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MAGNITUDE DEPENDENCE AND ATTENUATION CHARACTERISTICS OF PEAK GROUND ACCELERATION

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SUMMARY

A mathematical relationship for the attenuation of peak horizontal acceleration on the basis of the theory of earthquake fault model was proposed for the purpose of making possible the physical consideration of the magnitude and the distance coefficients. The regression analyses were carried out on earthquakes which occured in two areas using this mathematical relationship. Based on the resultant regression coefficients, the period dependence of the magnitude coefficient and the Q value (the quality factor of anelastic attenuation), and the regional difference in the Q value and the source spectrum were discussed.

INTRODUCTION

Many empirical attenuation relationships for peak horizontal acceleration have also been proposed in Japan since Kanai's formula was presented. According to those results, the magnitude and the distance coefficients for each empirical formula are fairly different. Consequently predicted values for each empirical formula are widely scattered even under the same condition. However, there has been little discussion about the cause of difference in those coefficients, because the mathematical relationships used for modeling the attenuation of peak acceleration have no precise physical basis. Recently, empirical relationships for response spectrum of strong ground motions have been re-examined on the basis of the theory of earthquake fault model in order to introduce the physical meaning into the mathematical relationship (Ref.1). Re-examination of this kind is very important also with regard to the attenuation relationship for peak acceleration because it makes possible the physical consideration of the magnitude and the distance coefficients.

In this study, the correlation between observed values and Kanai's formula was examined first. From this examination, the period of a wave showing peak acceleration in addition to earthquake magnitude and distance was adopted as a new parameter for estimating peak acceleration. The mathematical relationship among those parameters for regression analyses was constructed on the basis of the theory of earthquake fault model. Lastly, based on the resultant regression coefficients, the period dependence of the magnitude coefficient and the Q value, and the regional difference in the Q value and the source spectrum were examined.

The data used in the analyses were obtained at GL-3.5m, Koto-ku, Tokyo (Ref. 2), in which earthquakes with focal depth less than 80 km were selected. The data base consists of 112 earthquakes whose epicenters are shown in Fig. 1.

COMPARISON BETWEEN OBSERVED VALUES AND KANAI'S FORMULA

The correlation between observed values and Kanai's formula (Ref.3) was examined first as shown in Fig.2. The predominant period Tg in Kanai's formula was set to 0.5 sec. Peak acceleration was defined in this study as the root of the square sum of two horizontal peak accelerations. The size of an open circle in the figure represents the earthquake magnitude. Although there are considerable scatters in the correlation at first sight, the scatters can be classified relatively clearly by the size of earthquake magnitude. This fact suggests that earthquake magnitude takes an important part in the correlation. Figure 3 shows the relation between the ratios of observed value to Kanai's formula and earthquake magnitudes. The result shows the tendency of the ratios to decrease with earthquake magnitude in the range of $M_{JMA}<6.0$, while increasing slightly in the range of $M_{JMA}>6.0$. This tendency suggests that the magnitude coefficient in the range of $M_{JMA}<6.0$ is smaller than 0.61 and that in the range of $M_{JMA}>6.0$ slightly larger than 0.61 because the magnitude coefficient for Kanai's formula taken as the denominator is 0.61.

PERIOD OF A WAVE SHOWING PEAK ACCELERATION

It has been pointed out that the period of a wave showing the maximum amplitude becomes longer with earthquake magnitude (e.g.Ref.4). In Fig.4 an example is given that indicates that the period of a wave showing peak acceleration is fairly different according to the size of earthquake magnitude. On the other hand, from numerous recent studies on empirical relationships for response spectrum of strong ground motions, it has been made clear that the magnitude dependence becomes larger with the period (e.g.Ref.1). Accordingly, the cause of difference in the magnitude dependence shown in Fig.3 was guessed to be the period of a wave showing peak acceleration.

Tsujiura (Ref.5) and Yamaguchi et al. (Ref.4) pointed out the regional difference in the property of earthquakes which occured in the Kanto district. Based on their results, our data were grouped into four areas, A,B,C and others as shown in Fig.1. The relation between the period of a wave showing peak acceleration and earthquake magnitude for each area is shown in Fig.5. As pointed out by Tsujiura and Yamaguchi et al., the result indicates the regional difference precisely, that is, area A has the property of so called "Softness" compared with area C, and area B has the intermediate property. Although the periods for each area increase with earthquake magnitude, the change is not continuous. On the whole, periods near 0.2 sec are very frequent in the range of $M_{JMA}<6.0$ and periods near 0.9 sec in the range of $M_{JMA}>6.0$. Accordingly, it was suggested that the magnitude dependence is closely related to the period of a wave showing peak acceleration. And the periods of high frequency coincide with the ground characteristics of the site (Ref.2), therefore it was also suggested that the period of a wave showing peak acceleration on the ground surface depends strongly upon the ground characteristics.

REGRESSION MODEL

In order to introduce the period of a wave showing peak acceleration into the mathematical relationship for the attenuation of peak acceleration, a theoretical acceleration spectrum of seismic shear waves on the ground surface was applied. It can be written as follows (e.g.Ref.1);

$$A(T) = \frac{R\theta\phi\ \pi Mo(T)}{\rho Vs^3 XT^2} \exp\left(-\frac{\pi X}{QVsT}\right) Hg(T) \tag{1}$$

where Mo(T) is the source spectrum, $R\theta\phi$ is the radiation pattern, ρ is the density, Vs is shear wave velocity, X is the hypocentral distance, Q is the quality factor of anelastic attenuation and Hg(T) is the transfer function of seismic shear waves from the bedrock to the ground surface.

When taking the logarithm of equation 1,

$$\log A(T) = \log Mo(T) - \log XT^2 - \pi(QVs\ln 10)^{-1}XT^{-1} + \log\left(\frac{R\theta\phi\pi}{\rho Vs^3}Hg(T)\right) \quad (2)$$

By substituting the following relations into the above equation, on referring to the studies by Takemura (Ref.1) and Aki (Ref.6),

$$\log Mo(T) = (a+b\log T)M_{JMA} + \log P(T) \quad (3)$$

$$Q = QoT^{-n} \quad (4)$$

the following equation is obtained.

$$\log A(T) = (a+b\log T)M_{JMA} - \log XT^2 - \pi(QoVs\ln 10)^{-1}XT^{n-1} + \log\left(\frac{R\theta\phi\pi P(T)}{\rho Vs^3}Hg(T)\right) \quad (5)$$

Based on the above equation, the regression model was constructed as follows;

$$\log Amax = (a+b\log Tp)M_{JMA} - \log XTp^2 - cXTp^{n-1} + d + \Sigma e_i s_i \quad (6)$$

where Tp is the period of a wave showing peak acceleration, a,b,c,d and e_i are the regression coefficients, and s_i is the dummy variable related to the period Tp, which was set up as shown in Table 1.

REGRESSION RESULTS

According to the regional difference in the period of a wave showing peak acceleration, the regression analyses were carried out on earthquakes which occured in two areas. One is area A+B (because the data in area B were not sufficient) and the other area C. The regression coefficients of a,b and c, and the correlation coefficient were obtained for both areas as shown in Fig.6, when the n value which represents the period dependence of Q was varied from 0 to 1.0. The regression coefficients for area A+B change remarkably with n value. For area C, such remarkable change disappears, because the coefficient c for area C is very small compared with that for area A+B. On the other hand, the correlation coefficient depends little upon n value, for both areas. Accordingly, the n value is a very important factor for determining the regression coefficients for area A+B, although it is scarcely related to the correlation coefficient.

The JMA magnitude corresponds to the logarithmic spectral amplitude at about 4 sec (Ref.7), that is, the magnitude coefficient is 1.0 at the period of about 4 sec. The periods(Tm) when the magnitude coefficient takes a value of 1.0 were estimated by using the coefficients of a and b. Figure 7 shows the relation between the period Tm and the n value. The n value when the period Tm takes a value of about 4 sec is nearly 0.7 for area A+B and 0.7-1.0 for area C. This result agrees well with the results obtained from coda waves by Aki (Ref.6) and Sato (Ref.8). On the other hand, the coefficient c is 0.0041 for area A+B and 0.00023 for area C, when n takes a value of 0.7. The value of Qo in equation 5 was evaluated by using these values of c. Assuming Vs=3.5km/sec, Qo is 100 for area A+B and 1690 for area C. The value of Qo for area A+B is in good agreement with the results obtained from coda waves in the same area by Sato (Ref.8). The value of Qo for area C is very large compared with that for area A+B. According to Umino et al. (Ref.9), the subducting plate of high Q and high V exists beneath area C and the Q value was estimated to be about 1500. Our result is consistent with this value. Accordingly, it can be considered that seismic waves generated in area C arrived along the subducting plate of high Q and high V.

From these results, the following prediction equations in which the n value was set to 0.7 were obtained.

$$\log Amax = (0.78+0.33\log Tp)M_{JMA} - \log XTp^2 - 0.0041XTp^{-0.3} + C1(Tp) \quad \text{(for area A+B)} \quad (7)$$

$$\log Amax = (0.72+0.44\log Tp)M_{JMA} - \log XTp^2 - 0.00023XTp^{-0.3} + C2(Tp) \quad \text{(for area C)} \quad (8)$$

The constant terms of C1 and C2 are shown in Fig.8. The value of the constant terms for both areas is fairly different, particularly in the longer period range, which suggests that the source spectrum has the regional difference. This result corresponds to the regional difference in the period Tp. Figure 9 shows the period dependence of the magnitude coefficient obtained from the coefficients of a and b. The magnitude coefficient is fairly different according to the period Tp, which is consistent with the results pointed out in the regression analyses of response spectrum (e.g.Ref.1). The comparison of the observed and the calculated values is shown in Fig.10. The calculated values agree well with the observed.

CONCLUSIONS

A mathematical relationship for the attenuation of peak horizontal acceleration on the basis of the theory of earthquake fault model was proposed for the purpose of making possible the physical consideration of the magnitude and the distance coefficients. The period of a wave showing peak acceleration in addition to earthquake magnitude and distance was adopted as a new parameter in the mathematical relationship because it was found to be closely related to the magnitude dependence. Using this mathematical relationship, the regression analyses were carried out on two areas. One was the Kanto district and the other was off Ibaraki and Fukushima prefectures, Japan. Based on the resultant regression coefficients, the period dependence of the magnitude coefficient and the Q value (the quality factor of anelastic attenuation), and the regional difference in the Q value and the source spectrum were investigated. The results obtained are summarized as follows.

The magnitude coefficient depends strongly upon the period of a wave showing peak acceleration. Accordingly, the period of a wave showing peak acceleration, which is closely related to the ground characteristics besides earthquake magnitude, is a very important factor for estimating peak acceleration. The Q value also depends strongly upon the period. Assuming $Q=QoT^{-n}$, n takes a value of about 0.7 for both areas. This result is in good agreement with the value obtained from coda waves by Aki and others. On the other hand, the Qo value for both areas is fairly different. It was considered to be due to the difference in seismic attenuation structure. From the value of constant term, it was suggested that the source spectrum has the regional difference. This result corresponds to the regional difference in the period of a wave showing peak acceleration.

REFERENCES

1. Takemura, M. et al., "Theoretical Basis Relations about Response Spectra of Strong Ground-Motions", Transactions of AIJ, No.386, 1988
2. Abe, Y. et al., "Dynamic Behavior of Pile Foundation during Earthquake", 8WCEE, 1984
3. Kanai, K. et al., "Observation of Strong Earthquake Motions in Matsushiro Area, Part 1". Bull. Earthq.Res.Inst., 44, 1966
4. Yamaguchi, N. et al., "The Relationship between the Predominant Period and the Magnitude for the Earthquakes Which Occured in and near the Kanto District". ZISIN (II), 31, 1978
5. Tsujiura, M., "Regional Variation of Micro-Earthquake Spectrum - Kanto District -", ZISIN (II), 26, 1973
6. Aki, K., "Attenuation of Shear-Waves in the Lithosphere for Frequencies from 0.05 to 25 Hz", Phys.Earth Planet. Interiors, 21, 1980
7. Koyama, J. et al., "A Scaling Model for Quantification of Earthquakes in and near Japan", Tectonophysics, 84, 1982
8. Sato, H., "Regional Study of Coda Q in the Kanto-Tokai District, Japan", ZISIN (II), 39, 1986
9. Umino, N. and A. Hasegawa, "Three-Dimensional Qs Structure in the Northeastern Japan Arc", ZISIN (II), 37, 1984

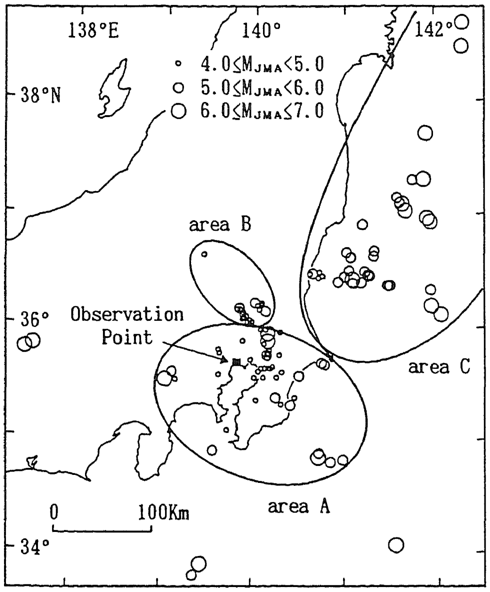

Fig.1 Epicenters of Used Data and Observation Point

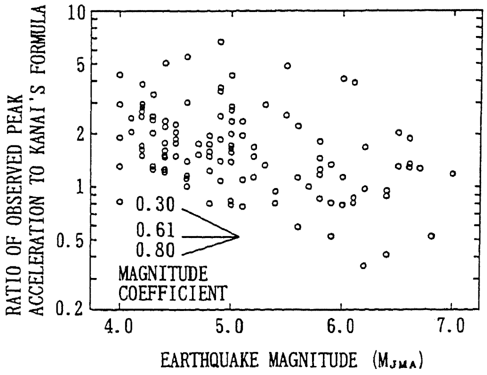

Fig.3 Relation between Ratios of Observed Value to Kanai's Formula and Earthquake Magnitudes

Table 1 Dummy Variable s_i

PERIOD RANGE (sec)	S1	S2	S3	S4	S5	S6	S7	S8	S9
0.1≦Tp<0.2	1								
0.2≦Tp<0.3		1				0			
0.3≦Tp<0.4			1						
0.4≦Tp<0.5				1					
0.5≦Tp<0.6					1				
0.6≦Tp<0.7						1			
0.7≦Tp<0.8			0				1		
0.8≦Tp<0.9								1	
0.9≦Tp<1.0									1

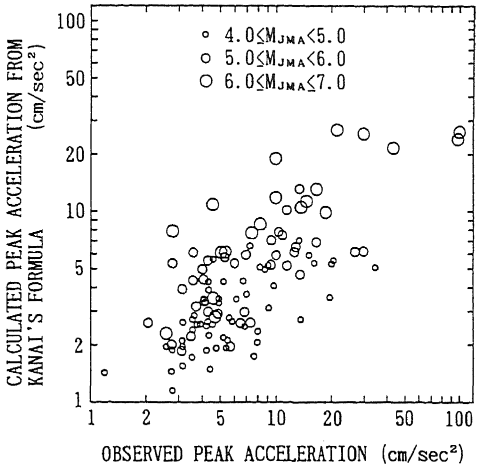

Fig.2 Correlation between Observed Values and Calculated Ones from Kanai's Formula

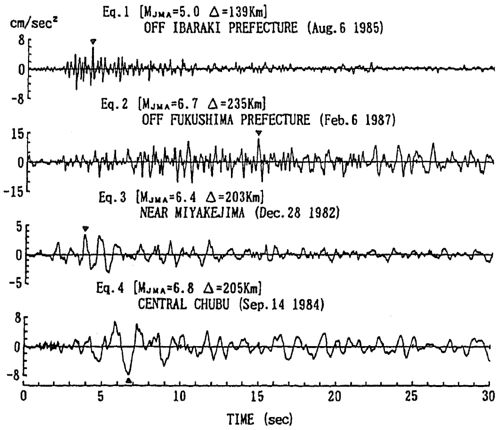

Fig.4 Accelerograms with Different Period of a Wave Showing Peak Acceleration

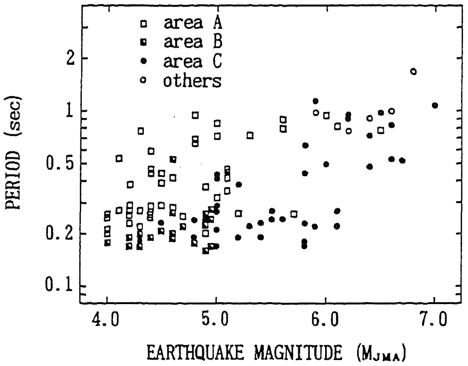

Fig.5 Relation between Periods of a Wave Showing Peak Acceleration and Earthquake Magnitudes

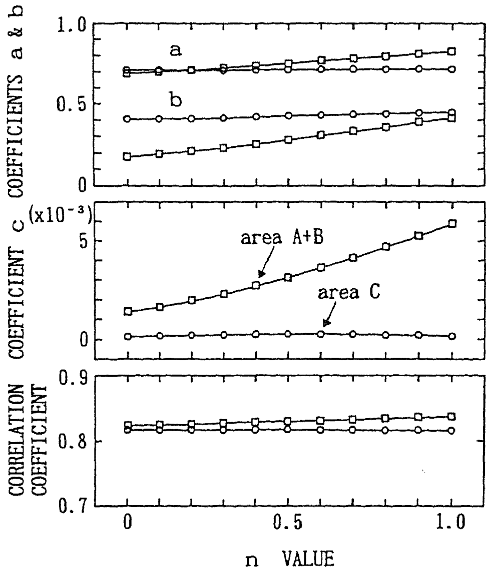

Fig.6 Relation of Regression and Correlation Coefficients to n Value

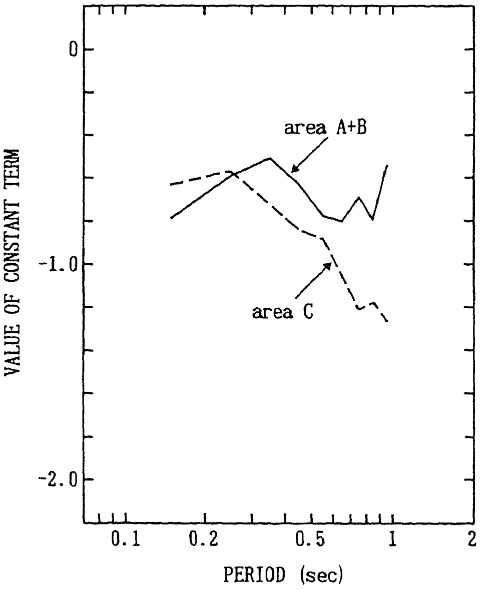

Fig.8 Value of Constant Term

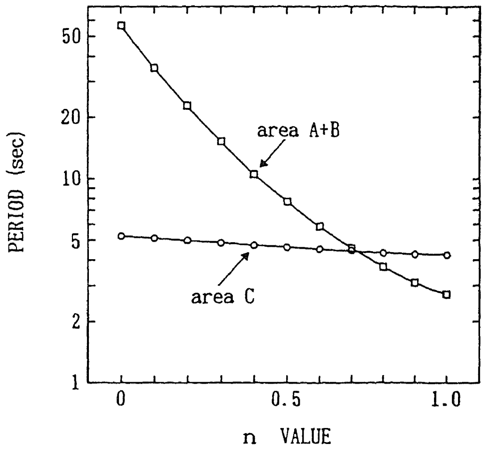

Fig.7 Relation between Period Corresponding to Magnitude Coefficient of 1.0 and n Value

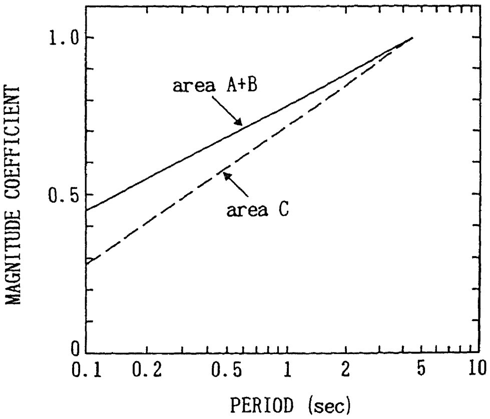

Fig.9 Period Dependence of Magnitude Coefficient

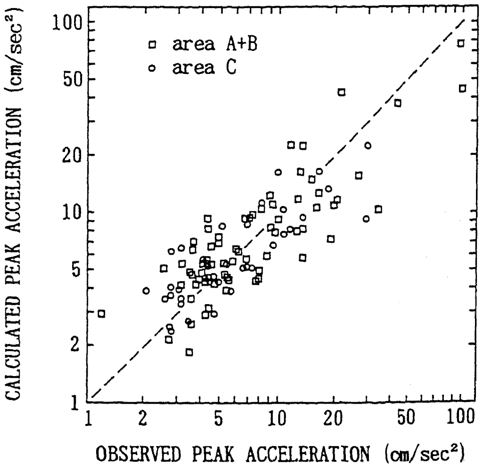

Fig.10 Correlation between Observed and Calculated Values