Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.II)

3-4-6

WAVE PROPAGATION IN SATURATED SOILS

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SUMMARY

A finite element analysis procedure for analysis of transient response of saturated elastic-plastic soils subjected to dynamic excitation is described. A generalization of Biot's equation of motion is stated in incremental form. Finite element implementation is based on use of one- and two- dimensional linear, bilinear, and biquadratic interpolation between nodal point values. The procedure is verified against exact solutions for two cases of one-dimensional wave propagation in a single material. An illustrative example of application to saturated soils is included.

INTRODUCTION

Biot's (Ref. 1) theory of dynamics of linear elastic saturated soils has been widely used. Several investigators have developed analytical (Refs. 2-4) and numerical (Refs. 5-9) solutions of Biot's equations.

Material non-linearity in two-phase media has been studied by several investigators (Refs. 10-13). Of these, Finn's "engineering approach" was found to be inadequate for dynamic response studies. The highly non-linear elastic-plastic, path-dependent soil behavior has been modelled by using incremental stress-strain relations for the solid phase in Biot's equations and relating the incremental effective stress to the incremental solid strain. Ghaboussi's liquefaction analysis involved computation of response and liquefaction potential of horizontally layered ground subjected to multi-directional excitation. Zieniewicz and his co-workers also studied layered saturated soils and compared the results with those obtained by Finn (Ref. 13). These studies were mainly concerned with predicting dynamic response of saturated media to base acceleration and the propagation characteristics of the stress waves in the two phases of the mixture were not highlighted.

In this paper, Biot's theory has been generalized to allow for non-linear solid stress-strain relations. The fluid was assumed to be linear elastic moving with small relative velocity so that d'Arcy's is applicable. A computer code allowed for bilinear, elastic-plastic as well as a "Cap" model for elastic-plastic work-hardening material behavior. Two-dimensional 4- and 8- noded quadrilateral elements were used. The code was tested against analytical solution for one-dimensional wave propagation in an elastic-plastic solid (Ref. 14,15). This specialization to a single material was accomplished by treating the fluid as a material of zero density and zero stiffness and setting the "drag" equal to zero.

GOVERNING EQUATIONS

Biot's Equations of motion for a fluid-solid mixture in a spatial domain of interest R bounded by S are (Ref. 7):

$$t_{ij} + \rho b_i = \rho \ddot{u}_i + (\bar{\rho}/n)\ddot{w}_i \tag{1}$$

$$\pi^*_{,i} + \frac{\bar{\rho}}{n} b_i = \frac{\bar{\rho}}{n}\ddot{u}_i + \frac{\bar{\rho}}{n^2}\ddot{w}_i + \frac{\mu}{k}\dot{w}_i \tag{2}$$

where t_{ij}, b_i, u_i, w_i are, respectively, components of the total stress, the body force per unit mass of the saturated soil, the soil displacement, and the mean displacement of the pore-water with respect to the soil. ρ, $\bar{\rho}$, n, k, μ are, resepectively, the saturated soil mass density, the partial mass density of the pore-water, the porosity, the permeability and the fluid viscosity. The kinematic relations are

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad ; \qquad \zeta = w_{i,i} \tag{3}$$

and the constitutive relations for linear elasticity are

$$t_{ij} = E_{ijkl}e_{kl} + \alpha M \delta_{ij}(\alpha e_{kk} + \zeta) \quad ; \qquad \pi^* = M(\alpha e_{kk} + \zeta) \tag{4}$$

For non-linear materials, Eqs.(4) will be written as relations between incremental quantities. The associated boundary conditions are (Ref. 7,9)

$$\left.\begin{array}{llll} u_i = \hat{u}_i & \text{on } S_1 \times [0,\infty) \; ; & w_i = \hat{w}_i & \text{on } S_3 \times [0,\infty) \\ t_{ij}n_j = \hat{t}_i & \text{on } S_2 \times [0,\infty) \; ; & \pi^* n_i = \hat{\pi}_i & \text{on } S_4 \times [0,\infty) \end{array}\right| \tag{5}$$

$\hat{u}_i$, $\hat{w}_i$, $\hat{t}_i$, $\hat{\pi}_i$ are values of u_i, w_i, t_i, π_i on S_1, S_3, S_2, S_4 respectively, and n_i are components of the outward normal to S. S_1, S_2 are complementary subsets of S and so are S_3, S_4.

The initial conditions for the problem are

$$u_i(0) = u_{io} \quad ; \quad w_i(0) = w_{io} \quad ; \quad \dot{u}_i(0) = \dot{u}_{io} \quad ; \quad \dot{w}_i(0) = \dot{w}_{io} \quad ; \quad \text{on } R \tag{6}$$

Finite element discretization of the linear problem stated above leads to the matrix set of equations of the form (Ref. 3,6,7)

$$\begin{bmatrix} M_{ss} & M_{sf} \\ M_{fs} & M_{ff} \end{bmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{w} \end{Bmatrix} + \begin{bmatrix} C_{ss} & 0 \\ 0 & C_{ff} \end{bmatrix}\begin{Bmatrix} \dot{u} \\ \dot{w} \end{Bmatrix} + \begin{bmatrix} \bar{K}_{ss} + \bar{K}_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix}\begin{Bmatrix} u \\ w \end{Bmatrix} = \begin{Bmatrix} P_s \\ P_s \end{Bmatrix} \tag{7}$$

where u, w are sets of nodal point values of u_i, w_i. The above equations can be symbolically written as

$$Mv + Cv + K^*v + K^{**}v = P \qquad \text{on } R \times [0,\infty) \tag{8}$$

Here $\bar{K}_{ss}$ and $\bar{K}_{ss}$ are, respectively, the effective soil stiffness and the effect of the fluid pressure on the total soil stiffness. v represents the set of nodal displacements of the soil and the relative displacements of the fluid. M, C, K^*, K^{**}, P represent matrices in Eq.(7).

INCREMENTAL FORMULATION

For a nonlinear problem, the dynamic balance equations may be written in the general form

$$F_I(t) + F_D(t) + F_S^*(t) + F_S^{**}(t) = P(t) \tag{9}$$

where F_I, F_D are respectively, the inertial and the damping force vectors, F_s^* represents the internal resisting force corresponding to effective stress in the soil and F_s^{**} is the force arising as the interaction of fluid and soil (Quantity M in Eq.(4)). $P(t)$ is the load vector representing applied forces and constraints, including those at the boundaries.

A short time Δt later the equation would be

$$F_I(t+\Delta t) + F_D(t+\Delta t) + F_S^*(t+\Delta t) + F_S^{**}(t+\Delta t) = P(t+\Delta t) \tag{10}$$

Assuming that

$$F_I(\tau) = M(\tau)\ddot{v}(\tau) \quad ; \quad F_D(\tau) = C(\tau)\dot{v}(\tau) \quad ; \quad F_S(\tau) = K(\tau)v(\tau) \tag{11}$$

and using Taylor series expansion about t to approximate Eq.(10), we obtain

$$M(t)\Delta\ddot{v}(t) + \Delta M(t)\ddot{v}(t) + C(t)\Delta\dot{v}(t) + \Delta C(t)\dot{v}(t) + K^*(t)\Delta v(t) + \Delta K^*(t)v(t)$$

$$+ K^{**}\Delta v(t) + \Delta K^* v(t) = P(t+\Delta t) - P(t) \tag{12}$$

Using equilibrium correction approach and assuming ΔM, ΔC, ΔK linear in $\Delta\ddot{v}$, Δv and Δv respectively,

$$\breve{M}(t)\Delta\ddot{v}(t) + \breve{C}(t)\Delta\dot{v}(t) + [\breve{K}^*(t) + \breve{K}^{**}(t)]\Delta v(t)$$

$$= P(t+\Delta t) - M(t)\ddot{v}(t) - C(t)\dot{v}(t) - [K^*(t) + K^{**}(t)]v(t) \tag{13}$$

$\breve{M}$, $\breve{C}$, $\breve{K}$ are not known beforehand. Therefore, an iterative process is required at each step. Standard time-domain integration procedures can be used to evaluate the incremental quantities.

APPLICATIONS

The procedure was implemented in a finite element code with a modular structure so that various stress-strain formulations could be introduced with ease. The computer program was used to solve two problems of one-dimensional wave propagation for which exact solutions are available. Fig.1 shows the finite element representation of a laterally restrained soil column subjected to a Heaviside step function load at one end and the bilinear representation of material behavior used by Belytschko (Ref. 14,15). Fig.2 shows a comparison of stress history obtained using the numerical procedure with the exact solution. Fig.3 shows the impulse loading used by Wood (Ref. 16). For Von Mises material the comparison of response is given in Figs.4 to 8. The procedure was applied to a fluid-saturated soil with the soil skeleton behavior following Singh's (Ref. 17) Cap model. Response to a heaviside step function loading of intensity 2.0 is given in Figs.9 and 10. Fig.9 shows soil stress profiles at $t = 1.6$, 4.0, and 8.0. Fig.10 shows the pore-water pressure profile at the same time steps.

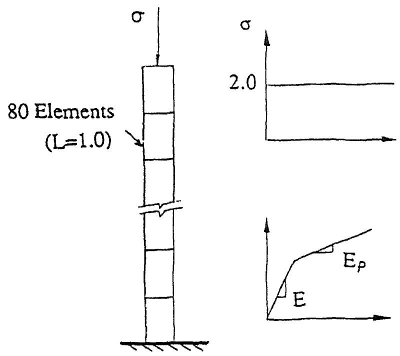

Fig. 1 Soil Column Under Step Load

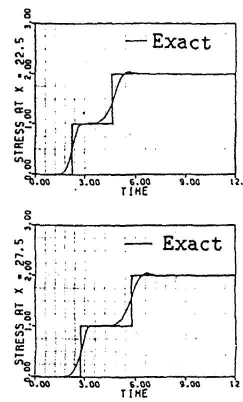

Fig. 2 Stress History

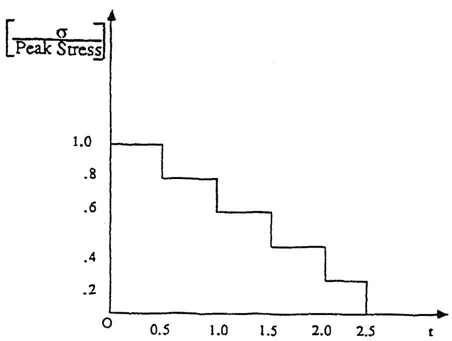

Fig. 3 Short Duration Stress Pulse

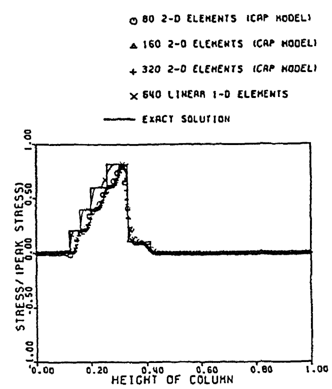

Fig. 4 Stress Profile at t = 4

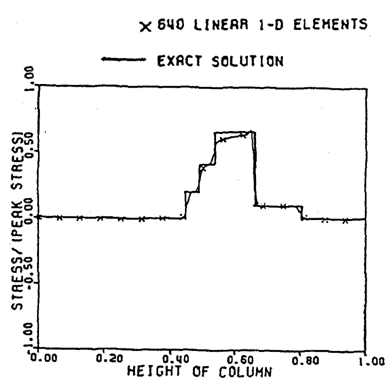

Fig. 5 Stress Profile at t = 8

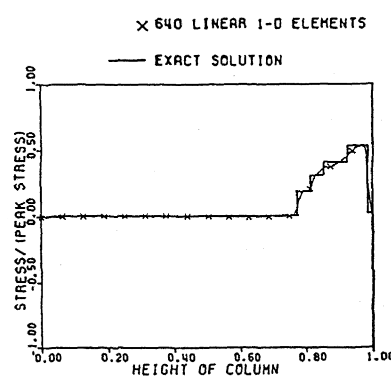

Fig. 6 Stress Profile at t = 12

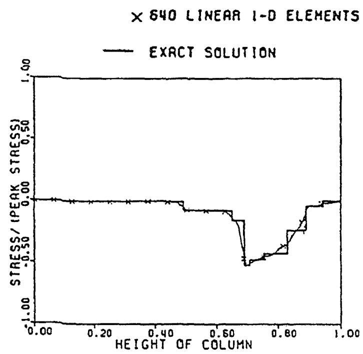

Fig. 7 Stress Profile at t = 16

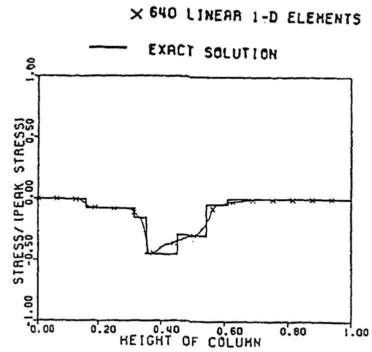

Fig. 8 Stress Profile at t = 20

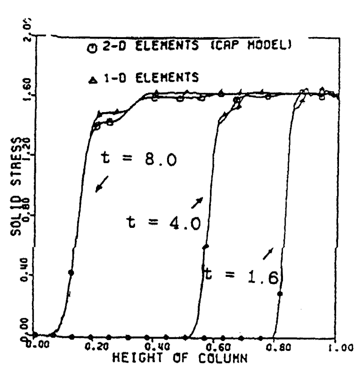

Fig. 9 Solid Stress Profiles at t = 1.6, 4.0 and 8.0

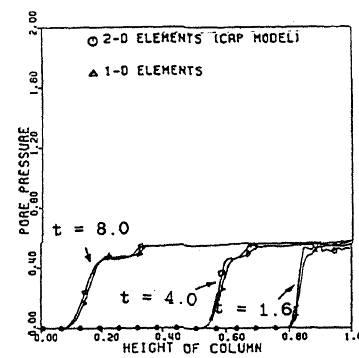

Fig. 10 Pore Pressure Profiles at t = 1.6, 4.0 and 8.0

DISCUSSION

A simple procedure for direct time-integration of equations of motion of saturated soils with non-linear soil behavior is described. Much further work needs to be done to make the method general and applicable to two- and three- dimensional transient response problems.

ACKNOWLEDGEMENTS

The work presented herein is part of a research program supported by the Air Force Office of Scientific Research, Air Force Systems Command USAF under Grant No. AFOSR-83-0055. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation herein. The Ohio State University Instruction and Research Computer Center provided the computational facilities.

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