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BASELINE CORRECTION OF ACCELEROGRAM DATA BY OPTIMAL SPLINE FUNCTION

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SUMMARY

In the digitized accelerogram data processing, the velocity and displacement resulted from a numerical integration usually include unreal drifts due to the long-period distortion contained in the original acceleration. A new method is presented in this paper for the baseline correction of accelerogram data using the spline function with variable knots. The baseline correction is formulated as an optimization problem. In the numerical examples, the proposed method is compared with other correction methods and discussed such as the parabolic baseline, the fixed-knot spline baseline, the band-pass filtering and the digital filtering.

INTRODUCTION

It is often inevitable that the numerically integrated time history data of velocity and displacement may include a remarkable drift from the zero baseline in the digitized data processing concerning the accelerogram analyses of observed or simulated earthquakes and vibration tests. This unnatural drift is resulted from the long-period distortion due to accumulated errors or noises of the instruments and numerical digitization of the accelerogram data. Methods of the accelerogram data correction to exclude an integrated drift may be roughly classified as a correction using parabolic baseline (Ref.1), a filtering in the frequency or time domain (Refs.2 and 3) and a direct modification based on optimal criteria (Ref.4).

This paper presents a new method for the baseline correction of accelerogram data using the spline function. The spline function is defined by the piecewise polynomials connected at several knots satisfying some continuity conditions as illustrated in Fig.1. The spline polynomials have the minimum curvature property and are the most successful approximating functions for practical applications. Since the spline function is the generalized extension of analytical polynomials, the baseline correction by the spline function may be suitably considered as an extension of Berg's method of the parabolic correction (Ref.1).

The baseline correction of accelerogram by the spline function is formulated as an optimization problem where the velocity power is to be minimized together with the other characteristic quantities expressed by the spline parameters subjected to the continuity conditions on the spline polynomials. When the spline knots are variable, the objective function will be multimodal and multivariable. Suboptimal solutions by a successive division procedure will be practical. For an accelerogram with a relatively long duration, numerical examples are demonstrated, and the proposed method is examined and compared with other methods.

BASELINE CORRECTION BY SPLINE FUNCTION

Let the accelerogram data be divided into n intervals as shown in Fig.2, and a velocity baseline v_k of the k-th interval ($k=1,2,..,n$) be expressed by using the polynomial of third order with parameters α_k, β_k, γ_k and δ_k as follows :

$$v_k = (\alpha_k \xi^3 + \beta_k \xi^2 + \gamma_k \xi + \delta_k) \tag{1}$$

where ξ is the nondimensional time parameter defined by $\xi = t/T_k$ $(0 \leqq t \leqq T_k)$ and T_k is the divided duration time of the k-th interval. Introducing another integration parameter ε_k, the baseline of displacement and acceleration d_k, a_k are given by

$$d_k = T_k \ (1/4 \ \alpha_k \ \xi^4 + 1/3 \ \beta_k \ \xi^3 + 1/2 \ \gamma_k \ \xi^2 + \delta_k \ \xi + \varepsilon_k) \ , \tag{2}$$

$$a_k = (3 \ \alpha_k \ \xi^2 + 2 \ \beta_k \ \xi + \gamma_k) \ / \ T_k \ . \tag{3}$$

The original acceleration A_k, velocity V_k and displacement D_k in the k-th interval will be corrected by these baselines as $A_k^* = A_k + a_k$, $V_k^* = V_k + v_k$ and $D_k^* = D_k + d_k$, respectively.

The continuity conditions between adjacent k-th and k+1st intervals on the spline functions of displacement, velocity, acceleration and the derivative of acceleration can be written as

$$e_k \equiv T_k \ (1/4 \ \alpha_k + 1/3 \ \beta_k + 1/2 \ \gamma_k + \delta_k + \varepsilon_k) - T_{k+1} \ \varepsilon_{k+1} = 0 \ , \tag{4}$$

$$f_k \equiv \alpha_k + \beta_k + \gamma_k + \delta_k - \delta_{k+1} = 0 \ , \tag{5}$$

$$g_k \equiv (3 \ \alpha_k + 2 \ \beta_k + \gamma_k) \ / \ T_k - \gamma_{k+1} \ / \ T_{k+1} = 0 \ , \tag{6}$$

$$h_k \equiv (3 \ \alpha_k + \beta_k) \ T_k^{\ 2} - \beta_{k+1} \ / \ T_{k+1}^{\ 2} = 0 \ . \tag{7}$$

where $k=1,2,..., n-1$. The continuity condition of Eq.(7) makes the baselines of displacement, velocity and acceleration the complete spline functions ot order 4, 3 and 2, respectively.

The spline parameters are determined so as to minimize the velocity power :

$$P = \sum_k \int_0^{T_k} \{ \ V_k(t) + v_k(t) \ \}^2 \ dt \tag{8}$$

and the sum of the weighted squares of the errors between some characteristic quantities Q_j and their specified or likely values $\bar{Q}_j$ ($j=1,2,..,s$)

$$Q = \sum_j q_j \ (\ Q_j - \bar{Q}_j \ )^2 \tag{9}$$

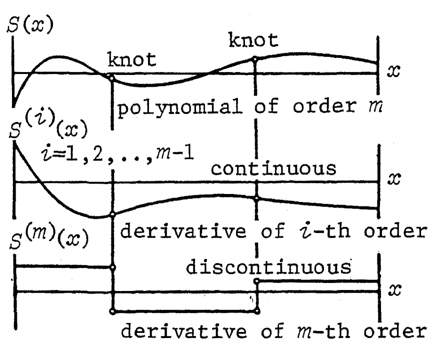

Fig.1 Spline Function

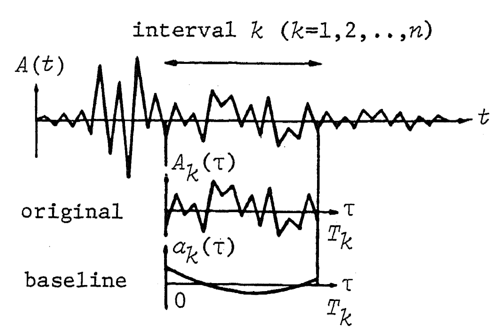

Fig.2 Baseline Correction by Spline Function

where q_j indicates the weighting factor in the minimization procedure. Characteristic quantities Q_j may be, for examples, the mean of displacement, the initial or terminal values of displacement, velocity or acceleration, and are well supposed to be expressed by the linear combinations of the spline parameters α_k, β_k, γ_k, δ_k and ε_k.

By introducing Lagrange multipliers ρ_k, λ_k, μ_k and ν_k (k=1,2,..,n-1) corresponding to the continuity conditions of Eqs. (4) to (7), the problem is reduced to the unconstrained minimization of the following function :

$$U = P + Q + \sum_k (\rho_k e_k + \lambda_k f_k + \mu_k g_k + \nu_k h_k) \tag{10}$$

When the spline knots are fixed, the stationary conditions of U with respect to the spline parameters α_k, β_k, γ_k, δ_k and ε_k (k=1,2,..,n) and Lagrange multipliers ρ_k, λ_k, μ_k, ν_k (k=1,2,..,n-1) lead to the simultaneous linear equation with (9n-4) unknowns and the solution can be obtained straightforward (Ref.6).

OPTIMAL SPLINE BASELINE

As the spline functions are piecewise analytic, they can behave only locally with little influence on the other intervals and present good approximating functions without ill-conditioned troubles such as a heavy vibration frequently appeared in the Lagrange interpolation. To make the best use of the advantage of spline functions, it is very important to select the proper position of spline knots. When the spline knots are also variable in the foregoing formulation, the minimization of multimodal and multivariable function will be needed and it will be usually difficult to reach the global optimum.

A suboptimal but practical solution is obtained by the iterative algorism (Ref.5) where the successive improvement in knots' position are carried out as illustrated schematically in Fig.3. Let P_k (k=1,2, ..,n) be the velocity power of each interval

$$P_k = \int_0^{T_k} \{ V_k(t) + \nu_k(t) \}^2 \, dt \tag{11}$$

and $t=t_i$ (i=0,1,..,n) be the knot position where $t_0<t_1< \dots <t_n$ and t_0 and t_n are supposed to be fixed.

Step 1 : Knot Position Compare the adjacent velocity powers P_k and P_{k+1}, and move the boundary knot $t=t_k$ to t_k^* by the following way. If $P_k > P_{k+1}$, then $t_k^* = t_k - \Delta t$ or else if $P_k < P_{k+1}$, then $t_k^* = t_k + \Delta t$ where Δt is the positive constant. Calculate new spline parameters and if the value of the function U by Eq.(10) could be decreased, then replace t_k with t_k^*. Repeat this replacement successively for i=1,2,..,n-1.

Step 2 : Convergence If at least one knot position is replaced among i=1,2,..,n-1, then repeat step 1 again from i=1 to n-1. If all the knot position are remained unchanged, the final knot positions $t_1, t_2, .., t_n$ will result in an optimal spline baseline. The convergence of this method will be examined in the following examples.

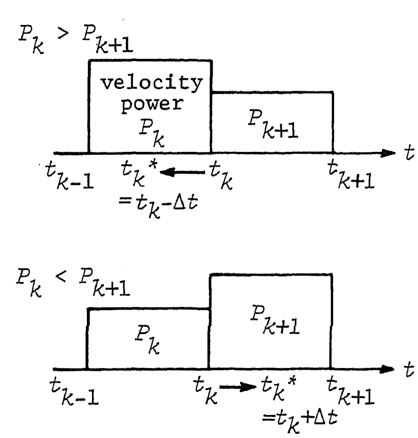

Fig.3 Replacement of Spline Knot

NUMERICAL EXAMPLE

As a demonstrative example problem, an accelerogram recorded at 1968 Tokachi-Oki Earthquake in Japan is considered here and the proposed method is compared with other methods such as the parabolic baseline and the filtering corrections. This record has a relatively long duration time and consequently exhibits unreal drift in the numerically integrated velocity and displacement as shown in Fig.4. For the convenience of the comparison, the maximum of the acceleration is set to be 100 gals.

Fixed-Knot Spline Figure 5 illustrates the corrected acceleration, integrated velocity and displacement and acceleration baseline by the parabolic correction which is one of fixed-knot splines with only one interval. Figure 6 demonstrates in the same way the correction results by another fixed-knot spline baseline where the whole duration is divided into eight intervals by equally spaced spline knots. The velocity power is minimized considering the intial and terminal zero displacement.

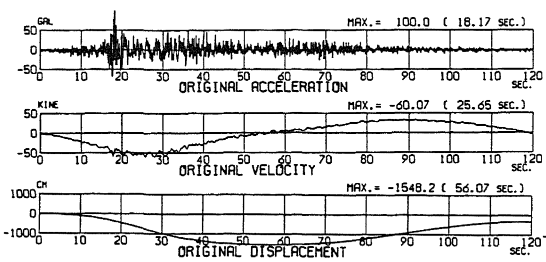

Fig.4 No Correction

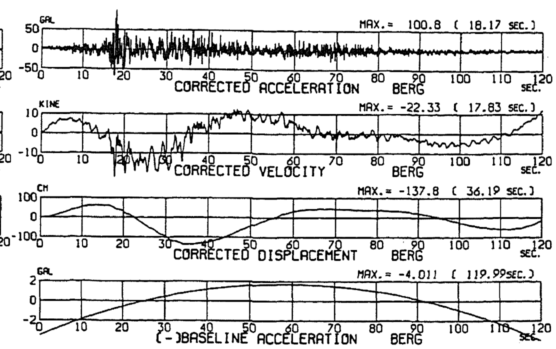

Fig.5 Parabolic Baseline Correction

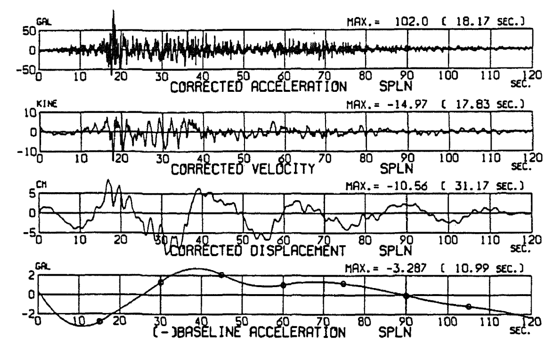

Fig.6 Fixed Knot Spline Correction

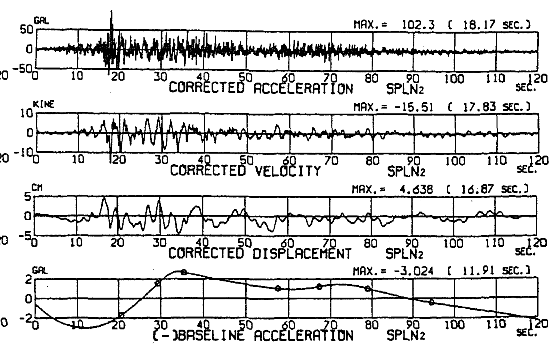

Fig.7 Optimal Spline Correction

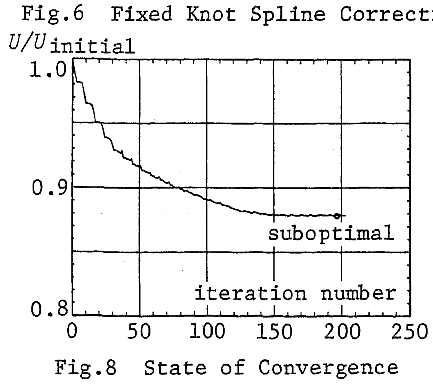

Fig.8 State of Convergence

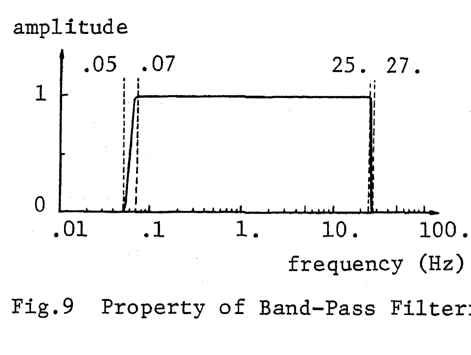

Fig.9 Property of Band-Pass Filtering

Errors in the corrected accelerations are at most 4 % and invisible, but the differences of the integrated values between original and corrected are notable. The parabolic method can exclude the great drifts of the integrated velocity and displacement to a certain extent, but unnatural vibrations with long period are included in both the velocity and the displacement. It would be unreasonable to apply the only one analytical polynomial function to the data of a relatively long duration. On the other hand, the fixed-knot spline method with several spline knots can exclude these defects, but the example still indicates the vibrating features in the resulted displacement.

<u>Optimal Spline</u> Corrected results by the proposed optimal spline method are shown in Fig.7. The number of divided intervals is equal to that by the fixed eight knots example, and the initial positions of the spline knots are also based on that example. The shift increment of the knot position Δt is constant throughout the iteration cycle and set to be 0.5 sec. Convergence of the proposed iterative method is not examined mathematically but exemplified here numerically in Fig.8. The number of iteration cycle is counted as each spline knot is considered for the comparison of the velocity powers of the adjacent intervals. The value of the objective function U, therefore, dose not necessarily decrease at every iteration.

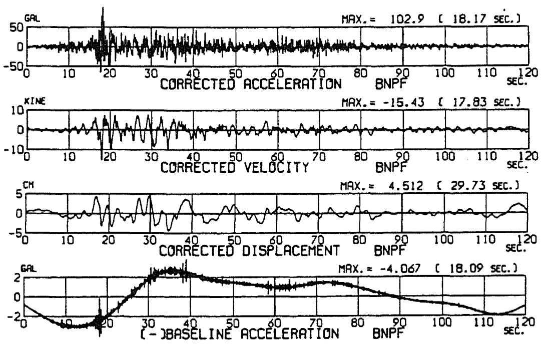

Fig.10 Band-Pass Filtering

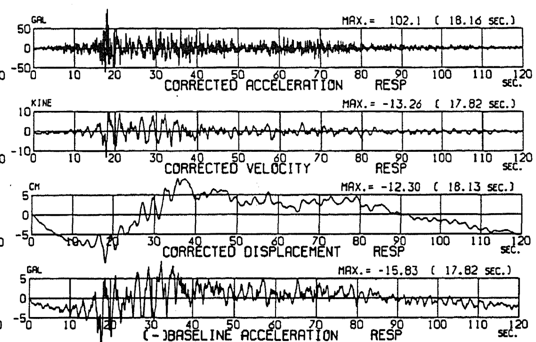

Fig.11 SDOF Digital Filtering

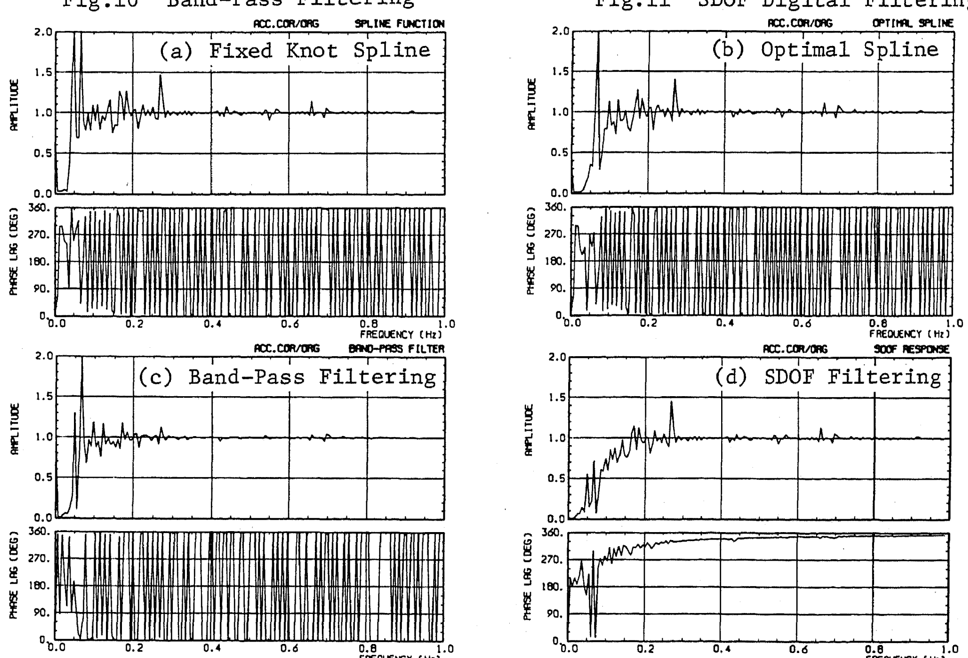

Fig.12 Equivalent Filtering Property

From the comparison of Fig.7 with Fig.6, the slight change of the acceleration baselines yields a striking difference in the displacements. It should be understood that the positions of the spline knots play an important role. The proposed optimal spline method, the solution being local optimal dependent on the initial knots condition, is exemplified to be reasonable and successful in the practical application even for the accelerogram with a relatively long duration.

Correction by Filtering In the practical field, filtering methods have been preferably applied to the calculation of velocity and displacement from the accelerogram. The filtering properties such as frequency bands are determined through the engineering practices. The band-pass filtering by the frequency property indicated by Fig.9 corrects the original accelerogram data as sketched in Fig.10. Correction results by this band-pass filtering show the remarkable resemblance with those by the optimal spline correction. The last example of the comparative method is a digital filtering by SDOF oscillator. Through the SDOF ocsillator with the fundamental frequency of 0.1 Hz and the damping ratio of 0.707 critical, the corrected velocity and displacement are obtained by the relative responses as shown in Fig.11.

In these filtering examples the nominal acceleration baseline defined by the difference between the corrected and original acceleration, $A^*(t)-A(t)$, is also sketched in Figs.10 and 11. Conversely, Fig.12 demonstrates the equivalent filtering property which is defined by the Fourier complex ratio of the corrected to the original, $\tilde{A}^*(\omega)/\tilde{A}(\omega)$ where the symbol $\sim$ indicates the Fourier transform with respect to time parameter. It is of interest that both the baseline and the equivalent filter by the proposed correction method have a close resemblance to those by the band-pass filtering.

CONCLUDING REMARKS

A new method has been presented in this paper for the baseline correction of the accelerogram data using the spline function with variable knot. A general formulation has been made for the baseline correction by the spline function and an iterative algorism for highly nonlinear optimization problem with variable knot spline has been introduced. Numerical examples have demonstrated that the conventional parabolic method for the baseline correction is extended and improved by the fixed knot spline methods. It should be emphasized that the knot position is of great consequence in the spline functions and that the variable knot spline method is more efficient. It should be reasonablly concluded that the proposed optimal spline method is very successful and practical even for the accelerogram data with a relatively long duration.

REFERENCES

1. Berg, G.V. and Housner, G.W., "Integrated Velocity and Displacement of Strong Earthquake Motion", B.S.S.A., 51, 175-189, (1961).
2. Trifunac, M.D., "Zero Baseline Correction of Strong-Motion Accelerograms", B.S.S.A., 61, 1201-1211, (1971)
3. Trifunac, M.D. et al., "Analysis of Errors in Digitized Strong-Motion Accelerograms", B.S.S.A., 63, 157-187, (1973).
4. Trujillo, D.M. and Carter, A.L., "A New Approach to the Integration of Accelerometer Data", Earthquake Eng. Struct. Dynamics, 10, 529-535, (1982).
5. Ichida, K. and Yoshimoto, F., Spline Functions and Their Application (In Japanese), Kyoiku Shuppan Company, 92-103, (1979).
6. Nagase, T. et al., "Baseline Correction of Accelerogram Data by Spline Function" (In Japanese), Summaries of Tech. Papers, 1985 Annual Meeting, Archit. Inst. Japan, 79-80, (1985).