Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.II)

3-3-4

SEISMIC RESPONSE OF A SOIL DEPOSIT WITH A LINEAR VARIATION OF THE Vs PROFILE

Leonardo LOJELO[1] and Tito SANO'[2]

[1]Tecno Idro Meteo, Rome, Italy
[2]ENEA-DISP, Rome, Italy

SUMMARY

A closed form solution for the equation of motion of a soil layer having a linear variation of the Vs profile versus depth, is obtained, assuming vertically propagating shear waves. The aforesaid characteristic represents very well real situations on alluvium deposits. A mathematical expression of the amplification function between an homogeneus halfspace and the surface of the soil layer, having the above mentioned characteristic, is determined. Through such expression sensitivity analyses of key parameters can be performed. Finally a comparison between analytical and numerical solution is presented.

INTRODUCTION

The evaluation of the seismic ground motion, expected at the free field surface during an earthquake, is one of the most important step in the seismic design of critical structures (major hazard industrial plants, long span bridges, dams, offshore platform, etc.), in seismic microzonation and in liquefaction analyses. The proposed characteristics of the seismic motion at a site (amplitude frequency content, duration) should be consistent with the seismotectonic enviroment - that is the source mechanism and the wave propagation path - and with the local soil condition. This last is often the most important and a key parameter in order to evaluate the motion at the free field surface at a specific site. The phenomenon is well known and can be studied through the amplification function. The amplification function is the ratio between the amplitude of the motion at the surface of a soil layer over an halfspace and the amplitude of the motion that would occur on an hypothetical outcropping of the same halfspace (Fig. 1). The amplification function of a soil layer is strictly correlated to the variation of the shear wave velocity versus depth (Vs profile). In many cases alluvium deposits are characterized by a Vs profile that can be well described by a linear variation versus depth. Assuming an elastic medium with a linear Vs profile and the simplified hypothesis of vertically propagating shear waves, an analytical solution for the amplification function can be obtained. A closed form solution allows both an easy programming an a personal computer and sensitivity analyses about the influence of the parameters governing the soil amplification phenomenon.

SOLUTION OF THE DYNAMIC EQUILIBRIUM EQUATION

Let's assume a soil layer of depth H, resting over an halfspace (elastic rock), as indicated in Fig. 2, and a vertically propagation of the shear waves. The dynamic equilibrium equation for the soil element shown in Fig. 2 is:

$$\frac{\partial}{\partial z}\left(G \frac{\partial U(z,t)}{\partial z}\right) = \rho \frac{\partial^2 U(z,t)}{\partial t^2} \qquad (1)$$

where G is the shear modulus, function of the soil mass density ρ and the shear wave velocity Vs $(G=\rho V_s^2)$ and $U(z,t)$ is the horizontal displacement.

Assuming ρ=costant, Vs varying linearly according to:

$$V_s = V_{so} \left(1 + \lambda Z/H\right) \qquad (2)$$

(being Vso the shear wave velocity at the surface), and a steady state armonic motion:

$$U(z,t) \quad u(z)\, e^{iwt}$$

$$\frac{\partial^2 U(z,t)}{\partial t^2} = -w^2 \cdot U(z,t) \qquad (3)$$

where w is the circular frequency of the vertically propagating shear waves, the equation (1) can be written:

$$\frac{d^2 u}{d\tau^2} + \frac{du}{d\tau} + \gamma^2 u = 0 \qquad (4)$$

where: $\gamma=(w/V_{so})H/\lambda$, $r=\ln(1+\lambda z/H)$

the solution of equation (4) is:

$$u = \left(A_1\, e^{ip\tau} + A_2\, e^{-ip\tau}\right) \Big/ \sqrt{1+\lambda Z/H} \qquad (5)$$

with: $p=\sqrt{\gamma^2 - 1/4}$

In equation (5) the first member represents the vertically travelling up wave, the second one is the coming back wave. The amplitude of both waves is not costant with depth - as in the case of Vs=costant - but descreases according to the square root of $1+\lambda z/H$. Assigning the boundary condition, that is the shear stresses equal to zero at the surface and the relative displacements equal to zero at the boundary between the layer and the halfspace, the following trascendental equation can be written:

$$tg\,\alpha = 2\alpha/\tau_1 \qquad (6)$$

with $\alpha = p\tau_1 \qquad \tau_1 = \ln(1+\lambda)$

Taking into account the definitions given for p and γ, the vibration modes are obtained from equations (6) and (5):

$$U_n = \frac{1}{\sqrt{1+\lambda z/H}}\left(\cos(p_n\tau) + \frac{1}{2p_n}\sin(p_n\tau)\right) \qquad (7)$$

while the natural frequencies can be well approximated by:

$$f_n = (2n-1)\frac{V_{so}}{4H}\,\frac{\lambda}{2\ln(1+\lambda)}\left(1+\sqrt{1+\frac{8\ln(1+\lambda)}{(2n-1)^2\pi^2}}\right) \qquad (8)$$

Parametric analyses, performed in order to compare equation (8) with the exact solution - obtained from equation (6) - have shown that the error is lover than 10%, even for high values of λ, namely λ =5.

SOIL AMPLIFICATION

The solution (5) of the dynamic equilibrium equation, taking into account the boundary conditions, gives the motion for the soil layer:

$$U(z,t) = A\left[e^{-1/2+ip\tau} + e^{-1/2-ip\tau}\,(ip-1/2)/(ip+1/2)\right]e^{i\omega t} \qquad (9)$$

Assuming vertically propagating shear waves with a costant velocity equal to Vsh, the solution of the dynamic equilibrium equation for the halfspace is, as well known /1/,

$$U_h = A_h\left(e^{i\omega z/V_{sh}} + A'_h\,e^{-i\omega z/V_{sh}}\right)e^{i\omega t} \qquad (10)$$

Assigning the congruency between the displacements and the shear stresses at the interface between the soil layer and the halfspace, it is possible to evaluate the amplitude of the travelling up (Ah) and the coming back (A'h) waves in the halfspace as a function of the amplitude of the waves in the soil layer. Finally, considering that at the outcropping halfspace A'h=Ah, the amplification function can be expressed in modulus, as:

$$|AF| = \frac{\sqrt{1+\lambda}}{\sqrt{\left(\cos(p\tau_1)+\sin(p\tau_1)/2p\right)^2 + \left(q_0(1+\lambda)\gamma\sin(p\tau_1)/p\right)^2}} \qquad (11)$$

where $q_o = V_{so}/\rho_h V_{sh}$ is the impe dance ratio and ρ_h is the mass density of the halfspace. Equation (11) doesn't take into account the material damping of the soil. If the material damping D is introduced considering a complex expression for the shear modulus:

$$G^* = G\,(1 + 2iD) \tag{12}$$

for small values of the material damping ($D \ll 1$), the following approximated expression for the amplification function is obtained from equation (11):

$$|AF| = \frac{\sqrt{1+\lambda}}{\cosh\beta\,\sqrt{R_e^2 + I_m^2}}$$

$$R_e = \cos\alpha + \sin\alpha/2p + \frac{D\gamma^2\cos\alpha\ \mathrm{tgh}\,\beta}{2p^3} - q_o\gamma\frac{1+\lambda}{p}\left(\cos\alpha\ \mathrm{tgh}\,\beta - D\sin\alpha\right)$$

$$I_m = \sin\alpha\ \mathrm{tgh}\,\beta + \frac{1}{2p}\left(\frac{D\gamma^2\sin\alpha}{p^2} - \cos\alpha\ \mathrm{tgh}\,\beta\right) + q_o\gamma\frac{1+\lambda}{p}\left(\sin\alpha + D\cos\alpha\ \mathrm{tgh}\,\beta\right) \tag{13}$$

$$\beta = \alpha\frac{D\gamma^2}{p^2} \qquad \alpha = p\,\tau_1$$

For low frequencies and small material damping ($\alpha D \ll 1$) the equation (13) can be semplified, assuming cosh β =1 and $\mathrm{tgh}\,\beta = \beta$:

$$|AF| = \frac{\sqrt{1+\lambda}}{\sqrt{\left(\cos\alpha + \frac{\sin\alpha}{2p}\right)^2 - \left(q_o\gamma\sin\alpha\,\frac{1+\lambda}{p}\right)^2 + D q_o\gamma\frac{1+\lambda}{p}\left(2\alpha - \sin\alpha + \frac{2\sin^2\alpha}{8p^3}\right)}} \tag{14}$$

Figure 3 shows the comparison between equation (13), (14) and a numerical solution (Shake code /2/), applied at the same test case. No differences pratically exist, betweeen equation (13) and Shake solution, while the approximation of the equation (14) decreases as the frequency increases (that is when the hypothesis $\alpha D \ll 1$ is less effective). The maximum values of |AF| are obtained at the natural frequencies given by equation (6):

$$|AF|_{max} = \left(\sqrt{q_o^2\,(1+\lambda) + 2D\,q_o\,\gamma_n\,\tau_1}\right)^{-1} \tag{15}$$

While the minimum values can be approximated, assigning $\cos\alpha$=1 and sen α =0:

$$|AF|_{min} = \sqrt{1+\lambda} \Big/ \sqrt{1 + 2\,D\,q_0\,\gamma_n\,\tau_1} \qquad (16)$$

If D=0, |AF| min is always greater that one when the shear wave velocity increases with depth ($\lambda \neq 0$), while it is always equal to one when the shear wave velocity is costant with depth ($\lambda = 0$). Figure 4 shows the comparison between a soil layer with material damping and Vs linearly increasing and a "dynamic-equivalente layer" with the same damping, the first natural frequency but Vs=costant. It is the wothwhile to note that the maximum and the minimun values of the amplification function of the soil layer are always greater than those obtained for the amplification function of the "dynamic-equivalent layer". It can be concluded that when the shear wave velocity increases versus depth, the wave amplitude decreases and the radiation damping effect is less than in the case of Vs=costant.

CONCLUSION

Alluvium deposits are, in many cases, well characterized by a linear variation of the shear wave velocity versus depth. The amplification function between an outcropping halfspace and the surface of a soil layer, resting over the halfspace and with Vs linearly increasing, can be obtained in a closed form solution, performing a transformation of the spatial coordinate of the dynamic equilibrium equation. As a consequence of the elastic non-homogeneity, the shear wave amplitude decreases propagating away from the surface. The amplification function shows a reduced effect of th radiation damping, if the non-homogeneous soil layer is compared with an homogeneous dynamic-equivalent layer. The simplified assumption of an homogeneous layer over an halfspace can lead, in many cases, to unconservative results in evaluating the seismic ground motion at a specific site. The proposed closed form solutions for the amplification function and for the natural frequencies of a soil layer with a linearly increasing Vs-profile seem to be more appropriate and equally easy-to-use dealing with soil amplification problems (design of critical structures, seismic microzonation liquefaction analyses).

REFERENCES

1. Roeset, J.M., Soil amplification of earthquakes, Numerical methods in geotechnical engineering, edited by Desay and Cristian, Mc Graw H.U., 1977.
2. Schnabel, P.B., Lysmer, J., Seed, H.B. SHAKE: a computer program for earthquake response analysis of horizontally layered sites, Univ. Calif. rep. EERC 72-12, Berkeley, December 1972.

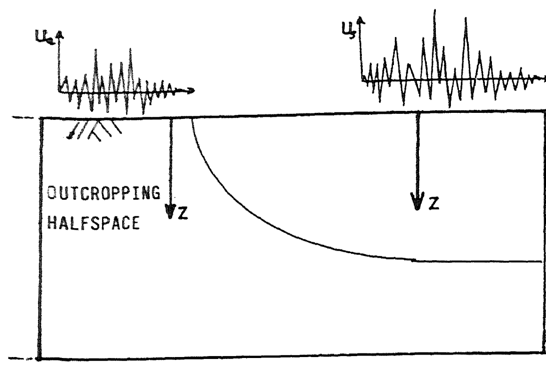

Fig. 1 - The soil amplification problem

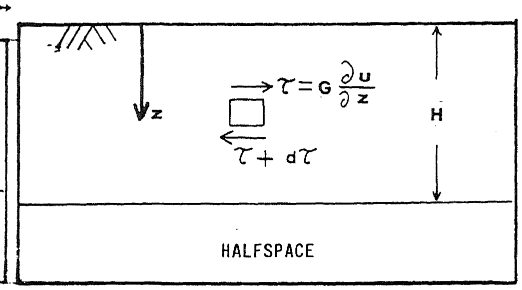

Fig. 2 - The soil layer over the halfspace

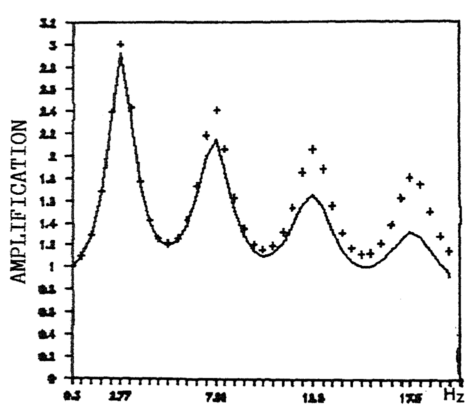

Fig. 3 - Comparison between Shake solution, eq. 13 and eq. 14

- Shake and eq. 13
++ eq. 14

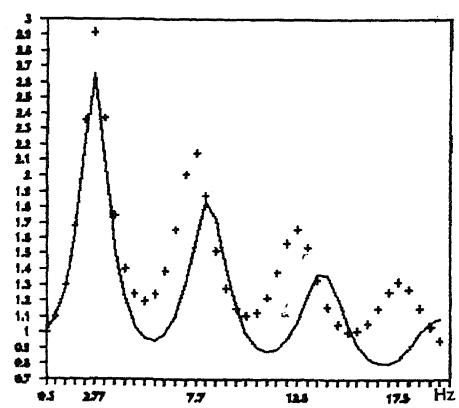

Fig. 4 - Comparison between Vs linearly increasing (+) and Vs costant (-)