0$=140(1/sec.), $\alpha$=0.063(1/m)
(b) $U_0$=140(1/sec.), $\alpha$=0.083(1/m)

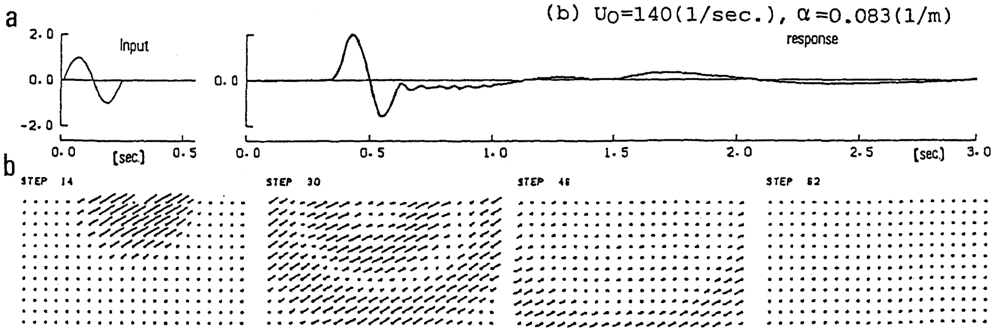


Fig-6. Time history of the half space soil
(a) Input wave at point O on the free surface and response wave at point P in the soil
(b) Response of half space medium

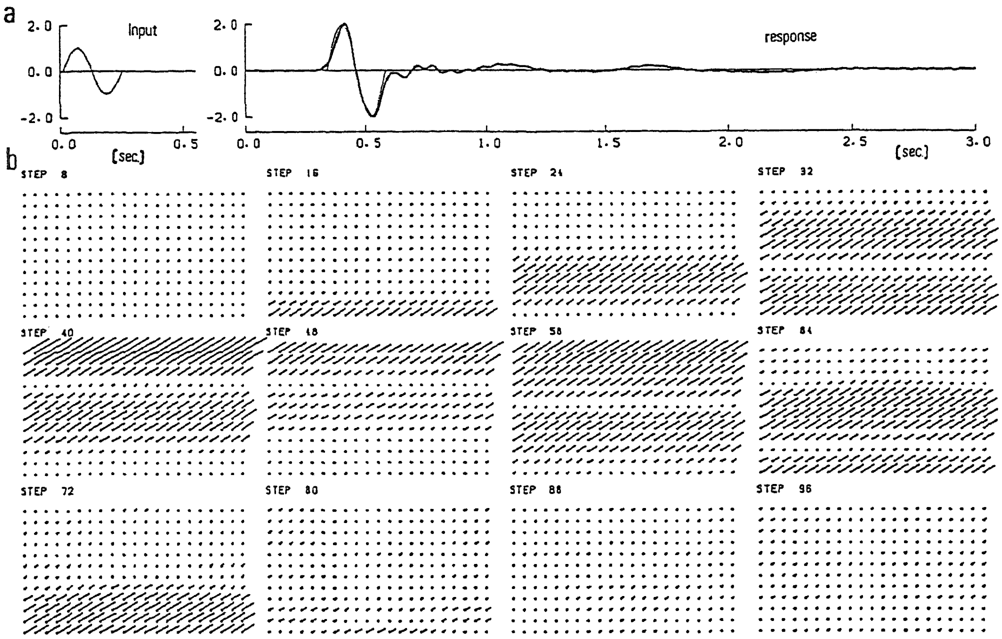


Fig-7. Time history of the half space soil
(a) Input wave at the bottom and response wave at a center of free surface
(b) Response of half space medium

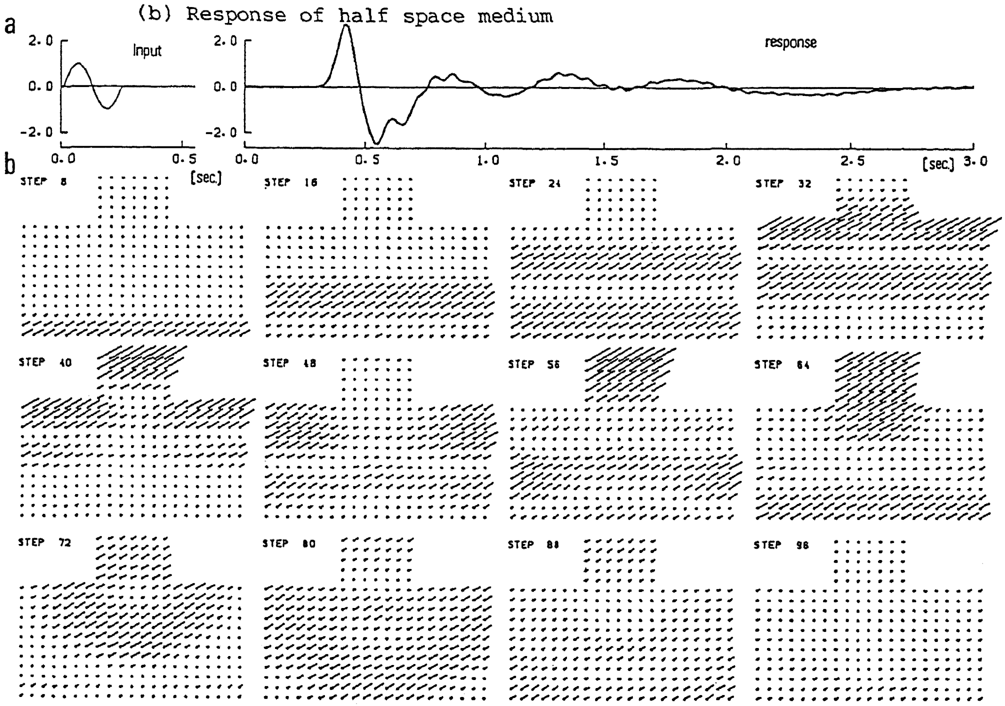


Fig-8. Time history of the bank and surrounding soil topography
(a) Input wave at the bottom and response wave at the top of the bank
(b) Response of the bank and surrounding soil topography