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REGRESSION ANALYSIS ON FOURIER AMPLITUDE SPECTRA OF SEISMIC GROUND MOTIONS IN TERMS OF EARTHQUAKE MAGNITUDE, HYPOCENTRAL DISTANCE AND SITE CONDITION

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SUMMARY

A response spectrum S(T) of seismic waves is empirically expressed as a regression equation with earthquake magnitude M and hypocentral distance X to estimate the average value of S(T). The regression equation is rewritten by using the Fourier amplitude spectrum to discuss physical meanings of the regression coefficients on the basis of the theory of earthquake fault-model. The regression coefficients can be expressed as simple functions of some physical parameters, and the values of these functions, which are calculated theoretically, are generally consistent with those of regression coefficients obtained from the observed records in the period range from 0.1 to 2.0 sec.

INTRODUCTION

The regression analysis is usually done to obtain the average value of response spectra for observed seismic ground motions. The regression equation is empirically expressed in the analysis as a function of earthquake magnitude M and hypocentral distance X (or epicentral distance D). In Japan, the regression equation, which is modified from the equation for the determinatation of magnitude in Japan Meteorological Agency (JMA) scale (Ref. 1), is usually expressed as follows:

$$\log S(T)=a(T)M-b(T)\log X+c(T), \qquad (1)$$

where S(T) is a response spectrum and a(T), b(T) and c(T) are regression coefficients at a period T. Although the regression equation such as eq.(1) is used by many investigators (Ref. 2-4), it is difficult to discuss the propriety of their results. One of the most important reasons is that the physical meanings of regression coefficients are not clear.

The purpose of this paper is to investigate the physical meanings of the regression coefficients on the basis of the theory of earthquake fault-model. Takemura et al. (Ref. 5) derived the relation among the regression coefficients and the physical parameters for the regression equation of the response spectrum S(T). In this paper, the regression equation will be rewritten by using the Fourier amplitude spectrum, because the Fourier spectrum can be easily understood by the theory of earthquake fault-model.

THEORY

The theory of Takemura et al. (Ref. 5) is summarized as follows:

In an infinite homogeneous elastic medium, Fourier amplitude spectrum of the far-field displacement for S-wave Us(T) is given by Kanamori (Ref. 6) as follows:

$$Us(T)=\frac{R_{\theta\phi}}{4\pi\rho Vs^3 X}\,Mo(T), \tag{2}$$

where ρ is a density, Vs is a velocity of S-wave, X is a hypocentral distance and $R_{\theta\phi}$ is a radiation pattern coefficient for the double couple point source. Mo(T) is a source spectrum and is agreement with a seismic moment Mo in the very long period range. In consideration of the quality factor Qs of S-wave along the propagating path and the transfer function Hg(T) from the bedrock to the observation point, Fourier amplitude spectrum F(T) is rewritten for acceleration as follows:

$$F(T)=\frac{R_{\theta\phi}\pi}{\rho Vs^3 X}\cdot\frac{Mo(T)}{T^2}\cdot\exp\left(\frac{-\pi X}{QsVsT}\right)Hg(T) \tag{3}$$

To study the physical meanings of the coefficient a(T) in eq. (1), we must consider the relation between earthquake magnitude M and source spectrum Mo(T). According to Aki (Ref. 7), magnitude M corresponds to the logarithmic source spectrum at the period of which observed seismic waves are used for the magnitude determination. Therefore, the relation between M and Mo(T) is assumed as,

$$Mo(TM)=P10^{M}, \tag{4}$$

where P is a constant and TM is the determinant period of magnitude. The magnitude used in this paper is in the JMA scale. TM of the JMA scale is assumed to be 4 sec, taking into account Takemura and Koyama (Ref. 8). Solid curves in Figure 1 shows a scaling model of source spectrum (Refs. 5 and 7) on which the magnitude in the JMA scale is assumed to be the logarithmic amplitude of source spectrum at 4 sec. It is found that the amplitude of source spectrum is proportional to T^n in the short period range. In order that the logarithmic spectral amplitude at the observation point may be proportional to M as like eq. (1), n must be satisfied with the equation as follows:

$$n=kM \quad (k>0). \tag{5}$$

Combining eqs. (4) and (5), the relation between Mo(T) and M can be written as,

$$Mo(T)=P10^{M}(T/TM)^{kM}. \tag{6}$$

According to eq. (6), the decay rate of source spectrum in the short period range is increasing with magnitude M. The propriety of this assumption will be discussed later.

Substituting eq. (6) into eq. (3), logarithm of F(T) can be expressed as,

$$\log F(T)=a(T)M-(b(T)X+\log X)+c(T), \tag{7}$$

where a(T), b(T) and c(T) are represented as follows:

$$a(T)=1-k\log(TM/T), \tag{8}$$

$$b(T)=\pi/(QsVsT\ln 10), \tag{9}$$

$$c(T)=\log\frac{Hg(T)}{T^2}+\log\frac{R_{\theta\phi}\pi P}{\rho Vs^3} \tag{10}$$

The regression equation (7) could be derived based on the earthquake fault-model, which is different from eq. (1).

ANALYSIS AND RESULTS

The regression coefficients of eq. (7) obtained from observed data are compared with the results from eqs. (8) to (10), respectively. The data for the analysis are 85 accelerograms in horizontal component observed at three observation points, OHJI1, OHJI2 and SNM in Tokyo (Table 1). The available period range of these data is 0.05 to 5 sec which is evaluated by the method of Ohta et al. (Ref. 9). 45 radial componets and 40 transverse components are calculated and Fourier amplitude spectra of strong shaking parts are analyzed which almost correspond to S-waves. The magnitude-hypocentral distance distribution of the data is shown in Figure 2. It is assumed in the regression analysis that a(T) and b(T) of eq. (7) are common to three observation points and that c(T) depends on the components and on the observation points. Results of the analysis are summarized as follows:

Regression coefficient a(T) Figure 3 shows the regresion coefficient a(T). In this figure, a dashed line shows the value obtained from eq. (6), when k=0.43. The regression coefficient a(T) is consistent with the dashed line in the range from 0.1 to 2 sec. This result indicates that the source spectrum is proportional to $T^{2.2}$ for M=5, to $T^{2.6}$ for M=6 and to $T^{3.0}$ for M=7. According to other studies (Refs. 10,11 and 12), source spectra of earthquakes with magnitude M smaller than 7 are proportional to about T^2 in the short period range, irrespectively of M. If we adopt these results, the values of n from the regression coefficient are overestimated for earthquakes with M larger than 6.0. This point will be discussed in detail later.

Regression coefficient b(T) Figure 4 shows the regression coeficient b(T). The Qs-value is estimated using eq. (9) is shown in Figure 5, which is calculated from the value of the envelope of b(T), in spite of its large variations. Vs is assumed to be 3.0km/sec. The Qs-value is inversely proportional to the period T. The frequency dependence of Qs-value coincides with those obtained from S-waves and coda-waves in the active seismic region by other investigators (Refs. 13, 14 and 15), though the absolute value of Qs is larger than these results (Fig. 6). The reason of the large Qs value may be that b(T) is commonly determined for the events in the different seismic regions around Tokyo.

Regression coefficient c(T) Theoretical value of c(T), which is expressed as eq. (10) involves a transfer function Hg(T) from the bedrock to the observation point. The transfer function is calculated with the theory of multiple reflection of SH-waves. Table 2 shows the velocity structure at each site. The velocity and damping coefficient structures of the shallower parts from 0 to -100m at OHJI and from 0 to -27 m at SNM are determined so as to coincide the theoretical spectral ratios among observation points of vertical array with the spectral ratios of observed seismic waves in consideration of the results of PS logging (Ref. 16), and the structures of the deeper parts are assumed being based on Shima et al. (Ref. 17). The other constants in eq. (10) are assumed as follows: $\rho=2.8g/cm^3$, Vs =3.0km/sec, $R_{\theta\phi}=0.6$, and $P=5x10^{18}$ dyne cm (=P1) or $3.3x10^{20}$ dyne cm (=P2). P1 is estimated from the scaling model in Fig. 1 (Ref. 8), and P2 is estimated from the source spectrum at 1 sec estimated from large earthquakes in the world by Koyama et al. (Ref. 18) and from n=0.43M obtained in the present study.

Figure 7 shows the regression coefficient c(T) at OHJI1 in transverse component compared to calculated value using eq. (10). The regression coefficient c(T) is explained by the calculated one with P=P1 in the period range longer than 0.5 sec, while it is explained by that with P=P2 in the period range shorter than 0.5 sec. The same results are obtained at other observation points and for radial component. To solve this problem, we have to study about the transfer function Hg(T) from the bedrock to the observation point.

DISCUSSION

In the previous section, it is indicated that the values of n are overestimated from the regression coefficient a(T) for earthquakes of M larger than 6.0. To discuss this point, we calculate the source spectra for some earthquakes with different magnitudes, using the Qs-value and the transfer function Hg(T) obtained from regression coefficients b(T) and c(T). Figure 8 shows the source spectra of the 1978 Off-Miyagi earthquake (M7.4), the 1975 Off-Fukushima earthquake (M6.0) and the 1975 SW of Ibaragi earthquake (M5.0). Dashed lines show the theoretical values obtained from regression coefficient a(T) and P=P1. The source spectra for the earthquakes with M=6.0 and M=5.0 are almost consistent with the theoretical values. But for the Off-Miyagi earthquake with M=7.4, the average decay rate of the source spectrum is smaller than the theoretical values in the period range from 0.1 to 4 sec. The average value of magnitudes M for analyzed earthquakes is about 5.5 (Fig. 2). The results in Fig. 8 suggest that the assumption of $\log F(T) \propto M$ in eq. (7) is admitted only arround the average value of M. Therefore, it must be noticed that we cannot extrapolate to estimate the spectra of strong ground motions due to larger earthquakes by using the regression equation with a(T)M.

CONCLUSIONS

Regression coefficients of spectra are studied and their physical meanings are clarified on the basis of the theory of earthquake fault-model in an infinite medium. Being based on these studies, the new regression equation is proposed. The new equation is under the assumption that the logarithmic amplitude of spectrum is in proportion to magnitude M, though the propriety of this assumption is not clear. It must be noticed that we extrapolate with a large error to estimate the spectra of strong ground motions due to large earthquakes, using the regression equation with a(T)M.

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Table 1 Summary of Data at Each Observation Point

observation point	geology	depth of point	number of horizontal records		
			radial	transverse	total
OHJI1	deluvial layer	GL-30m	19	17	36
OHJI2	deluvial layer	GL-100m	14	12	26
SNM	deluvial layer	GL-27m	12	11	23

Table 2 Physical Parameters of the Ground at Observation Sites

OHJI

thickness of layer(m)	density ρ (g/cm^3)	S-wave velocity Vs (m/sec)	damping coefficient $h(f)=h1f^{-\alpha}$	
			h1	α
12	1.5	90	0.082	0.424
8	1.6	180	0.082	0.424
6	1.7	190	0.255	0.424
4	1.9	360	0.255	0.424
13	1.9	360	0.255	0.424
11	1.8	380	0.071	0.424
19	1.9	550	0.071	0.424
27	1.9	500	0.071	0.424
100	2.0	500	0.025	0
1500	2.0	700	0.025	0
1400	2.3	1500	0.017	0
	2.5	3000	0.007	0

SNM

thickness of layer(m)	density ρ (g/cm^3)	S-wave velocity Vs (m/sec)	damping coefficient $h(f)=h1f^{-\alpha}$	
			h1	α
1	1.4	140	0.143	0.767
7	1.4	140	0.143	0.767
3	1.5	280	0.174	0.767
5	1.9	370	0.174	0.767
9	1.5	240	0.174	0.767
2	1.9	430	0.174	0.767
75	1.9	430	0.025	0
100	1.9	500	0.025	0
1500	2.0	700	0.025	0
1400	2.3	1500	0.017	0
	2.5	3000	0.007	0

▶ shows the location of observation point. f shows the frequency and h1 shows the damping coefficient at 1 Hz.

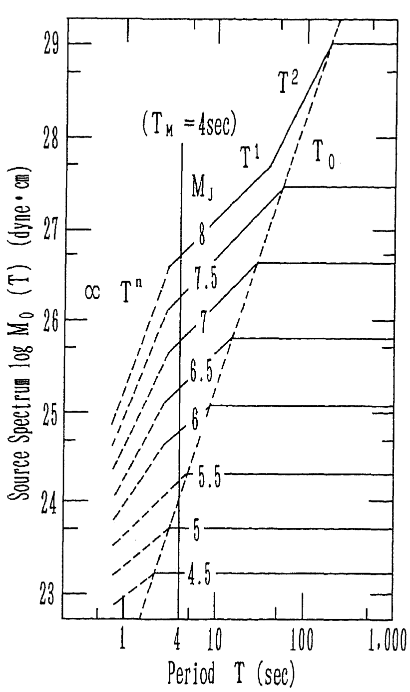


Fig. 1 Scaling Model of Source Spectrum (Ref. 5).

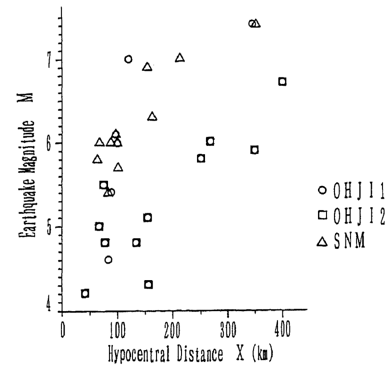


Fig. 2 Magnitude-Hypocentral Distance Distribution of the Data.

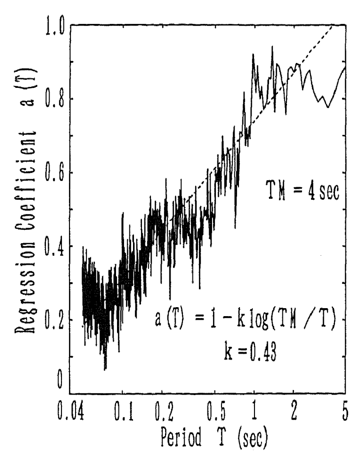


Fig. 3 Regression Coefficient a(T).

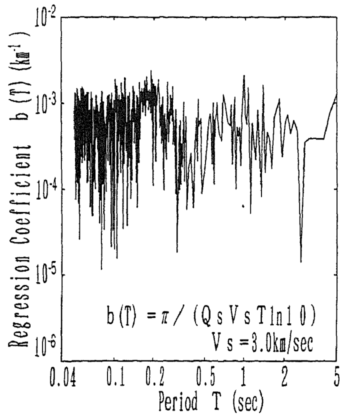


Fig. 4 Regression Coefficient b(T).

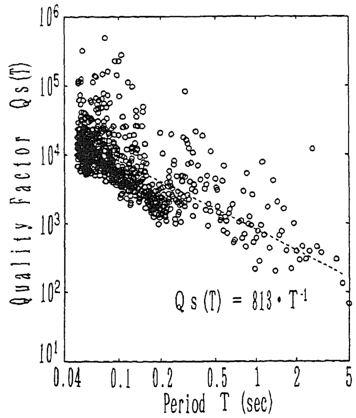


Fig. 5 Qs-values Obtained from b(T).

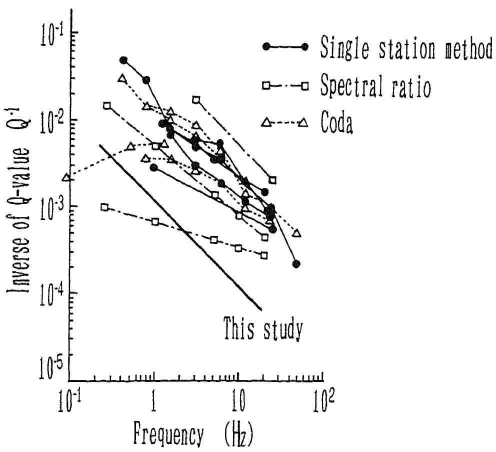


Fig. 6 Comparison of Q^{-1}-values (Ref. 11).

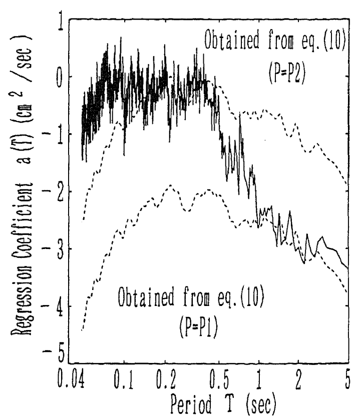
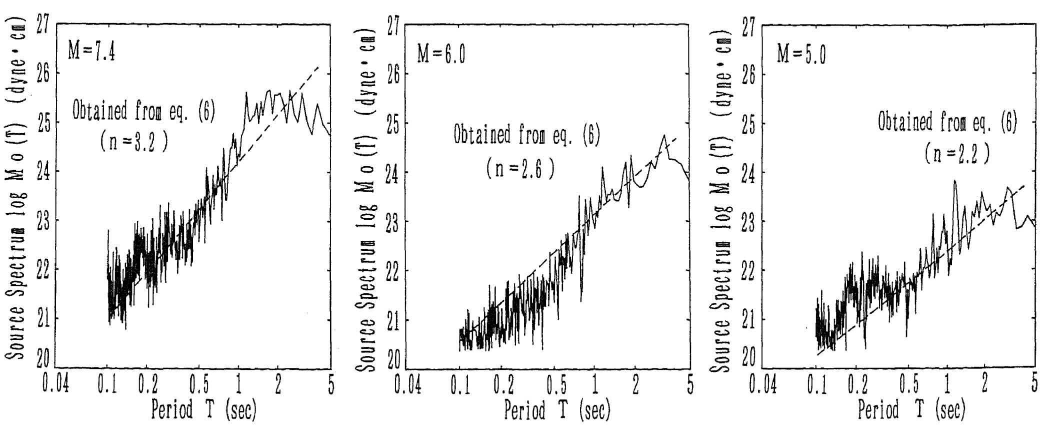


Fig. 7 Regression Coefficient c(T) at OHJI1 in Transverse Component.



a) The Off-Miyagi (M7.4) b) The Off-Fukushima (M6.0) c) The SW of Ibaragi (M5.0)

Fig. 8 Source Spectra of Earthquakes with Different Magnitudes