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REGRESSIONAL PRINCIPLE AND RELATIONSHIP BETWEEN EARTHQUAKE INTENSITY AND ACCELERATION

Yuxian Hu[1] and Jun-Rong Huo[2]

[1]Institute of Geophysics (IG) and Institute of Engg. Mechanics (IEM), State Seismological Bureau (SSB), Beijing and Harbin respectively, China

[2]IEM, SSB, Harbin, China

SUMMARY

A new regressional principle for multiple random parameters is suggested. The suggested regressional principle together with the currently used ones are tested by the intensity-acceleration conversion relation derived from attenuation relations for single earthquakes. A third parameter of magnitude or distance should be added to the conversion. The suggested principle applies also to other regressional problems in engineering seismology, such as the relations of earthquake magnitude and the fault rupture length, of body-wave and surface-wave magnitude, and of one parameter of ground motion to the other.

INTRODUCTION

The regressional analysis of intensity and peak acceleration is an old topic of study for the past 40 years. After careful studies, many investigators drew negative conclusions of such a functional relationship because of very large scattering of data. Nevertheless, some relations between intensity and acceleration thus obtained have been used in building design code and ground motion assessment because only intensity data are available in the regions. Such conversion relations have been derived for a long time by regressing lga on I (Ref. 1). One reason ever given for this routine principle is that intensity I is given and thus free from error. According to this reasoning, if it is to find intensity for given acceleration, I should be regressed on lga and two regressional relations are obtained from one set of observed data of both I and a (Refs. 2,3,4). Authors try to suggest some new regressional principles which consider the randomness of both I and a simultaneously and to provide a reasoning to support their suggestion.

REGRESSIONAL PRINCIPLES

For the general case of r normalized deterministic variables $X_i(i=1,...,r)$ and m normalized random variables $Y_i(i=1,...,m)$, the regressional function may be written as

$$\sum B_i X_i + \sum C_i Y_i = 0 \qquad (1)$$

and the suggested regressional principle is

$$J = \sum [\sum (\delta_i^p \Delta Y_{ij})^2] = \min. \tag{2}$$

with δ_i = variance of Y_i from randomness, p = the weighting index, and $\Delta Y_{ij} = Y'_{ij} - Y_{ij}$, where Y'_{ij} and Y_{ij} are respectively the jth observed and regressed values of the variable Y_i.

It is generally agreed that the deterministic variables X_i will not participate directly in least squares of Eq.(2), but the participation of random variables Y_i is subjected to question. Different selections of variable $Y_i (i=1,...,m)$ to be included in Eq. (2) result in different regressional principles as given in next section.

REGRESSIONAL RELATIONS BETWEEN INTENSITY AND ACCELERATION

When there are only two random parameters, intensity I and peak acceleration a, after normalization by introducing $Y_1 = (\lg a - \overline{\lg a}) / \sigma_{\lg a}$ and $Y_2 = (I - \overline{I}) / \sigma_I$, where σ^2 is the variance of data, the regressional relation of Eq. (1) becomes

$$Y_1 = KY_2 \quad \text{or} \quad Y_2 = Y_1/K \tag{3}$$

The regressional principles often used or suggested here are as follows.

Method 1 (often used): $J_1 = \sum (Y'_1 - KY'_2)^2 = \min.$, $K = \rho$ (4)

Method 2 (occasionally used): $J_2 = \sum (Y'_2 - Y'_1/K)^2 = \min.$, $K = 1/\rho$ (Refs. 2,5) (5)

Method 3 (suggested here): $J_3 = J_1 + J_2 = \min.$, $K = \text{sign}\,[\rho]$ (6)

Method 4 (suggested here): $J_4 = \lambda^{2p} \cdot J_1 + J_2 = \min.$, $\lambda = \delta_1/\delta_2$, and K is the real root of the following equation (p=2 is used in the present paper)

$$\lambda^{2p} K^3 (K - \rho) + (K\rho - 1) = 0 \tag{7}$$

Method 5: Kendall's principle of maximum likelihood (Ref. 6)

$$K = \left\{ 1 - \lambda^2 + [(1-\lambda^2)^2 + 4\lambda^2 \rho^2]^{1/2} \right\} / (2\rho) \tag{8}$$

Method 6: see the following section.

CHECK OF RESULTS

Method of Checking In order to check the adequacy of the regressional principles, as a standard of check, the conversion relation of intensity and acceleration

$$\lg a = (C_0 - B_0 C_1/B_1) + I\, C_1/B_1 \tag{9}$$

may be obtained from the attenuation laws of intensity and acceleration on the basis of data from the same earthquake

$$I = B_0 + B_1 \lg(R + R_o) \tag{10}$$

$$\lg a = C_0 + C_1 \lg(R + R_o) \tag{11}$$

by eliminating the distance term. Eq. (9) is taken as method 6. For the purpose of checking, authors collected (I, a) data from 14 earthquakes over the world as listed in Table 1, with some results of regression given in Figs. 1 and 2.

Analysis of Results Following conclusions are observed from the analysis of all numerical results listed in Tables 1 and 2.

Firstly, ratio λ of the standard errors δ_1 and δ_2 respectively of Y_1 and Y_2 varies in the range of 0.65 to 1.54 with an average very close to 1.0 ; the randomness of any of the two variables should not be ignored, and the regressional methods 1 and 2 are not adequate.

Secondly, the correlation coefficient ρ of lga and I varies from 0.45 to 0.83. When ρ = 0.45, as given in Table 2 for some Japanese earthquakes, acceleration a/g for Japan intensity V (approximately VIII for the 12 grade scales) is 0.05 by method 1 and 1.0 by method 2, a difference of 20 times.

Thirdly, when the ratio λ is unity, regressional methods 3, 4 and 5 give identical result as that from the attenuation laws (method 6). Since the ratio λ is very close to unity when data from many earthquakes considered together, the suggested method 3 is recommended to substitute the currently used method 1.

CONVERSION OF INTENSITY TO ACCELERATION

First of all, authors do not consider that I-a conversion is a good approach to find acceleration, because acceleration is only one of many independent factors affecting intensity. Compromising with the current approach, the first author has suggested a modified I-a conversion function by adding a third parameter, the magnitude M or distance R (Ref. 7), which can be easily obtained by eliminating the R terms or the M terms from the attenuation laws of intensity and acceleration of the same region, such as Eq's (10) and (11). One result obtained for the Western United States is given in Fig. 3 and Table 3. It can be seen that the peak acceleration can be as high as 1g or 2g for intensity IX and 3g or 4g for intensity X. Such high accelerations are incorrect results from incorrect extrapolation to an unreal focal distance, smaller than the focal depth or even of negative values as shown in Fig. 3 by dotted part of lines.

It is not only realistic but also more reasonable to introduce a third parameter M or R in the conversion relation to consider indirectly other parameter of ground motion, such as spectrum or duration. Because the ground motion of an earthquake of small magnitude at close distance is rich in high frequency content, the peak acceleration is large but the velocity and duration are not large; on the contrary, the ground motion of an earthquake of large magnitude at far distance usually has large duration and velocity but smaller acceleration. A ground motion with high acceleration but small velocity and short duration may correspond to same intensity as a ground motion with low acceleration but high velocity and long duration.

OTHER REGRESSIONAL PROBLEMS IN ENGINEERING SEISMOLOGY

In addition to I-a conversion, other similar problems of regression of multiple random variables also exist in engineering seismology and many other fields, for which the regressional principle suggested here can also apply.

Bolt (Ref. 1) and Mark (Ref. 3) have challenged the adequacy of the routine regressional principle in studying the relation between earthquake magnitude M and the fault rupture length L. The purpose of their studies is to find out possible maximum magnitude from the fault rupture length or the linear dimension of the seismic gap; but when used in seismic hazard analysis, the purpose is to estimate the fault rupture length for known earthquake magnitude. For both cases, the rupture length should be that of the source, not the rupture on ground surface. Fig. 4 gives the results from different regressional methods mentioned in the present paper for a set of China data of magnitude M and linear dimension of the area surrounded by aftershock foci. The differences between methods are also significant.

Ohsaki et al (Ref. 4) have studied relations between two parameters of the ground motion at the same point and the parameters considered include acceleration, velocity, displacement and Housner spectral intensity of the horizontal and vertical components. They used separately methods 1 and 2 and presented both results. Since the correlation coefficients of the parameters they studied are on average greater than those in the present paper, the differences of methods 1, 2 and 3 for their cases should be less than those of the present paper. One example for the relation between peak horizontal displacement D_h and the vertical acceleration A_v is

$$A_v = 15.98\ D_h \text{ (method 1)},\quad 25.71\ D_h \text{ (method 2)},\quad 20.27\ D_h \text{ (method 3)}$$

The differences are also considerable here.

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Table 1 Earthquake and Related Data

No	Earthquake	Data	Country	M_L	H (km)	Epicentral Dist. R (km)	Range of Acceleration (cm/s/s)	Range of Intensity	No of Records	Normalized Acceleration δ_{lga}	Normalized Intensity δ_I	Data σ_{lga}	σ_I
1	San Fernando	71.02.09	USA	6.6	13	9-185	5-1148	IV-VIII	180	0.601	0.707	0.322	0.817
2	Coyote Lake	79.08.06	USA	5.9	6	5- 73	30- 420	V-VII	26	0.592	0.649	0.298	0.769
3	Imperial Valley	79.10.15	USA	6.6	12	1-160	10- 810	IV-VII	70	0.387	0.589	0.447	0.996
4	Livermore	80.01.24	USA	5.5	8	11- 93	5- 260	IV-VI	23	0.748	0.742	0.393	0.824
5	Morgan Hill	84.04.24	USA	6.2	8	5-131	10-1290	IV-VII	126	0.731	0.733	0.425	0.702
6	No. 611	74.02.10	Japan	5.3	40	1- 82	10- 90	I-III(J)	22	0.741	0.613	0.273	0.656
7	No. 626	74.05.09	Japan	6.9	10	26-340	10- 180	II-IV(J)	44	0.881	0.781	0.312	0.656
8	No. 635	78.01.14	Japan	7.0	0	17-214	10- 115	II-V(J)	46	0.740	0.633	0.302	0.642
9	No. 705	76.06.16	Japan	5.5	20	11-170	15- 158	I-IV(J)	34	0.745	0.739	0.286	0.776
10	No. 789	78.06.12	Japan	7.4	30	80-598	4- 430	II-V(J)	90	0.511	0.653	0.479	0.854
11	No. 877	80.06.29	Japan	6.7	10	10-103	10- 290	II-V(J)	58	0.627	0.766	0.406	0.907
12	No.1027	83.05.26	Japan	7.7	14	90-270	3- 399	I-V(J)	50	0.695	0.555	0.551	0.938
13	No.1130	84.08.17	Japan	7.1	33	47-425	13- 270	I-IV(J)	28	0.612	0.622	0.330	0.990
14	Montenegro	79.04.15	Yugoslavia	7.2	40	20-260	6- 470	V-IX	50	0.645	0.419	0.546	1.323

(J) Japanese intensity

Table 2 Regressional Results of I-a Conversion

Eqk. No.	Slope of Regressional Line: Method 1 K1	2 K2	3 K3	4 K4	5 K5	6 K6	λ	ρ_{Ia}
1	0.274	0.566	0.394	0.425	0.423	0.445	0.850	0.696
2	0.222	0.677	0.388	0.411	0.415	0.411	0.912	0.573
3	0.367	0.549	0.449	0.504	0.490	0.512	0.657	0.817
4	0.334	0.682	0.477	0.475	0.475	0.473	0.943	0.683
5	0.397	0.923	0.605	0.606	0.606	0.606	0.998	0.656
6	0.208	0.842	0.415	0.366	0.345	0.355	1.208	0.498
7	0.298	0.760	0.476	0.446	0.444	0.361	1.128	0.626
8	0.209	1.055	0.468	0.419	0.387	0.408	1.169	0.447
9	0.165	0.824	0.369	0.367	0.365	0.365	1.008	0.448
10	0.428	0.736	0.561	0.615	0.604	0.637	0.782	0.763
11	0.291	0.690	0.447	0.495	0.495	0.542	0.819	0.650
12	0.469	0.737	0.587	0.545	0.556	0.508	1.178	0.755
13	0.203	0.550	0.334	0.337	0.337	0.337	0.983	0.607
14	0.343	0.497	0.413	0.370	0.381	0.348	1.540	0.832

Table 3 Acceleration(a/g)-Intensity Conversion

M \ I	VI	VII	VIII	IX	X
5.5	0.10	0.29	0.84	—	—
6.0	0.06	0.17	0.49	1.42	—
6.5	0.04	0.10	0.29	0.84	—
7.0	0.02	0.06	0.17	0.49	1.42
7.5	0.012	0.04	0.10	0.29	0.84
8.0	0.007	0.02	0.06	0.17	0.49
China Scale	0.06	0.12	0.25	0.50	1.00

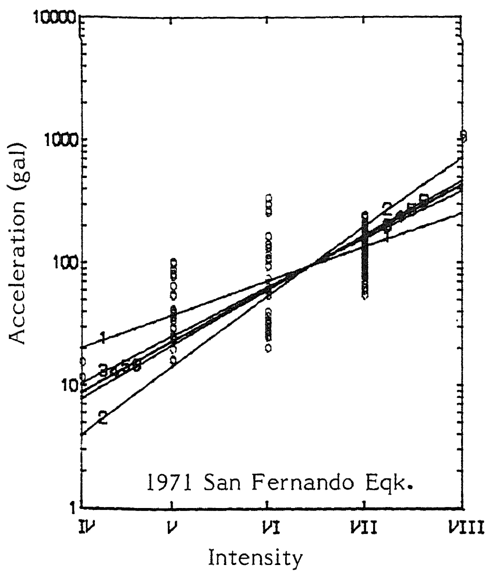

Fig. 1

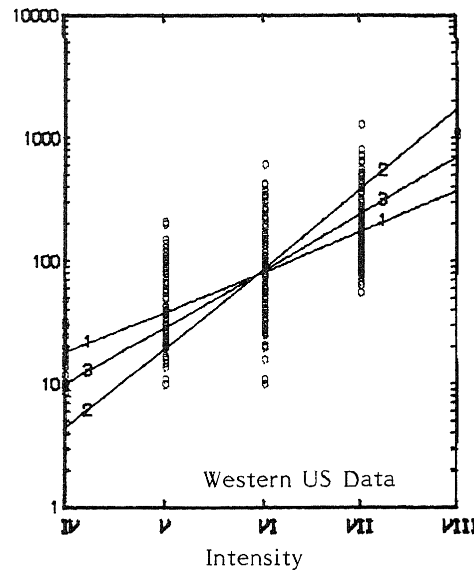

Fig. 2

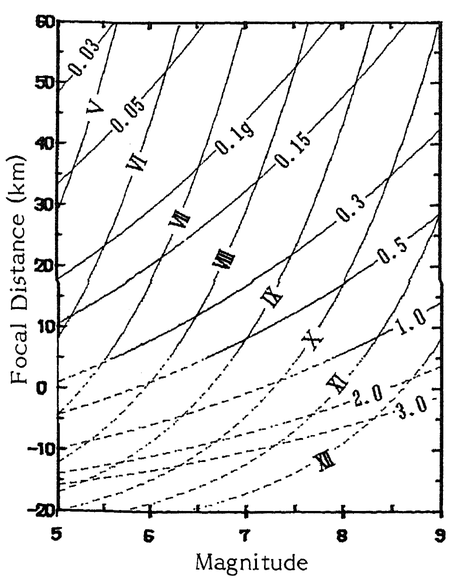

Fig. 3

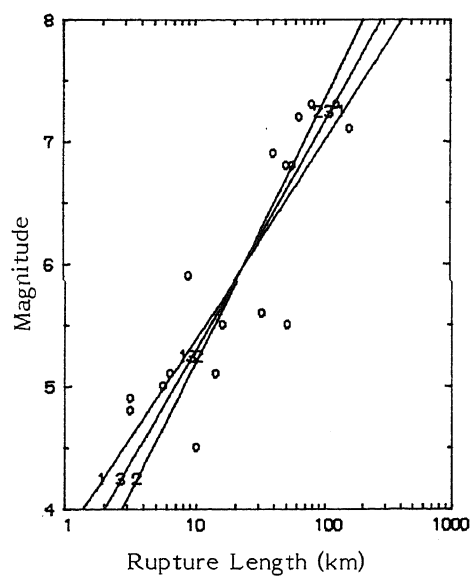

Fig. 4