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STOCHASTIC RESPONSE SPECTRA CONSIDERING TIME-DEPENDENT SEISMICITY

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SUMMARY

In the previous papers presented by the author, the response spectra have been calculated using a stationary Poisson model for earthquake occurrence. In this paper the stationarity assumption is relaxed by allowing the annual expected arrival rate of the earthquakes to be time-dependent. The time-dependent annual rate is obtained from the cumulative earthquake energy. It is used in a non-stationary Poisson model of earthquake occurrence in conjunction with the procedure outlined with the earlier papers to calculate stochastic response spectra for major cities in Japan. The results obtained from the analyses are discussed with reference to the influence of the time-dependent annual rate.

INTRODUCTION

In our previous study (Refs. 1,2) the response spectra at a given level of non-exceedance probability for a given site during a given lifetime was calculated using (1) the theory of first passage applied to the stationary random response of a linear SDOF system, (2) empirical equations relating the characteristics of the earthquake wave to the magnitude, M, and distance, Δ, and (3) probability distribution of M and Δ based on a local seismicity model.

In our previous study, we assumed that earthquakes occur in each epicentral region in accordance to a Poisson process whose mean is related to the average historical seismicity. However, the cumulative earthquake energy in each epicentral region changes with time and it must greatly influence the probability of earthquake occurrence with time. Therefore, it is necessary to consider the time-dependent characteristics of the occurrence rate when calculating stochastic response spectra. In this study, the occurrence of earthquakes is assumed to be a non-stationary Poisson process, in which the annual expected arrival rate of earthquakes (AEAR) is considered as time-dependent.

This paper discusses a procedure for evaluating the time-dependent AEAR based on the use of the cumulative earthquake energy. The time-dependent AEAR is used in the procedure, discussed in the previous papers by the author, to calculate the stochastic response spectra for a few major cities in Japan. The results of this method are discussed with special emphasis put on the effect of incorporating the time-dependent nature of the AEAR on the stochastic response spectra.

PROCEDURE TO OBTAIN TIME-DEPENDENT AEAR

A procedure for evaluating the time-dependent annual expected arrival rates of earthquakes (AEAR) is outlined below.

The energy, E_n, of an earthquake whose magnitude is M can be expressed as follows.

$$E_n = 10^{11.8+1.5M} \tag{1}$$

The probability density function of M which is bound by the lower and upper limits, m_L and m_U respectively, can be expressed as

$$f_M(m) = C_1 \beta e^{-\beta(m-m_L)} \tag{2}$$

where $\beta = b\ ln10$ (b denotes Gutenberg-Richter's b-value) and $C_1 = 1/[1-exp\{\beta(m_L-m_U)\}]$. From Eq.(1) and (2) the expected value of the energy of earthquakes whose magnitudes lie in the range $m_L \leq M \leq m_U$ is

$$E_{E_n} = \int_{m_L}^{m_U} E_n f_M(m)dm = \frac{C_1}{C_2}\beta e^{\beta m_L+27.17}(e^{C_2 m_U} - e^{C_2 m_L}) \tag{3}$$

where $C_2 = 1.5\ ln10 - \beta$.

In order to estimate the value of the time-dependent AEAR, $\nu(t)$, we assume the relations given by Eq.(4), (5) and (6).

$$\nu_I = a/E_{E_n} \tag{4}$$

$$\nu(t) = \nu_I\ g(t)/g_m \tag{5}$$

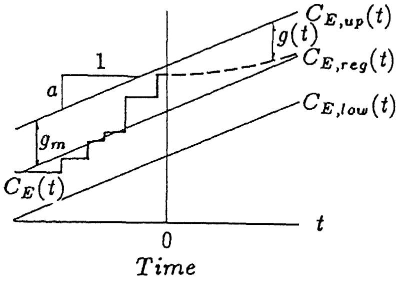
$$n = \int_0^t \nu(t)dt = \frac{g(0) + at - g(t)}{E_{E_n}} \tag{6}$$

where, ν_I is AEAR assuming time-independence, n is the expected earthquake occurrence rate in t years and the other notations are illustrated in Fig.1. Eq.(5) and (6) can be used to obtain a differential equation in $\nu(t)$ as

$$\dot{\nu}(t) + \frac{E_{E_n}\nu_I}{g_m}\nu(t) = \frac{a\ \nu_I}{g_m} \tag{7}$$

Applying $\nu(0) = \nu_I\ g(0)/g_m$ as a initial condition, we can solve for $\nu(t)$ and express it as

$$\nu(t) = \nu_I\{1 + (\frac{g(0)}{g_m} - 1)e^{-\frac{a}{g_m}t}\} \tag{8}$$



$C_E(t)$: Cumulative Earthquake Energy
$C_{E,up}(t)$: Upper Bound of $C_E(t)$
$C_{E,reg}(t)$: Regression Line of $C_E(t)$
$C_{E,low}(t)$: Lower Bound of $C_E(t)$
$g(t) = C_{E,up}(t) - C_E(t)$
$g_m = C_{E,up}(t) - C_{E,reg}(t)$
a : Slope of the Regression Line

Fig.1 Schematic Diagram Showing Cumulative Earthquake Energy

The average time-dependent AEAR during time 0 to t can then be expressed as

$$\nu = \frac{1}{t}\int_0^t \nu(t)dt = \nu_I + \frac{g_m}{E_{E_n}t}(\frac{g(0)}{g_m} - 1)(1 - e^{-\frac{a}{g_m}t}) \tag{9}$$

Eq.(9) can be used to define AEAR by substituting t = lifetime.

APPLICATION AND DISCUSSION

The above method and previous procedure (Refs. 1,2) are applied to a few sites and the results are discussed here. The calculations to obtain response spectra are carried out using numerical integration over magnitude and distance.

Application The non-exceedance probabilities of the acceleration response spectra for a lifetime of 30 years are obtained for three major cities in Japan; Tokyo (139.7°E, 35.7°N), Osaka (135.5°E, 34.7°N), and Fukuoka (130.4°E, 33.6°N). The various postulates that are made in constructing the seismicity model are:

(1) Based on the distribution of the epicenters of historical, damaging earthquakes (Ref. 3) six epicentral regions, R1 through R6, where the seismicities are inferred to be relatively high are set up as shown in Fig.2. The probability of earthquake occurrence is assumed to be uniform over each epicentral region.

(2) The cumulative earthquake energy, $C_E(t)$, of the historical, damaging earthquakes after 1700A.D. in each epicentral region is used. Fig.3 (a) to (f) show $C_E(t)$ of each epicentral region along with the regression line, upper bound and lower bound of $C_E(t)$. The upper and lower bounds are taken as the lines whose slopes are equal to that of the regression line and which pass through the farthest points of $C_E(t)$ from the regression line in the upper and lower direction, respectively.

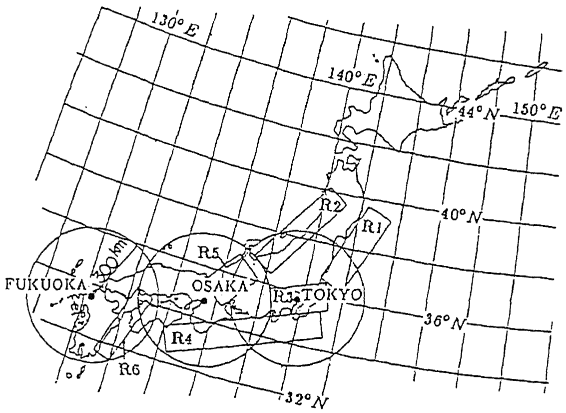
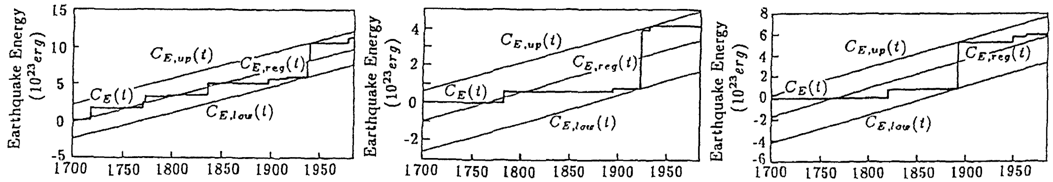


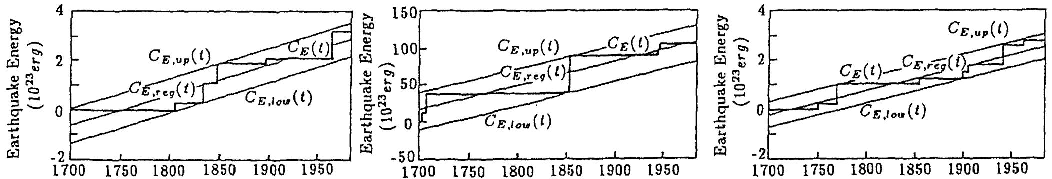
Fig.2 Epicentral Regions in Japan



(a) Region-1 (R1)

(c) Region-3 (R3)

(e) Region-5 (R5)



(b) Region-2 (R2)

(d) Region-4 (R4)

(f) Region-6 (R6)

Fig.3 Cumulative Earthquake Energy in Epicentral Regions R1 - R6

(3) The average time-dependent AEAR, ν, of each epicentral region, R1-R6, is obtained by using the cumulative energy plots shown in Fig.3. In the derivation, $b = 1$, $t = 30 years$ (that is equivalent to the lifetime) and the initial years ($t = 0$) are taken from 1700A.D. to 1980 with 10-year increments. The lower bound magnitude m_L is taken as 5.0 and the upper bound value m_U as the highest value which has been recorded historically in the region. The AEAR per unit area ($1km^2$) versus initial year is plotted for regions 1 to 6 in Fig.4.

(4) The range of influence is assumed to be within a radius of $300km$ of each site and the effect of seismic regions outside this radius is neglected. The epicentral regions, R7-R9, within a radius of $300km$ of each site and outside the specified regions, R1-R6, is taken to have uniform seismicity. The AEAR of each of the regions, R7-R9, is assumed to be time-independent.

(5) The damping ratio, ζ, for response spectra calculations is taken as 0.05, the lifetime as $30 years$ and the ground as hard ($Y_s = 0$).

Discussion The influence of the time-dependent seismicity on AEAR and stochastic response spectra are:

(1) The average time-dependent AEAR, ν , of each epicentral region varies with cumulative earthquake energy. The magnitude of AEAR decreases drastically after a big earthquake. [Fig.3 and Fig.4]

(2) At the present time (after 1950 A.D.), the cumulative earthquake energy and AEAR of most of the epicentral regions are relatively small [Fig.3 and Fig.4].

(3) The equi-probability response spectra (EPRS) shown in Fig.5 (a)-(i) of Tokyo are roughly similar to those of Osaka, but those of Fukuoka are much smaller.

(4) For Tokyo,the EPRS with $I_y = 1920$ (before the 1923 Kanto Earthquake) are very large and the EPRS with $I_y = 1950$ are much smaller. The EPRS with $I_y = 1980$ are between those of $I_y = 1920$ and 1950. These results show the dominant influence of the Kanto Earthquake which occurred in the region R3 [Fig.5 (a) to (c)].

(5) For Osaka, the EPRS increase as I_y approaches the present time, from a value of 1920 to 1980. The increase can be considered to be related to the increase in the cumulative earthquake energy in the region R5 [Fig.5 (d) to (f)].

(6) For Fukuoka, the influence of the time-dependent AEAR on EPRS is quite small [Fig.5 (g) to (i)].

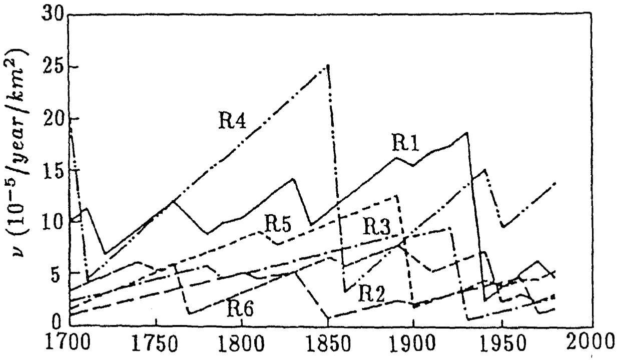
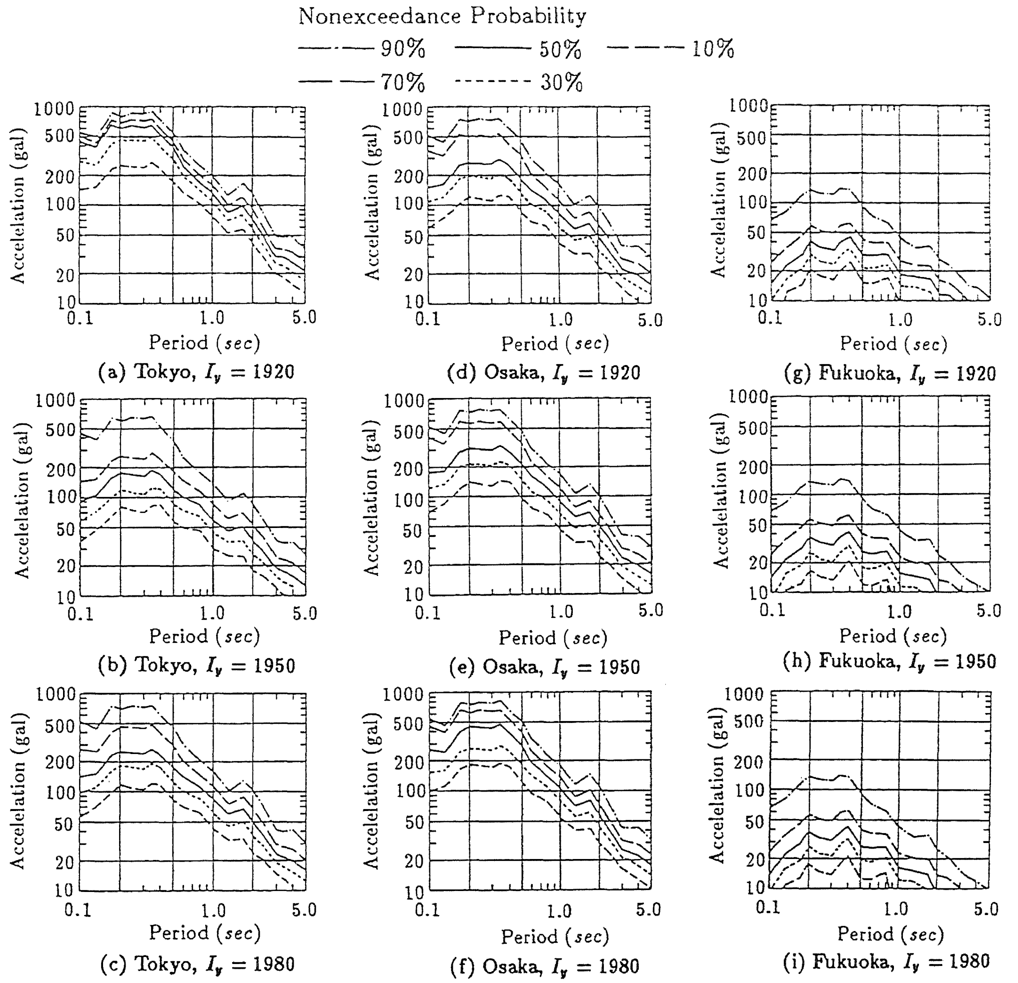
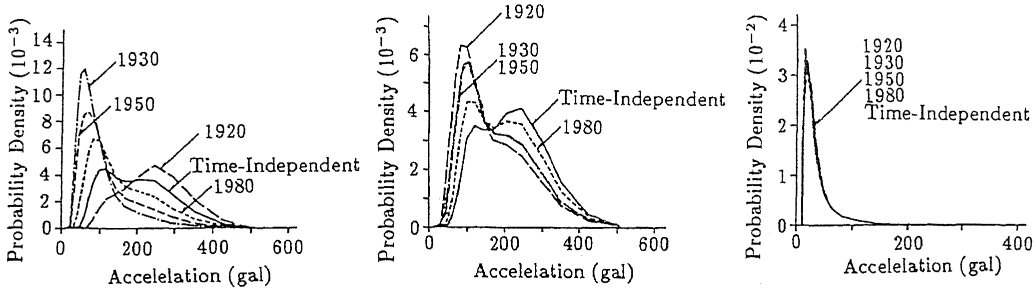


Fig.4 Annual Expected Arrival Rates of Earthquakes (AEAR) in Epicentral Regions R1 - R6



(a) Tokyo, $I_y = 1920$
(d) Osaka, $I_y = 1920$
(g) Fukuoka, $I_y = 1920$
(b) Tokyo, $I_y = 1950$
(e) Osaka, $I_y = 1950$
(h) Fukuoka, $I_y = 1950$
(c) Tokyo, $I_y = 1980$
(f) Osaka, $I_y = 1980$
(i) Fukuoka, $I_y = 1980$

Fig.5 Equi-Probability Response Spectra for Major Cities in Japan, Changing Initial Year, I_y, of Lifetime as a Parameter ($\zeta = 0.05$, $Lifetime = 30years$, $Hard\ Ground$)



(a) Tokyo
(b) Osaka
(c) Fukuoka

Fig.6 PDF of Maximum Response Spectra, Changing Initial Year, I_y ($T_0 = 0.6sec$, $\zeta = 0.05$, $Lifetime = 30years$, $Hard\ Ground$)

(7) Fig.6 (a), (b)and (c) show the probability density functions (PDF) of the effect of I_y on the maximum response spectra at the period $T_0 = 0.6$ sec for Tokyo, Osaka and Fukuoka respectively. In addition, the PDF calculated by assuming time-independent seismicity are plotted in Fig.6 for comparison. The influence of the time-dependent AEAR on PDF can be observed clearly for Tokyo and Osaka. In particular, paying attention to Tokyo shortly before and after the 1923 Kanto Earthquake, the PDF corresponding to $I_y = 1920$ is quite different from that of $I_y = 1930$.

CONCLUSIONS

A procedure to evaluate the time-dependent annual expected arrival rates of earthquakes (AEAR) is developed using the cumulative earthquake energy. The time-dependent AEAR is used in the author's previously published procedure to calculate the stochastic response spectra at a few major Japanese cities. The results from this analysis show the influence of the time-dependent AEAR on the response spectra. These can be summarized as follows:

(1) For $I_y = 1920$ and $I_y = 1930$, shortly before and after the 1923 Kanto Earthquake, the PDF of maximum response spectra of Tokyo are quite different. This is a good example which clearly illustrates the influence of the time-dependent seismicity on the stochastic response spectra.

(2) The stochastic response spectra of Osaka vary with I_y. As for Tokyo, this means that the cumulative earthquake energy of the epicentral regions close to the site strongly contributes to the stochastic response spectra.

(3) For sites where seismic risk is relatively high, the time-dependent characteristics of seismicity could strongly influence the stochastic response spectra. Therefore, the time-dependent characteristics of seismicity should be taken into account when estimating seismic forces for design.

The method proposed in this paper is useful and makes it feasible to discuss the effect of the time-dependent characteristics of seismicity on the probability density function of maximum response spectra.

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