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EFFECT OF DISTORTION OF SEISMIC WAVES ON GROUND STRAIN

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SUMMARY

The objective of this study is to develop a theory for the estimation of the spectra of the ground strain and the relative displacement between two points on the ground's surface during earthquakes when not only propagation but also distortion of seismic waves is considered. The power spectral density function of the seismic wave at each place is assumed to be the same. However, the correlation between the waves is assumed to decrease with an increase in the separation distance. It is then shown that the spectrum of the ground strain displays the predominance of lower frequencies than that of the ground velocity when the distortion is considered.

INTRODUCTION

In the case of structures with a great horizontal extent, such as bridges, tunnels and pipelines, the spatial variation of seismic ground motion significantly affects their earthquake responses. Therefore, in the earthquake-resistant design of these structures, consideration shall be given to the difference in seismic ground motion between each place along the structure. Such a difference is represented by the ground relative displacement or by the ground strain. And it is important to estimate precisely such values during future earthquakes. Although the problem of estimating the ground strain from the earthquake record has been studied by several investigators, most previous studies are based on the assumption that the wave form is not modified during the passage of the wave; i.e., the ground displacement in the x direction at a point x on the soil surface and at time t is assumed to be written as

$$u(x,t)=f(x-ct) \qquad (1)$$

in which the structure is considered to be lying along the x axis, and c is the apparent velocity of the wave as observed on the free-surface and along the x direction. Partial differentiation of this equation with respect to x or t yields the longitudinal strain as

$$\varepsilon = V/c \qquad (2)$$

in which V is the velocity of a soil particle. This equation is often used in earthquake-resistant design of underground structures. However, it should be noted that this equation is accurate only when the seismic waves propagate without distortion. The actual seismic wave, however, is modified as it propagates, and then the wave forms at different points should be considered to be different from each other.

Meanwhile, several investigators (Ref.1) estimated the value of the apparent velocity, c, from the ratio of the maximum recorded amplitude of the ground velocity to that of the strain in the underground pipelines by making use of Eq.(2). However, it should be noticed that this apparent velocity has the same meaning as that described above only when the wave propagates without distortion.

Distortion of seismic waves is considered to be caused by several reasons, however, they may be classified into the following three types:

1. Power spectra of seismic waves at different places are different from each other (Ref.2).
2. Even though such power spectra are equal, the correlation coefficients between motions at different places are not unity, and such processes are somewhat independent from each other.
3. Even though the power spectra are equal, and the correlation coefficients are unity, the phase velocity is a function of frequency, and then the waves are dispersive, as is the case of the surface wave.

In this paper, only the effects of distortion of type 2 are studied. And the ground relative displacement and the ground strain obtained when the distortion of the wave is considered are compared with those when it is neglected.

In what follows the seismic wave form is assumed to be of a stationary random process, and to be expressed as a function of both time and place. And the power spectral density function of the seismic wave at each place is assumed to be the same. However, the correlation between the waves is assumed to decrease with an increase in the separation distance between the two points, and to be given in terms of the cross-spectral function characterized by the associated parameter α . The spectra of the relative displacement and the average strain are then formulated, and their characteristics such as the predominant frequency are examined.

DEFINITION OF CORRELATION AND SPECTRAL DENSITY FUNCTIONS

Let the ground displacement at a point x on the ground surface and at time t be expressed as $u(x,t)$. The autocorrelation function is defined by

$$R_T(\tau)= E[u(x,t)u(x,t+\tau)] \qquad (3)$$

in which τ is the time lag, and E[] is the mean value over time t and place x. Another autocorrelation function can be defined by

$$R_X(x_0)= E[u(x,t)u(x+x_0,t)] \qquad (4)$$

in which x_0 is the separation distance between the two places. The displacement cross-correlation function is defined by

$$R_{XT}(x_0,\tau)= E[u(x,t)u(x+x_0,t+\tau)] \qquad (5)$$

The frequency spectrum $S_T(\omega)$ and the wave number spectrum $S_X(k)$ can be derived from the functions, $R_T(\tau)$ and $R_X(x_0)$, respectively, by Fourier transformation. And the cross-spectral density function is defined by

$$S_{XT}(x_0,\omega)= \frac{1}{2\pi} \lim_{\bar{T}\to\infty} \int_{-\bar{T}}^{\bar{T}} R_{XT}(x_0,\tau)\, e^{-i\omega\tau}\, d\tau \qquad (6)$$

Each of these three spectra is developed in terms of the two-sided forms.

HORIZONTALLY PROPAGATING WAVES WITH DISTORTION

When the seismic waves recorded at two places are compared, larger differences between the two wave forms are, in general, observed for larger separations and at higher frequencies (Ref.3). Therefore, the degree of wave distortion is assumed to be represented by a function of the product of the circular frequency, $|\omega|$, and the time of wave propagation between the separation distance, $|x_0|/c$. And by making use of such a function $A(|\omega||x_0|/c)$, which is frequently called the coherence function, the cross-spectral density function $S_{XT}(x_0,\omega)$ is expressed as

$$S_{XT}(x_0,\omega)= S_T(\omega)\exp(-i\omega\, x_0/c)A(|\omega||x_0|/c) \tag{7}$$

To simplify the mathematical development, the coherence function is assumed in this study to be given by the following expression based on the latest relevant research results (Ref.3).

$$A(|\omega||x_0|/c)= \exp[-\alpha|\omega||x_0|/(2\pi c)] \tag{8}$$

in which α is a parameter pertaining to the degree of distortion, and, hereafter, it will be referred to as the "distortion constant." Ishii (Ref.3) showed that this value ranges between 1.3 and 2.5 by analyzing recorded strong earthquake ground motions. An inverse Fourier transformation of Eq.(7) gives the cross-correlation function as follows.

$$R_{XT}(x_0,\tau)=\int_{-\infty}^{\infty} S_T(\omega)A\left(\frac{|\omega||x_0|}{c}\right)e^{i\omega(\tau-x_0/c)}\,d\omega \tag{9}$$

AUTOCORRELATION FUNCTIONS AND POWER SPECTRA OF GROUND STRAIN

The relative displacement is defined by

$$u_R(x,x_0,t)= u(x+x_0,t)-u(x,t) \tag{10}$$

in which $u(x,t)$ and $u(x+x_0,t)$ are the displacements at time t and at places x and $x+x_0$, respectively. And the average strain is defined by

$$s(x,x_0,t)= [u(x+x_0,t)-u(x,t)]/x_0 \tag{11}$$

Then the autocorrelation function of the relative displacement is obtained by use of Eqs.(5) and (10), leading to

$${}_R R_{XT}(x_0,\tau)= 2R_{XT}(0,\tau)-R_{XT}(x_0,\tau)-R_{XT}(-x_0,\tau) \tag{12}$$

The power spectrum of the relative displacement is derived from Eq.(12) by Fourier transformation.

$${}_R S_{XT}(x_0,\omega)= 2S_{XT}(0,\omega)-S_{XT}(x_0,\omega)-S_{XT}(-x_0,\omega)= 2\{S_T(\omega)-\mathrm{Re}[S_{XT}(x_0,\omega)]\} \tag{13}$$

in which Re[] indicates the real part.

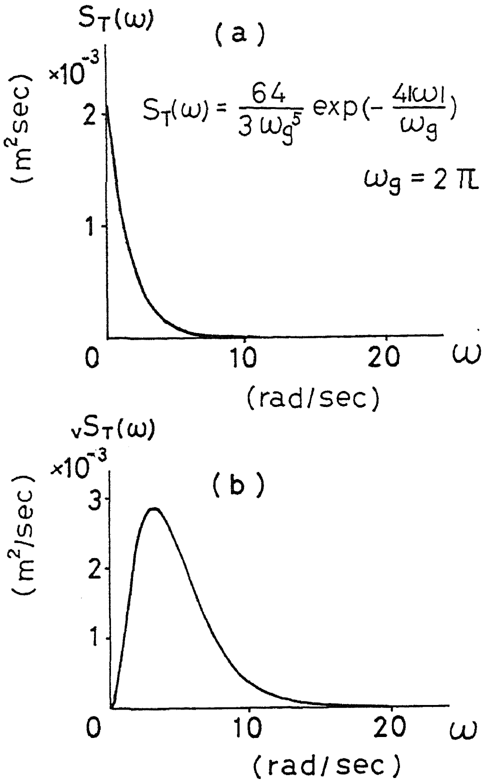
Meanwhile, the autocorrelation function and the power spectrum of the average strain are, respectively, given by

$${}_S R_{XT}(x_0,\tau) = {}_R R_{XT}(x_0,\tau)/ x_0^2 \tag{14}$$

$${}_S S_{XT}(x_0,\omega) = {}_R S_{XT}(x_0,\omega)/ x_0^2 \tag{15}$$

The root mean square (r.m.s.) values of the relative displacement and the average strain are, respectively, given by

$$\{E[u_R^2(x,x_0,t)]\}^{1/2} = [{}_R R_{XT}(x_0,0)]^{1/2} =\left[\int_{-\infty}^{\infty} {}_R S_{XT}(x_0,\omega)d\omega\right]^{1/2}$$
$$= \left[\!\left[2\int_{-\infty}^{\infty}\{S_T(\omega)-\mathrm{Re}[S_{XT}(x_0,\omega)]\}d\omega \right]\!\right]^{1/2} \tag{16}$$



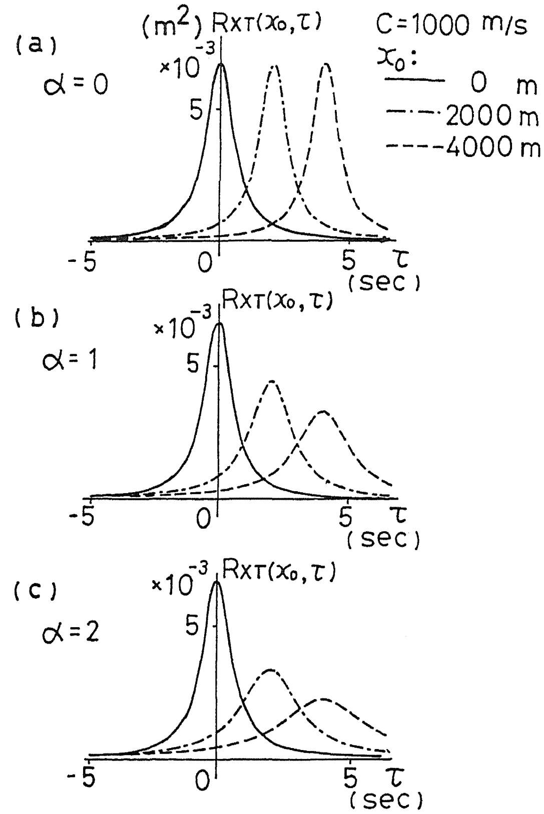


Fig. 1 Power Spectral Density Functions of (a) Displacement and (b) Velocity

Fig. 2 Cross-correlation Functions of Displacement

$$\{E[s^2(x,x_0,t)]\}^{1/2} = [{}_SR_{XT}(x_0,0)]^{1/2} = [\int_{-\infty}^{\infty} {}_SS_{XT}(x_0,\omega)d\omega]^{1/2}$$
$$= \frac{1}{x_0}\left[2\int_{-\infty}^{\infty}\{S_T(\omega)-\mathrm{Re}[S_{XT}(x_0,\omega)]\}d\omega\right]^{1/2} \quad (17)$$

THEORETICAL EXAMINATION OF CHARACTERISTICS OF RELATIVE DISPLACEMENT AND AVERAGE STRAIN

In order to simplify the mathematical development, the frequency spectrum of the ground acceleration, ${}_AS_T(\omega)$, is assumed to be the following function proposed by Goto and Kameda (Ref.4).

$${}_AS_T(\omega) = \omega^4 S_T(\omega) = \frac{64}{3\omega_g}(\omega/\omega_g)^4\exp(-4|\omega|/\omega_g) \quad (18)$$

Figs.1 (a)(b) show the frequency spectra of the ground displacement and velocity, $S_T(\omega)$ and ${}_VS_T(\omega)$, respectively. The predominant frequency of the ground is assumed to be 1Hz ($\omega_g=2\pi$) in Eq.(18), and the r.m.s. value of the ground acceleration of $1m/s^2$ is used for this analysis.

Substitution of Eqs.(8) and (18) into Eq.(9) yields

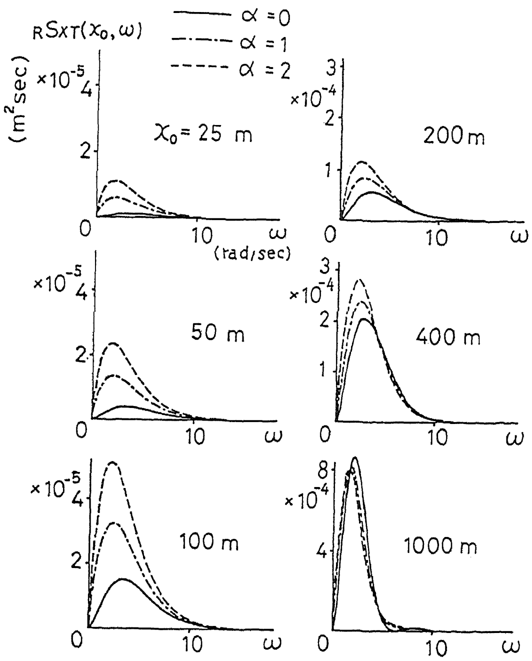


Fig. 3 Spectral Density Functions of Relative Displacement

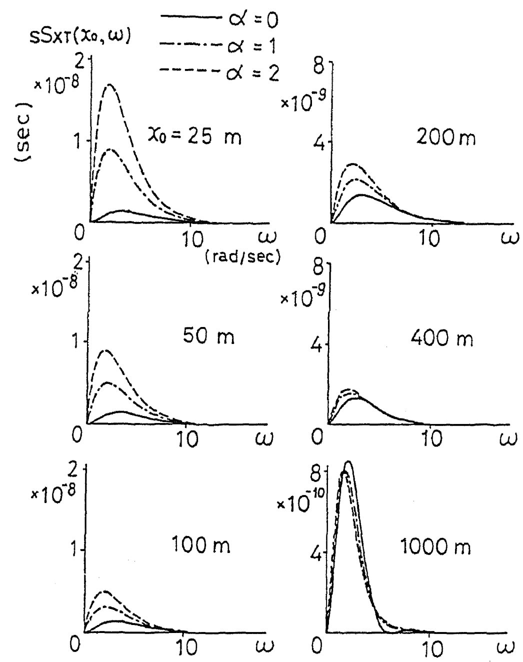


Fig. 4 Spectral Density Functions of Average Strain

$$R_{XT}(x_0,\tau) = \frac{128}{3\,\omega_g^5}\;\frac{\frac{4}{\omega_g} + \frac{\alpha}{2\pi c}|x_0|}{\left(\frac{4}{\omega_g} + \frac{\alpha}{2\pi c}|x_0|\right)^2 + \left(\tau - \frac{x_0}{c}\right)^2} \tag{19}$$

From Eqs.(7),(8),(12),(13) and (15)-(19), the power spectral density functions and the r.m.s. values of the relative displacement and average strain can be written as

$$_RS_{XT}(x_0,\omega) = x_0^2\;\; _SS_{XT}(x_0,\omega)$$

$$= \frac{128}{3\omega_g^5}\exp(-4|\omega|/\omega_g)\left[1-\exp\left(-\frac{\alpha|\omega||x_0|}{2\pi c}\right)\cos\frac{\omega x_0}{c}\right] \tag{20}$$

$$\{E[u_R^2(x,x_0,t)]\}^{1/2} = x_0\;\{E[s^2(x,x_0,t)]\}^{1/2}$$

$$=\left\{\frac{256}{3\,\omega_g^5}\left[\frac{\omega_g}{4} - \frac{\frac{4}{\omega_g} + \frac{\alpha}{2\pi c}|x_0|}{\left(\frac{x_0}{c}\right)^2 + \left(\frac{4}{\omega_g} + \frac{\alpha}{2\pi c}|x_0|\right)^2}\right]\right\}^{1/2} \tag{21}$$

NUMERICAL RESULTS

Calculated values for the correlation functions $R_{XT}(x_0,\tau)$ are shown in Fig.2. The full line curves of Fig.2 are the autocorrelation functions of the ground displacement (the separation x_0=0m), and the chain and dashed line curves are the cross-correlation functions for the separations of 2000m and 4000m, respectively. Fig.2(a) shows the case of waves without distortion (α=0), while Figs.2(b)(c) the cases with distortion (α=1,2). In the former case the maximum value of the cross-correlation function is equal to that of the autocorrelation function, while in the latter case it is smaller than that of the autocorrelation function. The larger the distortion constant or the separation distance is, the smaller the peak value becomes.

Fig.3 shows the spectra of the relative displacement for separations x_0 = 25, 50, 100, 200, 400, 1000m. The full line curves of Fig.3 are the results for the cases without distortion (α =0), and the chain and dashed line curves for the cases with distortion (α=1,2). In the case α =0, the predominant frequency decreases as the separation is increased. Indeed, the predominant frequencies of the relative displacement are

ω = 3.15, 3.14, 3.12, 3.04, 2.80, 2.01 rad/s,

respectively, for separation distances

x_0 = 25, 50, 100, 200, 400, 1000m.

In either case of α = 0, 1 or 2, the spectrum of the relative displacement becomes smaller as the separation is decreased. However, the spectrum for the case α =1 or 2 does not decrease as much as that for the case α =0 particularly at low frequencies.

Fig.4 shows the spectra of the average strain. In the case α =0, as the separation is decreased, the spectrum only becomes slightly larger (attention should be given to the differences in the scale of the ordinate). However, in the case α = 1 or 2, it becomes much larger particularly at low frequencies, and it becomes proportional to that of the displacement (Refs.5,6).

CONCLUSIONS

A theory is developed for estimating the spectra of the ground strain and the relative displacement between two points on the ground's surface during earthquakes when not only propagation but also distortion of seismic waves is considered. The power spectral density function of the seismic wave at each place is assumed to be the same. However, the correlation between the waves is assumed to decrease with an increase in the separation distance. The effect of the distortion of the wave form is remarkable for short separations. The spectrum of the ground strain is proportional to that of the ground velocity when the distortion is neglected. However, when it is considered, the spectrum becomes proportional to that of the displacement and displays the predominance of lower frequencies.

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