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AN APPLICATION OF VIBRATION ANALYSIS ON EXAMINATION ON BEHAVIORS OF LOVE WAVES

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SUMMARY

Due to the fact that Love waves are attributed to a kind of natural vibration of a surface layer and their nature can be understood by vibration experiment or by vibration mode analysis of a layer of finite length. This has been confirmed experimentally and analytically.

With this point as background, vibration analyses were applied to examine on behaviors of Love waves propagating across a discontinuity of a ground. Amplitude ratio of right part to left part of a model divided by a discontinuity depend on group velocity ratio. This quantity has influence on reflection and trasmission of Love waves.

INTRODUCTION

A study on seismic waves induced in a surface layer is to solve technical problems of such as earthquake resistance of any civil structure or lifeline system and its response to earthquake ground motion. For this reason, ground motion records obtained by seismological array observation have been closely analysed in the light of wave theory.

Meanwhile, there is another kind of useful approach to the study, in which either analytical or experimental models, or both, are used. In this approach, especially in an experimental one using a small-scale model, difficulty lies in both rpoducing ideal incident waves and detecting whole features of waves propagating in the model. In comparison with this kind of transient-wave experiment, vibration experiments can be conducted with less difficulties and serve directly to our understanding because standing waves are adaptable for our visual observation. It is also the case with a numerical analysis.

Thus, it is possible to substitute the vibration experiments for the wave ones, then, resulting merits would be more than appreciable. With this respect, we are encouraged by the well-known fact that surface waves such as Love waves and Rayleigh waves developing in a surface layer are essentially attributed to natural vibration of the surface layer. For this point of view, the authors carried out several experiments as well as analyses on Love waves and Rayleigh waves to confirm this fact, with satisfactory results as described in their previous papers(ref 1,2). This time, vibration analyses were conducted for investigate on behaviors of Love waves propagating across a discontuniuty of a ground(Ref.3).

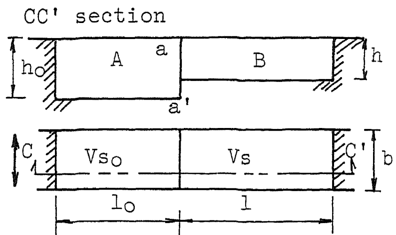
EXPERIMENTS

Every specimen used in this series of experiments is a model of an elastic layer on a rigid base. As model material, acrylamide gel(density: 10^3kg/m^3, Poisson's ratio:0.5) was used for the elastic layer. Due to low elasticity,this material is more siutable for this kind of experiment than materials of high elasticity used elsewhere such as alluminium alloy,plastics. And it can be become colorless and transparent by adding sodium acrylate. Alluminium plates were used for the rigid base models.

Configirations of models are shown in Fig.-1. Line aa' indicates a discontinuity. Vs means shear wave velocities. Suffix 0 refers to left-hand parts of models.

No.	Shear wave velocity(m/s) Vs_0	Vs	Dimension (mm) l_0	l	h_0	h	b
1	2.10		152.5				105
2	2.66	2.18	130	210	30		115
3	2.59	1.85					120
4	2.66	2.18	105	230			115
5	2.59	1.85					120
6	2.86		330	270	30	22.5	120
7	2.66		150		30	15	115

Shear wave velocities and demensins of the models, T a b l e - 1



Configuration of models
Fig.-1

Table-1 shows dimensions and shear wave velocities of the models. Each model was mounted on a shaking table and was excited in the horizontal direction indicated by arrow shown in Fig.-1. Edges of the models parallel(perpendicular) to the arrow are fixed(free). Various horizontal resonant responses produced on horizontal free surface were observed while the exciting frequency was gradually varied.

Photo.-1 shows an example of experimental results on model No.-1. For each observation,the phase velocity(C) of horizontal motion was calculated by multiplying frequency(f) and wavelength(λ) in the horizontal direction. Ratio of the phase velocity to the shear wave velocity(C/Vs) and wavelength to the model heights(λ/H)were shown in Table-2,in which(C/Vs)' obtained theoretically were also shown. Our experimental results agree well with theoretical ones.

Photo.-2 shows an example of results of model No.-6. Different shaped vibration modes are observed on both parts of the model. Amplitude ratio of resonant responses of each parts may depend on the behaviors of Love waves propagating across the discontinuity. In the next chapter,experimental results will be examined theoretically.

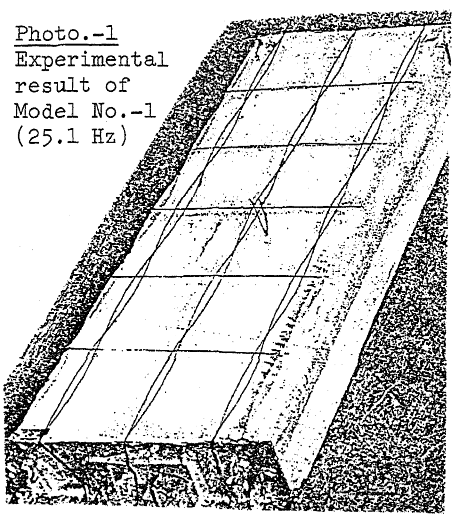


Photo.-1 Experimental result of Model No.-1 (25.1 Hz)

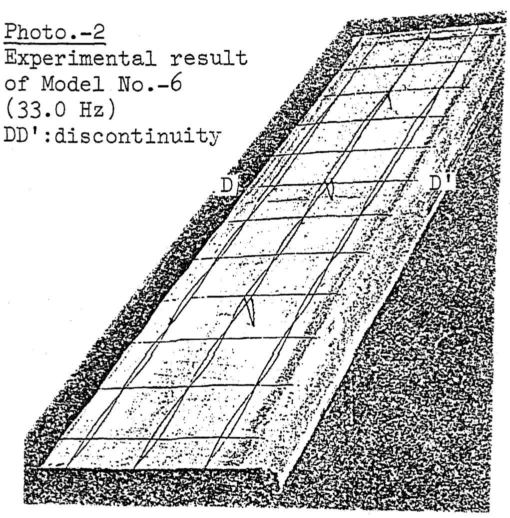


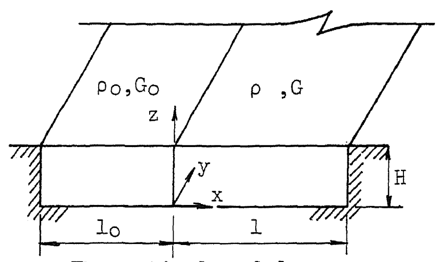
Photo.-2 Experimental result of Model No.-6 (33.0 Hz) DD':discontinuity

No.	Frequency f(Hz)	Wavelength λ (cm)	Phase velocity C(m/s)	Dimensionless wave-length (λ/H)	Dimensionless phase velocity C/Vs	(C/Vs)'(Theory)
1	18.0	61.0	11.0	20.3	5.24	5.17
2	20.5	20.3	4.16	6.77	1.98	1.97
3	25.1	12.2	3.06	4.07	1.46	1.43
4	31.0	8.71	2.70	2.90	1.29	1.23
5	36.6	6.78	2.48	2.26	1.18	1.15
6	42.2	5.54	2.33	1.85	1.11	1.10
7	53.7	61.0	32.8	20.3	15.6	15.3

Experimental results of the model No.-1
Table-2

THEORETICAL APPROACH

In order to examine the experimental results,vibration analysis was conducted for a surface layer with a uniform depth H rested on a rigid base. Both edge sides and bottom of the layer are fixed,and shear wave velocities are different between right-hand part and left-hand one(Fig.-2). Displacements are assumed to be produced in the y direction.



Theoretical model
Fig.-2

Taking account of boundary conditions and continuity conditions on aa', natural modes of the layer are obtained as follows,

$$v_o = A\cdot\sin((2n+1)\pi z/2H)\cdot(\tan k_o l_o\cdot\cos k_o x + \sin k_o x)\cdot\exp(i\omega t)$$
$$v = A\cdot\sin((2n+1)\pi z/2H)\cdot(\tan k_o l_o\cdot\cos kx + (k_o G_o/kG)\cdot\sin kx)\cdot\exp(i\omega t) \quad (1)$$

where v_o,v:displacements in the y direction,k_o,k:wave numbers,Go,G:rigidity of shear modulus,ω:circular frequency,n:0,1,2....,A:constant,suffix 0 refers to the left part of the model.

Using natural frequencies of the model,the expressions are obtained as next.

$$C_o = Vs_o(1 + ((2n+1)\lambda_o/4H)^2)^{\frac{1}{2}}$$
$$C = Vs\,(1 + ((2n+1)\lambda/4H)^2)^{\frac{1}{2}} \quad (2)$$

where C_o,C:phase velocities,λ_o, λ:wavelengths.

The above expressions mean phase velocities of Love waves propagating on an elastic layer on a rigid base. Now let re-examine the expression(1). It is well-known that $C_{go}\cdot C_o=Vs_o^2$($Cg\cdot C=Vs^2$),where C_{go} and C_g indicate group velocities(Ref. 4). Taking account of these relations,(1) is reduced to the followings.

$$v_o = B/(\rho_o C_{go}\cdot\cos k_o l_o)\cdot\sin((2n+1)\pi z/2H)\cdot\sin k_o(x + l_o)$$
$$v = B/(\rho C_g\cdot\cos kl)\cdot\sin((2n+1)\pi z/2H)\cdot\sin k(x - l) \quad (3)$$

whereρ_o,ρ :desities,B:constant.

When cos $k_o l_o$=1(cos kl=1),node produces at the discontinuity and the following is obtained.

$$(v_o/v) = (\rho\cdot C_g/\rho_o\cdot C_{go}) \quad (4)$$

The quantity in eq.(4) relates to reflection coefficient and transmission one of Love waves propagating across a discontinuity. When cos $k_o l_o$(cos kl) equals to zero, v_o becomes equal to v.

Now let us examine our experimental results in the light of the above theory. Phase velocities(C) of waves produced on right(B) and left(A) parts of the models divided by the discontunuity were calculated by the same way as the case of the model No.-1. The experimental results and ones calculated by the eq.(2) are shown in Table-3. Good agreement between experimental results and theoretical ones indicates that standing waves produced on the models are surely characterized by Love waves. Under several frequencies,resonant response produces only on one side of the model. Such phemonema indicate that Love wave propagating on one part reflects perfectly at the discontinuity.

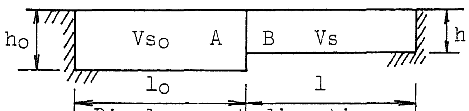
Observed values of (v_o/v) were compared with calculated ones by the eq.(4). Cos $k_o l_o$ and cos kl in the formula(3) are estimated 1 or 0,according to interval between discontinuity and node of wave with longer wave length than the others. As for the model No.-6 and No.-7,height of the left-hand layer and the right-hand one are different each other,unlike the theoretical model. However,almost all experimental results shown in Table-3 agree with theoretical ones. According to experimental and theoretical investigation,amplitude ratio of right-hand part to left one depend on group velocity ratio.

NUMERICAL APPROACH

For the purpose of re-examination on previously obtained result,eigen-value analyses were calculated by use of F.E.M.. Configurations,dimensions and shear wave velocities are shown in Fig.-3 and Table-4. Squarelike elements(1 x 1cm) were used. Natural modes were examined by the same manner as the experimental investigations. On Table-5,natural modes produced on right-hand part(B) and left-hand one(A) of the model were shown. The modes are characterized by fundamental mode of Love wave. It can be seen from Table-5 that amplitude ratios of A to B depend on group velocity ratios,as for model No.1 and No.2. Concerning model No.3, difference of heights between A and B has influence on natural modes.

	Shear wave velocity(m/s)		Dimension (mm)			
No.	V_{so}	V_s	l_o	l	h_o	h
1	2.80	5.60	300		80	
2	2.80		300		80	60
3	2.80		300		80	40

Shear wave velocities and dimensions Table-4



Displacement direction: perpendicular to the plane
Numerical model
Fig.-3

CONCLUSIONS

Vibration analyses were conducted on the models consisting two parts(left-hand:A,right-hand:B) divided by the discontinuity,in order to examine on behaviors of Love waves propagating across the discontinuity of the models. As results,amplitude ratio of A to B depend on group velocity ratios. The latter has influence on reflection and transmission of Love waves propagating through a discontinuity.

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Model (No.2)	Frequency f(Hz)		Wavelength (cm)	cos(k/l)	Phase velocity C	C(Theory)	Group velocity Cg(m/s)	Ratio of amplitude v_0/v	v_0/v(observed)
1	19.5	A	*						
		B	12		8.2	8.1			
2	23.4	A	34	1	8.0	8.0	0.88		
		B	18	1	4.2	3.9	1.8	0.8	1.0
3	27.6	A	16	0	4.4	4.4	1.6		
		B	12	0	3.3	3.1	1.4	1.0	1.0
4	32.8	A	12	1	3.9	3.8	1.8		
		B	8.8	1	2.9	2.7	1.6	1.1	1.0
5	39.4	A	8.7	1	3.4	3.3	2.1		
		B	7.2	1	2.8	2.5	1.9	1.1	1.0
6	46.4	A	6.5	1	3.0	3.0	2.4		
		B	5.3	1	2.5	2.4	1.9	1.3	1.0
(No.3)									
1	16.5	A	*						
		B	42		6.9	6.7			
2	20.3	A	*						
		B	15.5		3.2	3.2			
3	22.7	A	44	0	10	9.8	0.69		
		B	12	0	2.7	2.6	1.3	1.0	1.2
4	24.5	A	26	1	6.4	6.2	1.0		
		B	10.5	1	2.6	2.5	1.3	0.77	0.75
5	27.0	A	19	0	5.1	4.9	1.3		
		B	9.0	0	2.4	2.3	1.4	1.0	1.0
6	29.1	A	15	0	4.4	4.2	1.5		
		B	8.0	0	2.3	2.2	1.5	1.0	1.0
(No.4)									
1	20.4	A	*						
		B	46		9.4	8.6			
2	23.6	A	42	0	9.9	9.7	0.71		
		B	18	0	4.2	4.0	1.1	1.0	1.0
3	25.7	A	22	1	5.7	5.6	1.2		
		B	15	1	3.5	3.5	1.4	0.86	0.8
4	27.9	A	17	1	4.7	4.6	1.5		
		B	12	1	3.3	3.1	1.5	0.83	0.8
(No.5)									
1	16.4	A	*						
		B	46		7.5	7.3			
2	20.0	A	*						
		B	17		3.4	3.2			
3	22.2	A	*						
		B	13		2.9	2.7			
4	24.5	A	32	0	7.8	7.4	0.86		
		B	10.5	0	2.6	2.5	1.3	0.85	0.8
5	26.6	A	20	1	5.3	5.0	1.3		
		B	9.6	1	2.6	2.4	1.4	0.93	1.0
6	29.4	A	17	1	5.0	4.5	1.3		
		B	8.0	1	2.4	2.2	1.4	0.93	1.0
(No.6)									
1	23.8	A	66		16	16			
		B	*						
2	27.1	A	22		6.0	6.0			
		B	*						
3	29.4	A	18		5.3	5.2			
		B	*						
4	31.9	A	15	0	4.8	4.5	1.7		
		B	108	0	35	34	0.23	1.0	1.0
5	33.0	A	13	1	4.3	4.2	1.9		
		B	54	1	18	17	0.45	4.2	5.0
6	34.0	A	12	1	4.2	4.0	1.9		
		B	44	1	15	14	0.55	3.5	5.0
(No.7)									
1	24.6	A	30		7.4	7.2			
		B	*						
2	35.5	A	10		3.6	3.5			
		B	*						
3	42.5	A	8.3	0	3.5	3.2	2.0		
		B	60	0	26	27	0.27	1.0	1.0
4	43.9	A	7.5	1	3.3	3.2	2.1		
		B	30	1	13	14	0.54	4.0	4.0

Experimental results

Table - 3

Model (No.1)	Frequency f(Hz)		Wavelength (cm)	cos(kl)	Phase velocity C(m/s)	Group velocity Cg (m/s)	Ratio of amplitudes v_o/v(theory)	v_o/v(F.E.M.)
1	18.22	A	17.33	0	3.158	2.483		
		B	120.0	0	21.86	1.431	1.0	0.9807
2	20.14	A	15.33	1	3.087	2.539		
		B	55.00	1	11.08	2.831	0.8961	0.8900
3	22.43	A	13.00	0	2.990	2.622		
		B	40.00	0	8.972	3.495	1.0	0.9786
4	24.87	A	11.88	1	2.955	2.654		
		B	32.00	1	7.958	3.941	0.6374	0.654
5	27.79	A	10.40	1	2.890	2.713		
		B	26.00	1	7.225	4.340	0.6251	0.8144
6	30.04	A	9.550	0	2.869	2.733		
		B	22.70	0	6.819	4.599	1.0	0.8536
7	33.00	A	8.571	1	2.828	2.772		
		B	20.00	1	6.600	4.752	0.5833	0.5375
8	35.94	A	7.830	0	2.814	2.786		
		B	17.70	0	6.361	4.930	1.0	0.8752
9	38.16	A	7.330	0	2.797	2.803		
		B	16.33	0	6.232	5.032	1.0	0.7617
10	41.23	A	6.667	1	2.749	2.852		
		B	15.00	1	6.185	5.071	0.5624	0.4928
(No.2)								
1	11.78	A	35.50	1	4.182	1.875		
		B	134.4	1	15.83	0.4953	3.785	2.670
2	12.09	A	33.33	0	4.030	1.946		
		B	83.00	0	10.03	0.7812	1.0	1.035
3	12.56	A	30.80	1	3.868	2.027		
		B	58.00	1	7.285	1.076	1.884	1.872
4	13.29	A	27.75	1	3.688	2.126		
		B	43.20	1	5.741	1.365	1.558	1.655
5	13.94	A	25.50	0	3.555	2.206		
		B	36.00	0	5.018	1.562	1.0	1.174
6	14.74	A	23.50	1	3.464	2.263		
		B	30.67	1	4.521	1.734	1.305	1.515
7	15.59	A	21.60	0	3.367	2.328		
		B	27.00	0	4.209	1.863	1.0	1.175
8	16.42	A	20.00	1	3.284	2.387		
		B	24.00	1	3.941	1.989	1.200	1.333
9	17.36	A	18.40	0	3.194	2.454		
		B	21.60	0	3.750	2.091	1.0	1.233
10	18.25	A	17.33	1	3.163	2.479		
		B	19.75	1	3.604	2.175	1.138	1.227
11	19.24	A	16.14	0	3.105	2.525		
		B	18.08	0	3.475	2.256	1.0	1.251
12	20.18	A	15.20	1	3.067	2.556		
		B	16.87	1	3.386	2.315	1.104	1.186
13	21.19	A	14.25	0	3.020	2.596		
		B	15.71	0	3.329	2.355	1.0	1.255
14	22.17	A	13.57	1	3.008	2.606		
		B	14.75	1	3.270	2.397	1.087	1.143
(No.3)								
1	17.48	A	18.33	1	3.204	2.447		
		B	130.0	1	22.72	0.3450	7.039	4.650
2	17.72	A	18.00	0	3.190	2.458		
		B	77.00	0	13.64	0.5747	1.0	1.50
3	18.05	A	17.60	1	3.177	2.467		
		B	56.4	1	10.18	0.7701	3.205	2.250
4	18.65	A	17.00	1	3.171	2.473		
		B	41.00	1	7.647	1.025	2.410	3.279
5	19.38	A	16.00	0	3.101	2.528		
		B	32.33	0	6.266	1.251	1.0	2.126
6	19.98	A	15.50	0	3.097	2.532		
		B	28.20	0	5.634	1.391	1.0	1.464
7	20.74	A	14.67	1	3.043	2.576		
		B	24.50	1	5.081	1.543	1.669	2.233
8	21.62	A	14.00	0	3.027	2.590		
		B	21.50	0	4.648	1.687	1.0	2.182
9	22.36	A	13.50	1	3.019	2.597		
		B	19.78	1	4.423	1.773	1.465	1.590

Numerical results

Table - 5