EARTHQUAKE ANALYSIS OF CABLE-STAYED BRIDGES UNDER THE ACTION OF TRAVELLING WAVES

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SUMMARY

In this paper the influence of phase-difference effect on the earthquake response of the Yonghe bridge (Tianjin , China) is analysed.

According to the principle of Newmark's design spectrum a modification to the standard design spectrum of China-Code in the range of long periods ($T \geq 3.5s$) is suggested. A synthetic accelerogram compatible with the modified design spectrum is used in the earthquake analysis.

The result shows that, for the cable-stayed bridge with a "swim"-pattern, the part of dynamical response is much reduced, so that the phase-difference effect is still favorable, although the quasi-statical response should be added.

INTRODUCTION

The Yonghe bridge (Tianjin , China, Fig. 1), which was designed by taking the Pasko-Kennewick bridge (USA) as a prototype, is located in a high seismicity area. It will be the longest one of the newly built cable-stayed bridges in China with a main span of 260 m and a "swim"-pattern.

In general, the earthquake analysis of structures is based on the assumption that the ground, on which the structure stands, moves as a rigid base. Actually the finite propagation velocity of the surface waves in the ground will cause phase differences at various support points. These effects are naturally significant to the long-span structures, such as cable-stayed bridges.

In this paper the influence of phase-difference effect on the earthquake response of the Yonghe bridge is analysed.

EQUATION OF MOTION IN CONSIDERATION OF PHASE-DIFFERENCE EFFECT

In the case of cable-stayed bridge described diagrammatically in Fig. 2 the pylons 1 and 2 with a time-difference of ℓ/c are excited by the ground acceleration $\{ \ddot{\delta}_g \}$, that is the longitudinal horizontal ground motion excitation at two tower supports, namely

$$\{ \ddot{\delta}_g \} = \begin{Bmatrix} \delta_{g1} \\ \delta_{g2} \end{Bmatrix} \quad \text{with} \quad \ddot{\delta}_{g2} (t) = \ddot{\delta}_{g1} (t - \frac{\ell}{c}) \qquad (1)$$

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where c is the velocity of surface waves, which is about 100 to 200 m/s for clay soil.

The deformation of structure due to the non-coherent relative displacement of two towers can be expressed in the form

$$\{\delta_{OB}\} = (F_{OB})\ \{\delta_g\} \tag{2}$$

where (F_{OB}) is a matrix of influence function, which can be determined simply by statics[1].

The system-equation of motion in consideration of the phase-difference effect is written as

$$(M_B)\{\ddot{\delta}_B\} + (C_B)\{\dot{\delta}_B\} + (K_B)\{\delta_B\} = -\ (M_B)\{\ddot{\delta}_{OB}\} = -\ (M_B)(F_{OB})\{\ddot{\delta}_g\} \tag{3}$$

For a linear system the modal analysis can be used. Substituting the modal coordinates

$$\{\delta\ \} = (\Phi)\ \{Y\}$$

into equation (3) and noticing the generalized mass in normalized coordinate $M_n = 1$, we obtain the uncoupled modal equation in the form:

$$\ddot{Y}_n + 2\xi_n\,\omega_n\,\dot{Y}_n + \omega_n^2\,Y_n = -\ \{\Phi_n\}^T(M_B)(F_{OB})\{\ddot{\delta}_g\} = P_n(t) \tag{4}$$

If the velocity of surface waves $c \to \infty$, then $\ddot{\delta}_{g1} = \ddot{\delta}_{g2} = \ddot{\delta}_g$ and $(F_{OB})\{\delta_g\} \to \{I_x\}\ \delta_g$, the equation of motion will return to the following well-known form without considering the phase-difference effect

$$\ddot{Y}_n + 2\,\xi_n\,\omega_n\,\dot{Y}_n + \omega_n^2\,Y_n = -\ \{\Phi_n\}(M_B)\{I_x\}\,\ddot{\delta}_g = -\ \Gamma_n\,\ddot{\delta}_g$$

Therefore, the matrix $(F_{OB}) = (F_{OB1}, F_{OB2})$ has the nature as follows:

$$\{F_{OB1}\} + \{F_{OB2}\} = \{I_x\}$$

The total displacement of nodals as the response of the system are:

$$\{\delta\}_{tot} = \{\delta_B\} + \{\delta_{OB}\} = (\Phi)\ \{Y\} + (F_{OB})\{\delta_g\} \tag{5}$$

in which,

$\{\delta_{OB}\}$ quasi-statical relative nodal displacement due to the opposite displacement of supports --- quasi-statical response;

$\{\delta_B\}$ dynamical relative nodal displacement caused by it --- dynamical response.

The total internal forces in the structure can be expressed accordingly:

$$\{S\}_{tot} = \{S_B\} + \{S_{OB}\} \tag{6}$$

MODAL PARTICIPATION FACTOR AND MODAL GROUND ACCELERATION IN CONSIDERATION OF PHASE-DIFFERENCE EFFECT

The modal equation of motion (4) may also be written in the compact form:

$$\ddot{Y}_n + 2\xi\,\omega_n \dot{Y}_n + \omega_n^2 Y_n = -(\Gamma_n^p)\{\ddot{\delta}g\} = -\Gamma_n\,\ddot{\delta}_{gn}^p \tag{7}$$

in which the matrix of participation factor reflecting the phase-difference of excitation is described as:

$$(\Gamma_n) = \{\Phi_n\}^T (M_B)(F_{OB}) = (\Gamma_{n1}^p,\ \Gamma_{n2}^p)$$

and moreover

$$\Gamma_{n1}^p + \Gamma_{n2}^p = \Gamma_n \tag{8}$$

where Γ_n is a participation factor for the identical excitation of all support points.

A weighted average of the ground acceleration from the different support points is introduced in the form:

$$\ddot{\delta}_{gn}^p = \frac{1}{\Gamma_n}(\Gamma_{n1}^p\,\ddot{\delta}_{g1} + \Gamma_{n2}^p\,\ddot{\delta}_{g2}) \tag{9}$$

It describes the modal ground acceleration in consideration of the phase-difference effect. The above-mentioned relation can naturally be generalized to the case with m-support excitation. Thus, equations (8) and (9) may be adapted as:

$$\sum_{i=1}^{m} \Gamma_{ni}^p = \Gamma_n \tag{10}$$

$$\ddot{\delta}_{gn}^p = \frac{1}{\Gamma_n}\sum_{i=1}^{m} \Gamma_{ni}^p\,\ddot{\delta}_{gi} \tag{11}$$

In the present case (two pylons, symmetrical structure) we have:

$$\Gamma_{n1}^p = \Gamma_{n2}^p = \frac{\Gamma_n}{2} \quad \text{for all antisymmetrical eigenforms;}$$

$$\Gamma_{n1}^p = -\Gamma_{n2}^p \quad \text{for all symmetrical eigenforms.}$$

Therefore, for the antisymmetrical eigenforms,

$$\ddot{\delta}_{gn}^p = \frac{1}{2}(\ddot{\delta}_{g1} + \ddot{\delta}_{g2}) \tag{12}$$

the equation of motion may be expressed as:

$$\ddot{Y}_n + 2\xi_n\omega_n\dot{Y}_n + \omega_n^2 Y_n = -\Gamma_n \cdot \frac{1}{2}(\ddot{\delta}_{g1} + \ddot{\delta}_{g2})$$

so that the influence of phase-difference effect can be recognized in a response spectrum of this ground acceleration.

For the symmetrical eigenforms, the equation of motion can be shown as:

$$\ddot{Y}_n + 2\xi_n\omega_n\dot{Y}_n + \omega_n^2 Y_n = -\Gamma_{n1}(\ddot{\delta}_{g1} - \ddot{\delta}_{g2}) \tag{13}$$

It is well-known that, in the case of coherent excitation, the only contribution of antisymmetrical eigenforms exists due to the antisymmetricity of horizontal ground motion. However, in the case of non-coherent excitation, not only the antisymmetrical eigenforms, but also the symmetrical eigenforms will make contributions to the earthquake response.

MODIFICATION TO THE STANDARD DESIGN SPECTRUM IN THE RANGE OF LONG PERIODS ($T > 3.5s$) AND ITS CORRESPONDING SYNTHETIC ACCELEROGRAM

A wide-band standard design spectrum provides in any case a basis for the safe and rational earthquake-resistant design due to the random character of future earthquake.

According to the principle of Newmark's design spectrum[2] a modification to China-Code spectrum [3] in the range of long periods ($T > 3.5s$, line CD in Fig. 3 is replaced by line CD') is recommended. Fig. 4 shows graphically a modification formula suggested by the author for the acceleration amplification factor of China-Code spectrum in the range of periods $T > 3.5s$ in comparison with the response spectrum of El Centro and also Tianjing earthquakes.

Fig. 5 shows a 20s long synthetic accelerogram, whose response spectrum is in good agreement with the modified design spectrum of China-Code (Fig. 6, $\Delta T = 0$).

RESPONSE SPECTRUMS IN CONSIDERATION OF PHASE-DIFFERENCE EFFECT AND HORIZONTAL DISPLACEMENT RESPONSE AT THE TOP OF PYLON

Three response spectrums with time-differences of $\Delta T = 260\,m/c = 1,2$ and 3s (c = 260, 130 and 86.7 m/s) to a synthetic ground accelerogram compatible with the China-Code spectrum are shown in Fig. 6.

It is clear that the spectral ordinates in consideration of the phase-difference effect become smaller, and therewith the dynamical displacements δ_B and internal forces δ_B are always less than those obtained without considering the phase-difference. This is understandable, because the average ground acceleration δ^p_{gn} is in any case smaller than δ_g . Nevertheless, the quasistatic response should added to the dynamic one.

From economical reasons the phase-difference effect is investigated on a simplified model of cable-stayed bridge. This model of system, called "PHERS", is shown in Fig.7, its dynamic behaviour agrees approximately with that of the real bridge system. We consider the combination of the first three eigenforms, in which the first and third eigenforms are antisymmetrical, and the second one is symmetrical. The results are summarized clearly in Fig. 8, using the horizontal displacement at the top of pylon d_k as a decisive characteristic magnitude.

It is interesting to note that the change of d_k in terms of the time-difference ΔT is a periodic function with a period nearly equal to the basic eigenperiod of 5.86s of the sys tem. Consequently, the phase-difference effect is favorable in general to the "swim"-variant with its long basic eigenperiod.

It can be seen that, the part of dynamical response (curve d_B^8 in Fig.8) is much reduced due to the non-coherent excitation of two supports. This reduction is mainly due to the first antisymmetrical eigenform with a predominant modal participation factor. The spectral displacement of 27.37cm under coherent excitation ($\Delta T=0$) reduces to about 15cm, when $\Delta T=3s$ (see Fig. 6). Therefore, the displacement response at the top of tower of 0.245cm reduces accordingly to 0.137cm (see Table 2). After the summation of the part of quasi-static response (curve d_{OB}^8 in Fig. 8), the total response is still smaller than that under coherent excitation.

CONCLUSIONS

The equation of motion of cable-stayed bridge under non-coherent excitation of supports is derived by using the matrix of influence function. Through the modal analysis the modal participation factor and the modal ground acceleration in consideration of phase-difference effect are obtained, so that the phase-difference effect can also be judged in a response spectrum of this modal ground acceleration.

According to the principle of Newmark's design spectrum a modification to the standard design spectrum of China-Code in the range of long periods ($T > 3.5s$) is suggested. It is safe and rational to carry out the earthquake analysis by using a synthetic accelerogram compatible with the modified design spectrum.

The result shows that, for the cable-stayed bridge with a "swim"-pattern the part of dynamical response is much reduced due to the non-coherent excitation of two supports, so that the phase-difference effect is still favorable, although the quasi-statical response should be added.

REFERENCES

(1) Li, K.H.: Structural Dynamics for Aseismic Design, (in Chinese) 1980

(2) Newmark, N.M.; Hall, W.J.: Procedures and Criteria for Earthquake-resistant Design, Building Science Series 46, Building Practices for Disaster Mitigation, National Bureau of Standard, 1973

(3) China-Code for Aseismic Design of Highway Bridges, (in Chinese) 1978

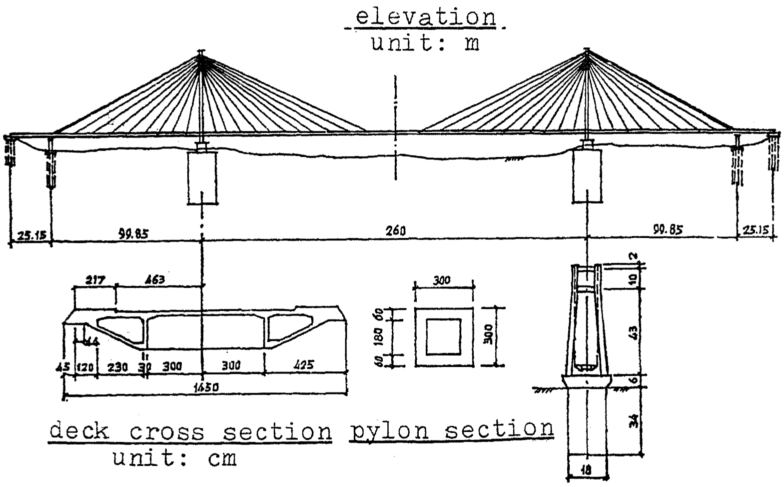

Fig. 1 Yonghe bridge (Tianjin)

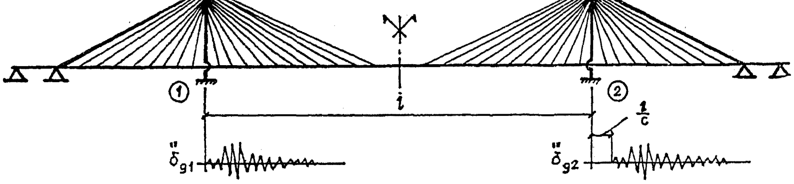

Fig. 2 Non-coherent excitation of cable-stayed bridges

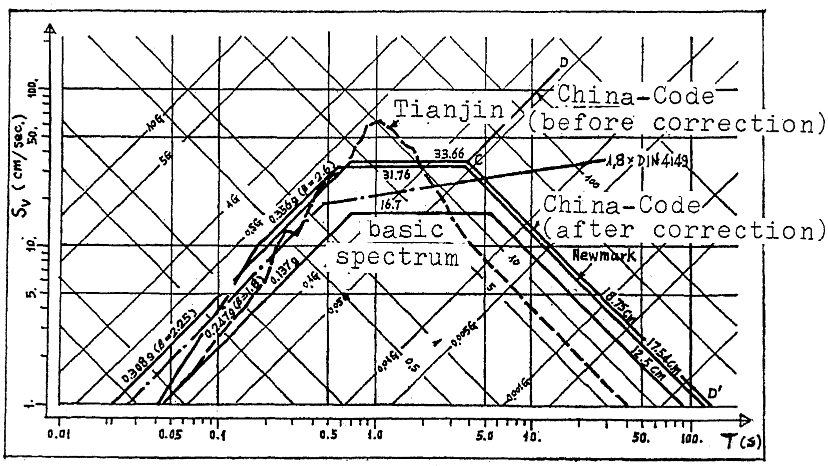

Fig. 3 Design spectrum of China-Code in comparison with Newmark's and Tianjin spectra

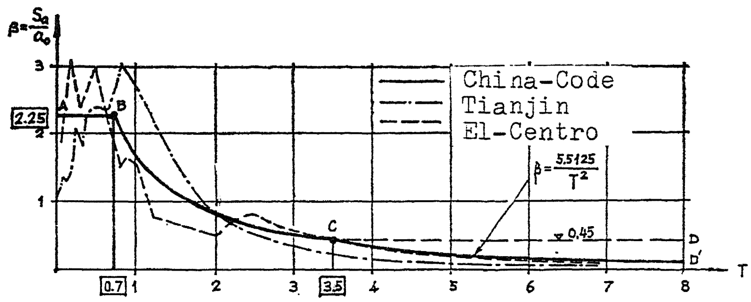

Fig. 4 Correction in the range of long period

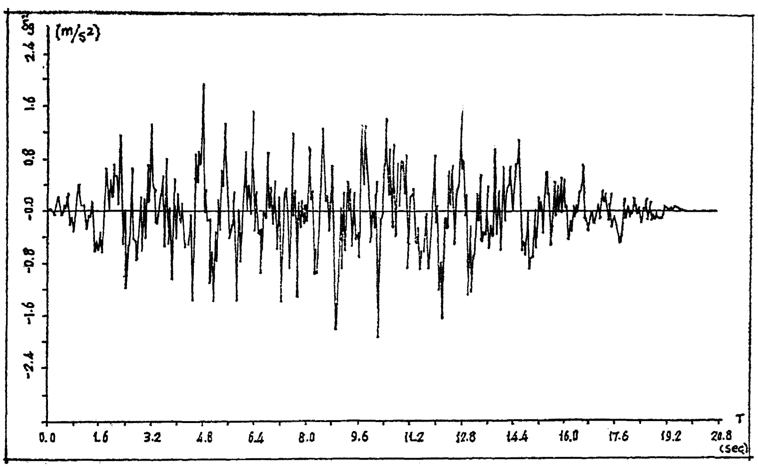

Fig. 5 Synthetic accelerogram compatible with corrected design spectrum of China-Code

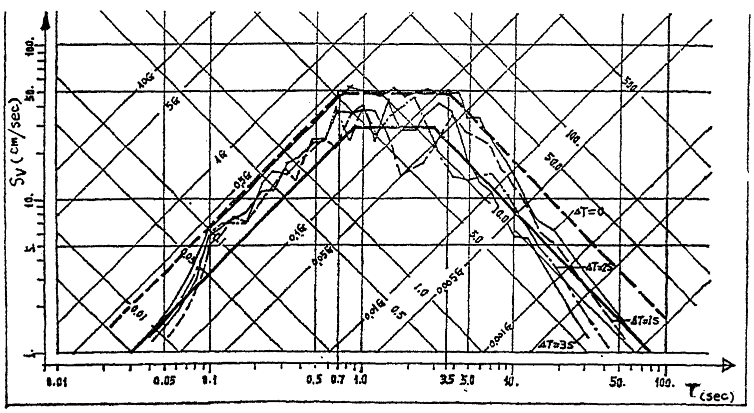

Fig. 6 Response spectra in consideration of phase-difference effect

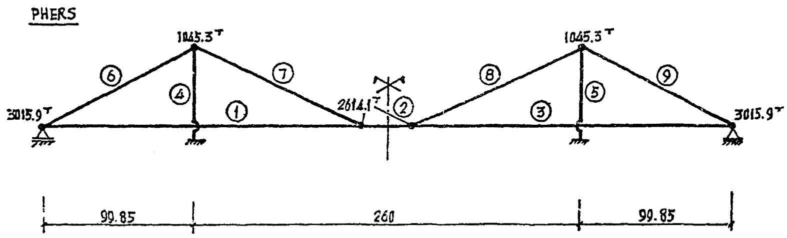

Fig. 7 Simplified substitutive model

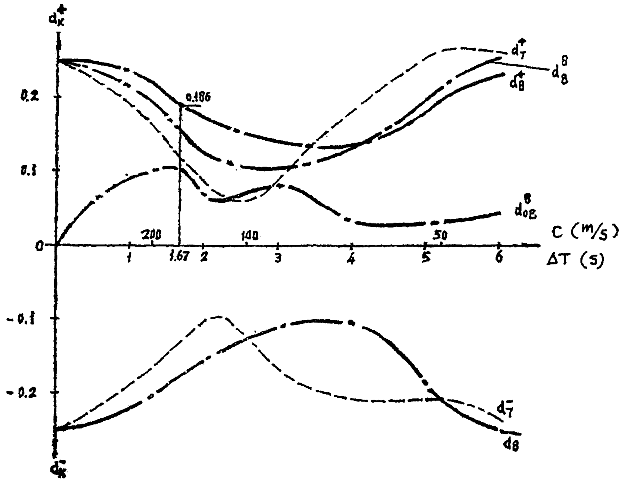

Fig. 8 Max. hor. displacement at the top of pylon vs. time-difference ΔT