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PREDICTION METHOD OF PHASE VELOCITY OF TRAVELING SEISMIC WAVES CAUSING CRITICAL RELATIVE DISPLACEMENT

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SUMMARY

The present investigation is concerned with a method to predict the apparent phase velocity of horizontally traveling seismic waves along the surface of the base rock in advance of seismic analyses, which causes critical relative displacement in a design of large structures. In order to obtain the prediction method, the conception of the traveling seismic waves is verified through the solutions of wave equations. The ratio of the relative displacement between any two points along the ground surface to the base motions is derived from the solutions. The method is applied to the seismic analyses of a bridge.

INTRODUCTION

Seismic analyses of structures are generally based on the assumption that the base rock motions are due to vertically incident S- or P-waves. This assumption is derived from the concept of the Snell's law, that is, as the seismic waves becomes nearer the ground surface, the phase velocity usually becomes lower, and moreover the angle of incidence becomes smaller. However, this assumption is incorrect, in case that the hypocenters are near the observation point, or the boundary of soil layers is not horizontal (Ref. 1). If seismic waves are inclined, the phase difference occurs along the ground surface, and the structures built on the ground are affected. A large and rigid foundation would move less than motions of the free field (Ref. 2). Meanwhile, in case of wide-span structures like a bridge, larger relative displacements are usually generated due to inclined waves than due to vertically incident waves (Ref. 3).

The analyses of structures due to the inclined incident seismic waves are generally taken with F.E.M., in which the inclined seismic waves are translated into the horizontally traveling seismic waves (Ref. 4), for the restriction of the boundary conditions at the bottom of the model. The apparent phase velocity of the horizontally traveling seismic waves is related to the angle of incident waves, which is the most important for these kinds of analyses. However, the velocity is not selected upon certain reasons, for the lacking of seismic observation data.

In this paper, first of all, the concept of the traveling seismic waves is verified, using theoretical methods with elastic wave equations (Ref. 5). Secondary, relation between the relative displacement at a certain distance along the ground surface and the apparent phase velocity of the traveling seismic waves is defined with the analyses above mentioned. The method to predict the phase velocity causing the critical relative displacements in a seismic design is formulated with these definitions.

The method is applied for the seismic analyses of a certain bridge. The analyses are taken with F.E.M., after the critical velocity is predicted with taking the above method. The results of the analyses have validated that the prediction method is very effective, especially for the relatively uniform soil layers.

DISPLACEMENTS DUE TO HORIZONTALLY TRAVELING SEISMIC WAVES

It is not easy to input the inclined waves to the base rock in the analyses of F.E.M., so it is necessary to transfer the inclined waves to the horizontal and vertical components of waves along the base rock. If this transference is accomplished, it becomes possible to define the amplification factors of the ground as the ratio of the amplitude on the ground surface to that of the waves along the base rock, in the same way which is usually used in analyses due to vertically incident waves. There are three components of wave motions. Two of them are orthogonal components of horizontal motions, and the other is vertical motions, which are represented as SH-waves, horizontal component of (SV+P)-waves, and vertical component of (SV+P)-waves, respectively. The model of a soil layer for the theoretical analyses is shown in Fig. 1, in which z=0 is the surface of the base rock, and z=H is the free surface.

Displacements due to Horizontally Incident SH-waves The potentials of incident and reflected SH-waves are written in the form, respectively

$$\Psi_0=B_0e^{i(pt-s_2z-qx)},\quad \Psi'_0=B'_0e^{i(pt+s_2z-qx)} \qquad \ldots (1), (2)$$

and the potentials in the soil layer is

$$\Psi_1=B_1e^{i(pt-s_1z-qx)}+B'_1e^{i(pt+s_1z-qx)} \qquad \ldots (3)$$

If displacements within a soil layer, and within a base rock are assigned v_1 and v_2 respectively, these are expressed as

$$v_1=-\frac{\partial\Psi_1}{\partial z},\quad v_2=-\frac{\partial(\Psi_0+\Psi'_0)}{\partial z} \qquad \ldots (4), (5)$$

The boundary conditions are

$$z=0\ ;\ v_1=v_2,\ \frac{\partial v_2}{\partial z}=0 \qquad \ldots (6), (7)$$

$$z=H\ ;\ \frac{\partial v_1}{\partial z}=0 \qquad \ldots (8)$$

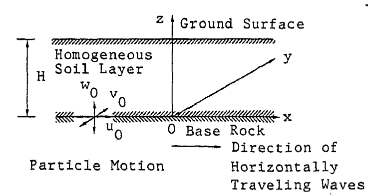

Fig. 1 Model for Theoretical Analyses

Substituting (4) and (5) to (6) through (8), the constants B_0, B'_0, B_1 and B'_1 are determined, and then if the incident SH-wave is assigned v_0, the amplification factor is obtained:

$$\frac{v_1}{2v_0}=\frac{e^{-is_1(H-z)}+e^{is_1(H-z)}}{e^{-is_1H}+e^{is_1H}} \qquad \ldots (9)$$

where,

$s_1^2 = q^2(c^2/V_s^2 - 1)$, $c = p/q$, $p = 2\pi f$, c; phase velocity, V_s; S-wave velocity

Displacements due to Horizontal Component of Incident (SV+P)-waves The potentials of incident and reflected SV-waves are, respectively

$$\Phi_0=A_0e^{i(pt-r_2z-qx)},\quad \Phi'_0=A'_0e^{i(pt+r_2z-qx)} \qquad \ldots (10), (11)$$

and the waves in the soil layer can be expressed as

$$\Phi_1=A_1 e^{i(pt-r_1z-qx)}+A_1' e^{i(pt+r_1z-qx)} \qquad \text{... (12)}$$

The potentials of incident and reflected SV-waves, and those in the soil layer are similarly expressed as (1), (2) and (3), respectively. If the horizontal and vertical displacements in a single soil layer are assigned u_1 and w_1 respectively, and those within the substratum are assigned u_2 and w_2 respectively, the relations between these waves and the potentials are

$$u_1=\frac{\partial\Phi_1}{\partial x}-\frac{\partial\Psi_1}{\partial z},\ w_1=\frac{\partial\Phi_1}{\partial z}+\frac{\partial\Psi_1}{\partial x},\ u_2=\frac{\partial(\Phi_0+\Phi_0')}{\partial x}-\frac{\partial(\Psi_0+\Psi_0')}{\partial z},\ w_2=\frac{\partial(\Phi_0+\Phi_0')}{\partial z}+\frac{\partial(\Psi_0+\Psi_0')}{\partial x}$$

... (13), (14), (15), (16)

The boundary conditions can be expressed as

$$z=0;\ u_1=u_2,\ w_1=w_2=0,\ \frac{\partial w_2}{\partial x}+\frac{\partial u_2}{\partial z}=0 \qquad \text{... (17), (18), (19)}$$

$$z=H;\ \lambda_1\left(\frac{\partial u_1}{\partial x}+\frac{\partial w_1}{\partial z}\right)+2\mu_1\frac{\partial w_1}{\partial z}=0,\ \frac{\partial w_1}{\partial x}+\frac{\partial u_1}{\partial z}=0 \qquad \text{... (20), (21)}$$

Substituting (12) through (16) into (17) through (21), and then if horizontal component of incident (SV+P)-wave is assigned u_0, the horizontal and vertical components of normalized amplification factors are obtained, respectively, as

$$\frac{u_1}{2u_0}=\left\{q^2\left(e^{-ir_1z}-\frac{X_1}{X_2}e^{ir_1z}\right)+s_1\left(\frac{X_3}{X_4}qe^{-is_1z}-X_5e^{is_1z}\right)\right\}/X_6 \qquad \text{... (22)}$$

$$\frac{w_1}{2u_0}=\left\{r_1q\left(e^{-ir_1z}+\frac{X_1}{X_2}e^{ir_1z}\right)-q\left(\frac{X_3}{X_4}qe^{-is_1z}+X_5e^{is_1z}\right)\right\}/X_6 \qquad \text{... (23)}$$

where,

$$s_1^2=q^2\left(\frac{c^2}{V_s^2}-1\right),\ r_1^2=q^2\left(\frac{c^2}{V_p^2}-1\right)$$

$$X_1=2r_1QS_1\cdot SH_1+q(P_1\cdot QS_1-RM_1\cdot SH_1)e^{-i(r_1+s_1)H}-q(P_1\cdot QS_1+RM_1\cdot SH_1)e^{-i(r_1-s_1)H}$$

$$X_2=-2r_1QS_1\cdot SH_1+q(P_1\cdot QS_1+RM_1\cdot SH_1)e^{i(r_1-s_1)H}-q(P_1\cdot QS_1-RM_1\cdot SH_1)e^{i(r_1+s_1)H}$$

$$X_3=q\left(e^{-ir_1H}-\frac{X_1}{X_2}e^{ir_1H}\right)P_1+r_1e^{is_1H}\left(1+\frac{X_1}{X_2}\right)SH_1,\ X_4=\left(e^{-is_1H}+e^{is_1H}\right)q\cdot SH_1$$

$$X_5=r_1+r_1\frac{X_1}{X_2}-q\frac{X_3}{X_4},\ X_6=(q^2-s_1r_1)-(q^2+s_1r_1)\frac{X_1}{X_2}+2s_1q\frac{X_3}{X_4}$$

$$P_1=\lambda_1(q^2+r_1^2)+2\mu_1r_1^2,\ SH_1=2\mu_1qs_1,\ RM_1=2\mu_1qr_1,\ QS_1=\mu_1(q^2-s_1^2)$$

Displacements due to Vertical Component of Incident (SV+P)-waves Substituting (12) through (16) to the boundary conditions

$$z=0;\ u_1=u_2=0,\ w_1=w_2,\ \frac{\lambda_2}{\mu_2}\left(\frac{\partial u_2}{\partial x}+\frac{\partial w_2}{\partial z}\right)+2\frac{\partial w_2}{\partial z}=0 \qquad \text{... (24), (25), (26)}$$

and also (20) and (21), and then if vertical component of incident (SV+P)-wave is assigned w_0, we can define horizontal and vertical components of normalized amplification factors, respectively, as

$$\frac{u_1}{2w_0}=\left\{s_1q\left(e^{-ir_1z}-\frac{X_7}{X_8}e^{ir_1z}\right)+s_1\left(\frac{X_9}{X_{10}}s_1e^{-is_1z}-X_{11}e^{is_1z}\right)\right\}/X_{12} \qquad \text{... (27)}$$

$$\frac{w_1}{2w_0}=\left\{r_1 s_1\left(e^{-ir_1 z}+\frac{X_7}{X_8}e^{ir_1 z}\right)-q\left(\frac{X_9}{X_{10}}s_1 e^{-is_1 z}+X_{11}e^{is_1 z}\right)\right\}/X_{12} \qquad \cdots (28)$$

where,

$$X_7=2q\cdot SH_1(q^2-s_1{}^2)+s_1\{2qr_1\cdot SH_1+P_1(q^2-s_1{}^2)\}e^{-i(r_1-s_1)H}$$
$$-s_1\{2qr_1\cdot SH_1-P_1(q^2-s_1{}^2)\}e^{-i(r_1+s_1)H}$$

$$X_8=2q\cdot SH_1(q^2-s_1{}^2)-s_1\{2qr_1\cdot SH_1-P_1(q^2-s_1{}^2)\}e^{i(r_1+s_1)H}$$
$$+s_1\{2qr_1\cdot SH_1+P_1(q^2-s_1{}^2)\}e^{i(r_1-s_1)H}$$

$$X_9=s_1\left(e^{-ir_1H}-\frac{X_1}{X_2}e^{ir_1H}\right)P_1+qe^{is_1H}\left(1-\frac{W_1}{W_2}\right)SH_1,\ X_{10}=(-e^{is_1H}+e^{-is_1H})s_1\cdot SH_1$$

$$X_{11}=q-\frac{X_7}{X_8}+\frac{X_9}{X_{10}}s_1,\ X_{12}=-(q^2-s_1r_1)+(q^2+s_1r_1)\frac{X_7}{X_8}-2s_1q\frac{X_9}{X_{10}}$$

Relative Displacements   The relative displacements between any two points are given by

$$(u_r,\ v_r,\ w_r)=2(u_1,\ v_1,\ w_1)\sin(\pi\cdot\frac{L}{c}\cdot f) \qquad \cdots (29)$$

where v_r, u_r and w_r are the relative displacements at a distance L due to incident SH-waves, horizontal component of (SV+P)-waves and vertical component of (SV+P)-waves, respectively.

Examples of Normalized Amplification Factors and Relative Displacements   The normalized amplification factors along the ground surface in case of Poisson's ratio ν=0.25 are shown for several parameters in Figs. 2 through 6. The normalized relative displacements along the ground surface due to incident SH-waves, and horizontal component of (SV+P)-waves are shown in Figs. 7 through 9, in case of L/H=4.8 and ν=0.25.

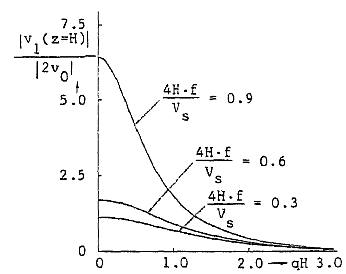

Fig. 2 $|v_1(z=H)|\ /\ |2v_0|$

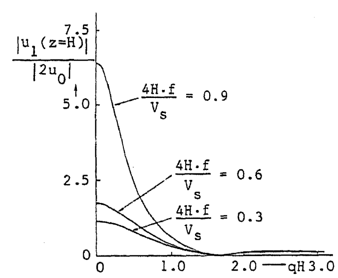

Fig. 3 $|u_1(z=H)|\ /\ |2u_0|$

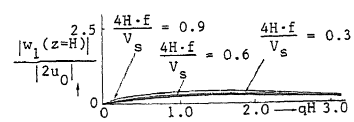

Fig. 4 $|w_1(z=H)|\ /\ |2u_0|$

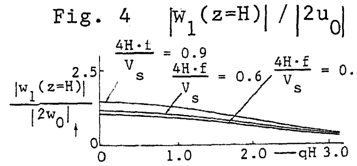

Fig. 5 $|w_1(z=H)|\ /\ |2w_0|$

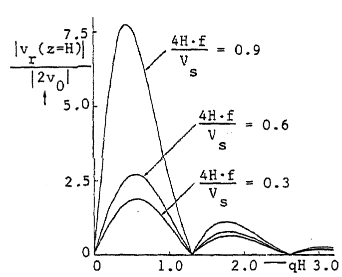

Fig. 7 $|v_r(z=H)|\ /\ |2v_0|$

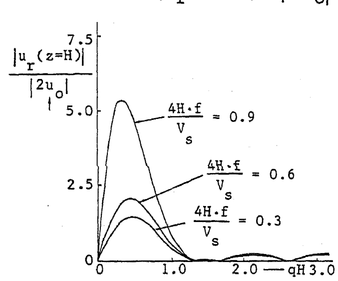

Fig. 8 $|u_r(z=H)|\ /\ |2u_0|$

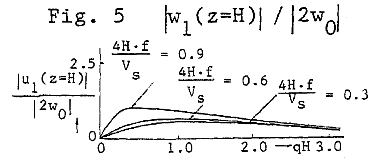

Fig. 6 $|u_1(z=H)|\ /\ |2w_0|$

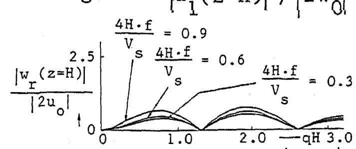

Fig. 9 $|w_r(z=H)|\ /\ |2u_0|$

APPLICATION OF THE METHOD TO SEISMIC ANALYSES

The seismic analyses of a bridge due to the vertically incident waves without phase difference along the base rock (case 1), and also due to the horizontally traveling waves with phase difference (case 2) are performed by the F.E.M. program. In these analyses, the piers are modeled to beam elements, and one end of a bridge is pin-hinged, and the other is roller (see Fig. 10). The typical soil conditions used for the model are shown in Table 1.

The ratios of the absolute AH and relative displacements RH to the input motions U_0 are shown in Figs. 11 and 12, respectively, which are obtained by the theory mentioned above for the averaged soil conditions shown in Table 2. The results analyzed under the same conditions by the F.E.M. are also shown in these figures, which show us the good agreement between the theory and the F.E.M. The mode shapes analyzed by the F.E.M. in case of horizontally traveling incident SH-waves are shown in Fig. 13.

Fig. 12 also shows us the critical phase velocity for the seismic analyses. The relative displacements is maximum at c=200 m/sec, in case of horizontal component of (SV+P)-waves. The apparent phase velocity of the traveling seismic waves should be greater than the shear wave velocity. However, this value is lower than the shear wave velocity at the base rock, so c=350 m/sec, which is shear wave velocity at the base rock, is selected as the critical velocity.

The input wave is shown in Fig. 14, which is obtained in performing the deconvolution method to the recorded waves, and in reducing the maximum acceleration to 100 gals (Ref. 6). The results of the analyses due to vertically and horizontally traveling incident seismic waves are shown in Figs. 15-(a) and (b), respectively. Tables 3 and 4 show us response accelerations at the top of each pier, and relative displacements between two adjacent piers respectively. These tables show us that the maximum response acceleration is generated in the analyses due to the vertically incident seismic waves, however the maximum relative displacement is generated in the analyses due to the horizontally traveling incident seismic waves, and also show us that the method to predict the critical phase velocity is very effective.

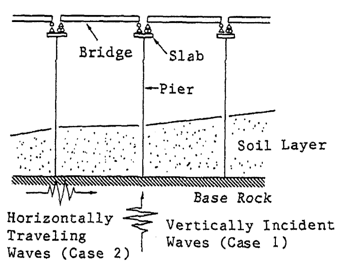

Fig. 10 Model for Seismic Analyses by F.E.M.

Table 1 Site Conditions

Thickness (m)	Soil Profile	Density (tf/m^3)	Vs (m/sec)	Damping Factor
5 - 7	Bay Mud	1.8	150	0.10
2 - 4	Alluvium	1.8	100	0.10
5 - 15	Tertiary	1.9	390	0.10

Table 2 Conditions of Calculations

Soil Properties	S-Wave Velocity	(m/s)	200.
	P-Wave Velocity	(m/s)	1430.
	Density	(tf/m^3)	1.8
	Poisson's Ratio		0.49
Depth of Soil Layer		(m)	15.
Frequency of Wave		(Hz)	0.9
Distance between Two Points		(m)	40.

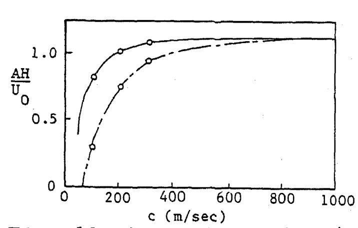

Fig. 11 Comparison of AH/U_0 by Theory and F.E.M.

Note:
o F.E.M.
——— SH-waves (Theory)
—·— Horizontal Component of (SV+P)-waves (Theory)

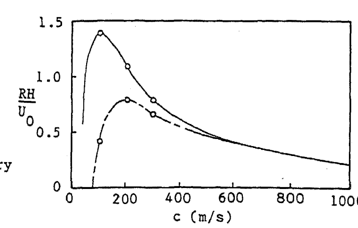

Fig. 12 Comparison of RH/U_0 by Theory and F.E.M.

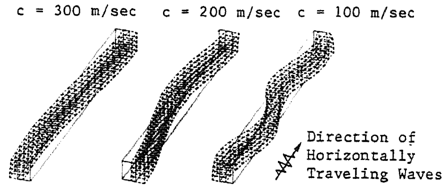

Fig. 13 Mode Shapes due to Traveling SH-waves

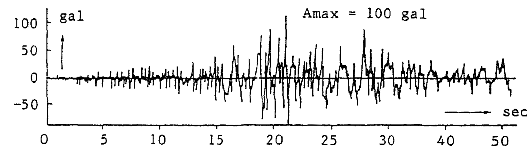

Fig. 14 Incident Seismic Wave

Table 3 Response Acc. at Top of Pier

Pier No.	Response Acc. (gal) Case 1	Case 2
P1	335	150
P5	265	95
P7	305	145
P11	395	155

Table 4 Relative Disp. between Two Adjacent Piers

Pier No.	Relative Disp. (cm) Case 1	Case 2
P1 - P2	2.2	2.7
P4 - P5	0.6	1.0
P6 - P7	0.0	1.1
P9 - P10	0.5	0.9

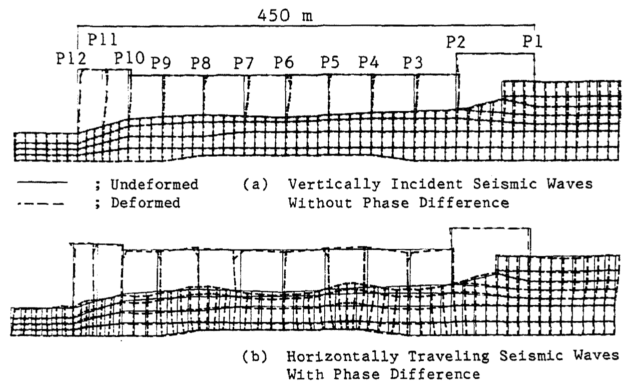

Fig. 15 Deformed Shapes due to Seismic Waves (a) Case 1, (b) Case 2

CONCLUSIONS

The method to easily predict the phase velocity of the horizontally traveling waves along the base rock is formulated, which may be critical in the seismic designs. The formulated method is made just by theoretical wave equations, so the formula may be improved by investigating the behavior of the ground with the seismic data which may be obtained by array observations.

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REFERENCES

1. Dezfulian, H. and Seed, H. B., "Response of Nonuniform Soil Deposits to Travelling Seismic Waves," Journal of the Soil Mechanics and Foundation Division, ASCE, SM1, Proc. Paper 7808, 1971.
2. Yamahara, H. and Shioya, K., "A Study on the Filtering Effect of Foundation Stab Based on Observational Records," Transactions of AIJ, No. 270, 1978.
3. Yamada, Y., Takemiya, H., Kawano, K. and Hirano, A., "Earthquake Response Analysis of High-Elevated Multi-Span Continuous Bridge on Dynamic Soil Structure Interaction," Proc. of J.S.C.E., No. 328, 1982.
4. Udaka, T., "Analysis of Response of Large Embankments to Traveling Base Motions," Ph.D. Dissertation, Department of Engineering, University of California, Berkeley, 1975.
5. Sezawa, K. and Kanai, K., "Damping in Seismic Vibrations of a Surface Layer due to an Obliquely Incident Disturbance," BERI, No. 14, 1935.
6. Kurita, E., Fukuhara, T. and Noda, S., "Strong-Motion Earthquake in Port Areas," Technical Note of the Port and Harbor Research Institute, Ministry of Transport, Japan, No. 458, 1983.