

EVALUATION OF SIMPLIFIED EARTHQUAKE ANALYSIS PROCEDURES  
FOR INTAKE-OUTLET TOWERS

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SUMMARY

The internal forces in uniform and tapered intake-outlet towers induced by earthquake ground motion, characterized by a smooth design spectrum, are computed for a range of fundamental vibration period by modal analysis and by the Montes-Rosenblueth procedure. Based on these results it is shown that the latter procedure is excessively conservative in many cases. The limitations of other available simplified analysis procedures, and the requirements for a reliable but simple analysis procedure for preliminary design of towers are identified.

INTRODUCTION

Intake-outlet towers should be designed to elastically resist the relatively frequent moderate intensity earthquakes; and may be permitted to yield significantly, but without collapse, in the event that very intense ground shaking occurs. An approximate analysis procedure that considers only the most important effects in the earthquake response of towers, and yet is simple to apply, is required in the preliminary stages of elastic design. The available simplified analysis procedures (Refs. 1-3) are evaluated in this paper.

VIRTUAL MASS OF TOWER

It has been established that the effects of surrounding water (Fig. 1) on dynamics of towers may be approximately represented by the added mass shown in Fig. 2 (Ref. 4). The virtual (or total) mass of the tower is

$$m(z) = m_o(z) + m_i(z) + m_a(z) \quad (1)$$

in which  $m_o(z)$  = the mass of the tower by itself,  $m_i(z)$  = the mass of water inside the tower, and  $m_a(z)$  = the added mass due to interaction with surrounding water.

The added mass  $m_a(z)$  presented in Fig. 2 is for cylindrical towers with a circular cross-section for a range of values of the ratio  $r_o/H$ , in which  $r_o$  = the outside radius; and  $H$  = the depth of surrounding water. Strictly speaking these results are valid only for towers with radius  $r_o$  constant over height; however they are even useful for towers with a variable radius. It is recommended that the added mass at any location,  $z$ , above the base be computed from the curve for  $r_o/H = r(z)/H$  pertaining to that location. This simply obtained approximate value checks well with accurate solutions based on a finite element analysis of the fluid domains surrounding variable-radius towers (Ref. 5).

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The inside water is treated as moving rigidly with the tower, an appropriate idealization for slender towers.

#### MODAL ANALYSIS

The earthquake analysis of a free-standing tower partially submerged in water proceeds in the standard manner, with its mass defined by the virtual mass  $m(z)$  instead of just its own mass  $m_o(z)$ . Thus, the maximum response of a tower to earthquake ground motion can be estimated from the design (response) spectrum corresponding to the ground motion by the following procedure:

1. Define structural properties: compute virtual mass  $m(z)$  and flexural stiffness  $EI(z)$ ; and estimate modal damping ratios  $\xi_n$ .
2. Compute the frequencies  $\omega_n$ , periods  $T_n = 2\pi/\omega_n$ , and mode shapes  $\phi_n(z)$ ,  $n=1, 2, \dots$ , of several natural modes of vibration of the tower.
3. Compute the maximum response in individual modes of vibration by repeating the following steps for the lower modes of vibration:

(a) Corresponding to period  $T_n$  and damping ratio  $\xi_n$ , read the ordinate  $S_{an}$  of the pseudo-acceleration from the design spectrum.

(b) Compute equivalent lateral forces from

$$f_n(z) = (L_n/M_n) S_{an} m(z)\phi_n(z) \quad (2)$$

$$\text{where } L_n = \int m(z)\phi_n(z) dz \quad \text{and} \quad M_n = \int m(z)\phi_n^2(z) dz.$$

(c) Compute the shear  $V_n$  and moment  $M_n$  at any section by static analysis of the tower subjected to the equivalent lateral forces:

$$V_n(z) = \int_z^{H_s} f_n(\zeta) d\zeta ; \quad M_n(z) = \int_z^{H_s} f_n(\zeta)(\zeta - z) d\zeta \quad (3)$$

4. Determine an estimate of the maximum shear  $V(z)$  and moment  $M(z)$  at any section by combining the modal maxima  $V_n(z)$  and  $M_n(z)$  in accordance with the following equation:

$$V(z) \doteq \sqrt{\sum_n V_n^2(z)} \quad ; \quad M(z) \doteq \sqrt{\sum_n M_n^2(z)} \quad (4)$$

#### MONTES-ROSENBLUETH PROCEDURE

This is a simplified analysis procedure developed by Montes and Rosenblueth for estimating earthquake forces in chimneys (Ref. 3). By using the virtual mass  $m(z)$  to define the mass, the procedure can be applied to free-standing intake-outlet towers in the following steps:

1. Construct two envelopes of the design spectrum: (a) flat spectrum and (b) hyperbolic spectrum with cut-off, as shown in Fig. 3. The ordinate of the flat spectrum, which represents a constant pseudo-acceleration, is equal to the maximum value over all periods less than the fundamental period  $T_1$ . The hyperbolic spectrum, which represents pseudo-acceleration varying with period as a hyperbola or a constant pseudo-velocity, passes through the ordinate of the design spectrum at  $T_1$ ; the spectrum is cut-off to a flat branch for all periods less than  $T_1/10$ .

2. Compute shears and moments in the tower associated with the flat spectrum from the following equations:

$$V(z) = 0.647(\tilde{S}_{a1}/g)W[1 - (z/H_s)^3] \quad (5a)$$

$$M(z) = 0.461(\tilde{S}_{a1}/g)WH_s[1 - (z/H_s)]^{3/2} \quad (5b)$$

where  $\tilde{S}_{a1}$  is the maximum value of  $S_a(T)$  over periods less than  $T_1$  and  $W$  is the total, virtual weight of the tower.

3. Compute shears and moments in the tower corresponding to the hyperbolic spectrum with cut-off from the following equations:

$$V(z) = 1.553(S_{a1}/g)W\{[1 - (z/H_s)^2]^{1/2} - 6.25(z/H_s)^2[1 - (z/H_s)]^2\} \quad (6a)$$

$$M(z) = 0.519(S_{a1}/g)WH_s[1 - (z/H_s)] \quad (6b)$$

4. The M-R (Montes-Rosenblueth) estimate of the shear (and moment) at any section is provided by the smaller of the two values for the shear (and moment) obtained in steps 2 and 3 (Fig. 4).

The approximate expressions for shear and moment in steps 2 and 3 were obtained in Ref. 3 from results of analysis of uniform towers for the two idealized spectra mentioned above. The M-R estimates are equal for two towers with the same total weight, independent of the weight and stiffness distribution.

#### PRESENTATION AND DISCUSSION OF RESULTS

The two types of towers analyzed are: (a) Uniform towers with mass per unit height  $m(z) = m$  and flexural stiffness  $EI(z) = EI$ , both constant over height; and (b) tapered towers with mass and stiffness decreasing linearly from the base ( $z=0$ ) to the top ( $z=H$ ) with  $m(H) = m(0)/9$  and  $EI(H) = EI(0)/10$ . The latter represents an extreme taper, more than usually encountered in real towers, chosen to cover all practical cases and, in part, to indirectly and roughly consider the variation in virtual mass that would be introduced by the added mass (Fig. 2). For the limited objectives of this paper, it is not necessary to explicitly include the added mass  $m_a(z)$  and the virtual, total mass  $m(z)$  is defined directly as described above.

The parameters selected for the ground motion are: maximum acceleration  $\bar{a}_g$ , velocity  $\bar{v}_g$ , and displacement  $\bar{d}_g = 1g, 48\text{in/sec},$  and  $36\text{in.}$ , respectively. Starting with these parameter values, the design spectrum is constructed by the procedures of Ref. 6 for 5% damping ratio and 84.1 percentile level of response (Fig. 5). The response results are presented in dimensionless form so that they are valid for ground motions of any intensity, provided  $\bar{a}_g, \bar{v}_g,$  and  $\bar{d}_g$  are in the ratio  $1g:48\text{in/sec}:36\text{in.}$

The internal forces (shears and moments) in two types of towers induced by earthquake ground motion characterized by the design spectrum of Fig. 5 were computed for a range of fundamental vibration periods by modal analysis and by the Montes-Rosenblueth Procedure. The natural frequencies and mode shapes of vibration required in modal analysis are available in text books as standard results for a uniform cantilever. For the tapered tower they were obtained by the Rayleigh Ritz method with the shape functions selected as the

mode shapes of a uniform tower. The variation of shears and moments with height in uniform towers with different fundamental vibration periods are presented in Fig.6; similar results for tapered towers are presented in Fig. 7. For three selected values of fundamental period  $T_1$ , the heightwise variation of shears and moments obtained by modal analysis and by Montes-Rosenblueth (M-R) procedure are presented in Figs.8-10 for uniform towers and in Figs.11-13 for tapered towers. The variation of base shear and moment with fundamental vibration period, obtained by the two analysis procedures, is presented in Figs. 14 and 15.

These response results demonstrate that the M-R estimates for shears and moments are reasonably good only for uniform towers with fundamental vibration period which is either rather long, near the long-period end of the constant pseudo-velocity branch of the spectrum; or short enough to fall on the constant pseudo-acceleration branch of the spectrum. In the first case, the M-R estimate is controlled by eq.6 associated with the hyperbolic spectrum; and in the latter case by eq.5 associated with the flat spectrum. At intermediate-periods the M-R estimate is excessively large, because the response is affected by both types of spectra, a situation illustrated in Fig. 4. The mass and stiffness distribution over height does not enter into the M-R estimate for shears and moments; only the total weight enters into eqs.5 and 6. When presented in dimensionless form the M-R estimate is thus identical for uniform and tapered towers. However, the results of modal analysis demonstrate that the dimensionless responses of a tapered tower are much smaller than those of a uniform tower. Thus the M-R estimate for the response of tapered towers is excessively conservative over the entire period range.

The presented response results demonstrate that, over a wide range of fundamental periods, excellent results for shears and moments in towers are obtained by considering only the first two vibration modes in modal analysis. For short period towers, with fundamental period shorter than the value at the corner of constant pseudo-acceleration and constant pseudo-velocity branches of the spectrum, only the first mode contributes significantly to the forces, and even the second mode need not be included. For towers with longer vibration periods, the one-mode analysis recommended in Ref.1 is inadequate because the second mode has significant contributions. The second mode response should be explicitly considered because, contrary to what was suggested in Ref.2, it can not be satisfactorily approximated by simply increasing the first mode response.

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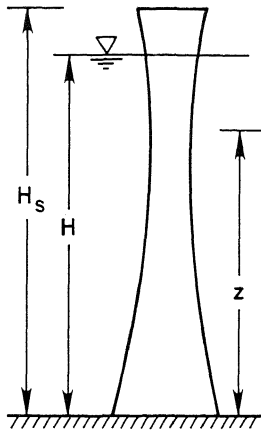


Fig. 1 Tower Submerged in Water.

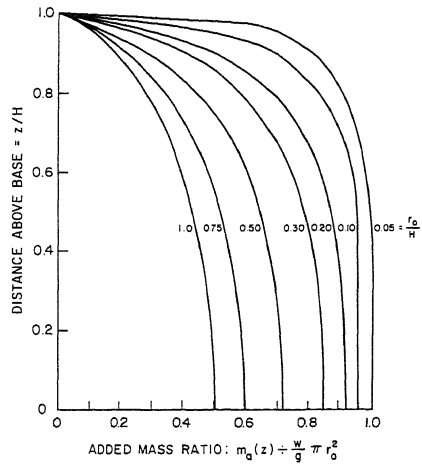


Fig. 2 Added-mass Representing Hydrodynamic Effects.

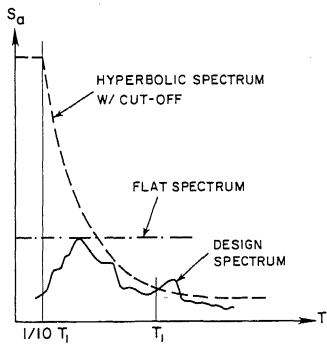


Fig. 3 Envelopes of Design Spectrum.

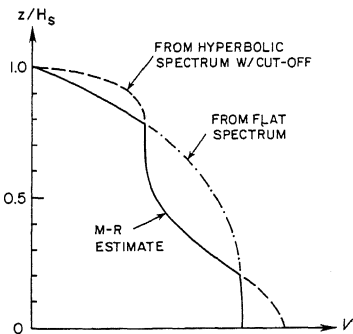


Fig. 4 Explanatory Sketch to Obtain M-R Estimate of Shears.

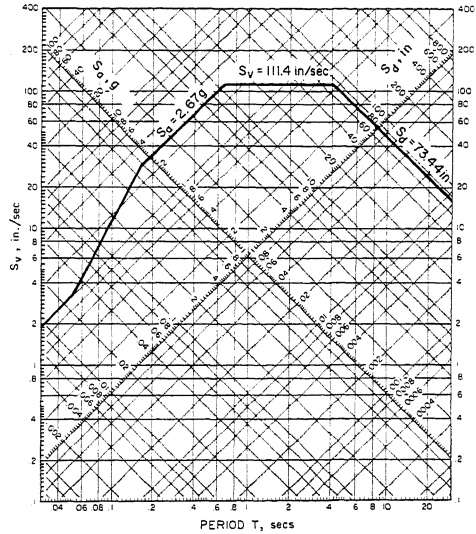


Fig. 5 Earthquake Design Spectrum.

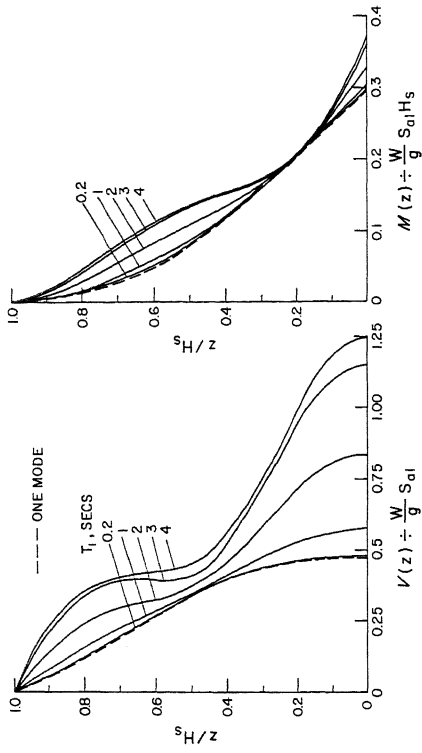


Fig. 6 Heightwise Variation of Shears and Moments in Uniform Towers.

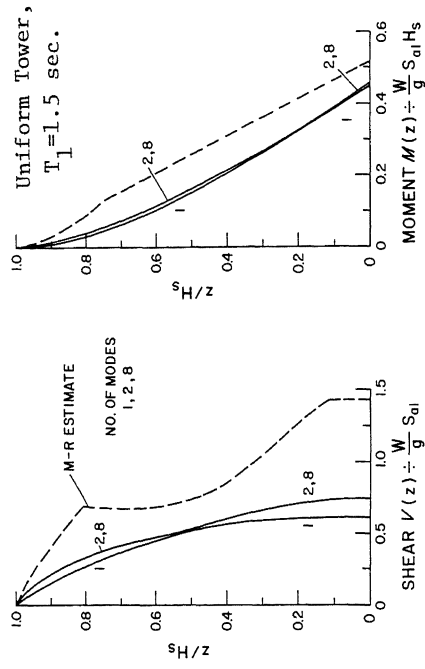


Fig. 7 Heightwise Variation of Shears and Moments in Tapered Towers.

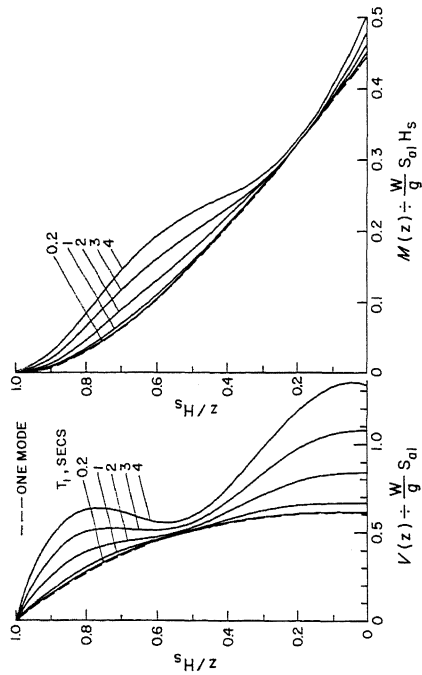


Fig. 8 Comparison of Shears and Moments from Modal Analysis and M-R procedure.

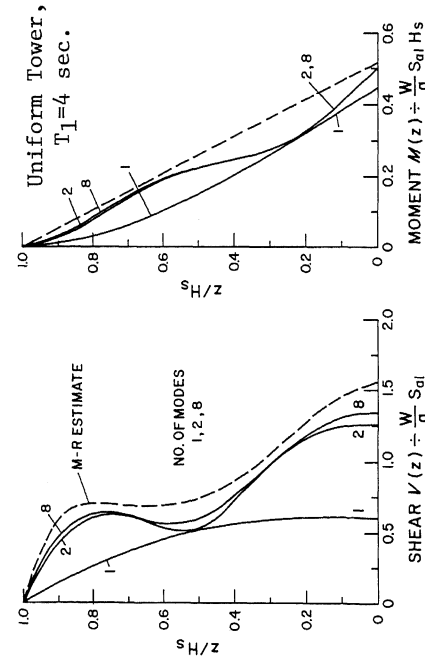


Fig. 9 Comparison of Shears and Moments from Modal Analysis and M-R Procedure.

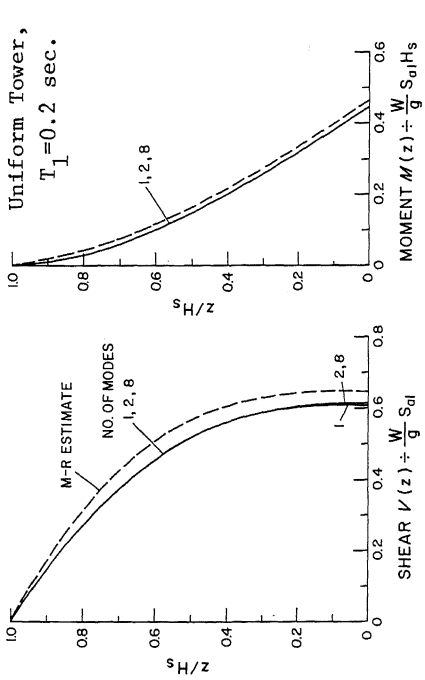


Fig. 10 Comparison of Shears and Moments from Modal Analysis and M-R Procedure.

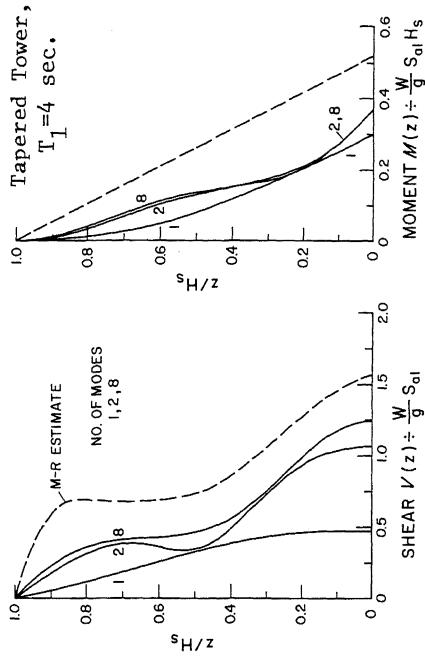


Fig. 11 Comparison of Shears and Moments from Modal Analysis and M-R Procedure.

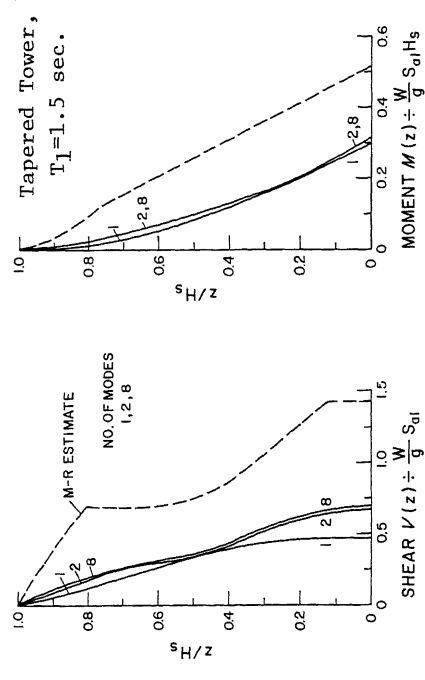


Fig. 12 Comparison of Shears and Moments from Modal Analysis and M-R Procedure.

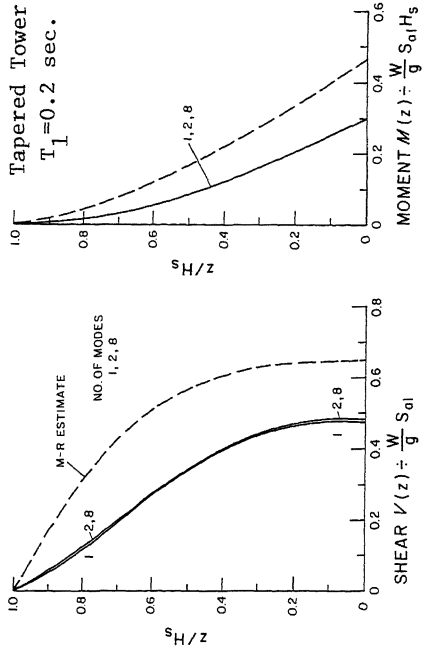


Fig. 13 Comparison of Shears and Moments from Modal Analysis and M-R Procedure.

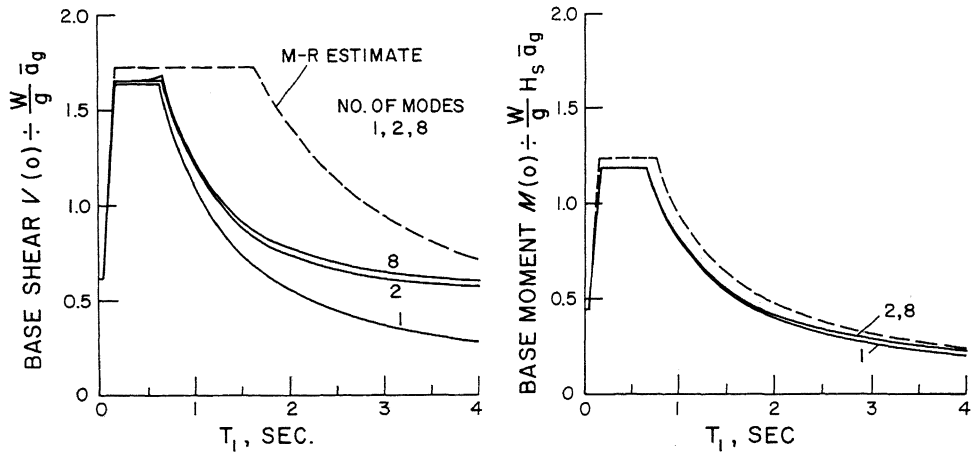


Fig. 14 Comparison of Base Shears and Moments in Uniform Towers from Modal Analysis and M-R Procedure.

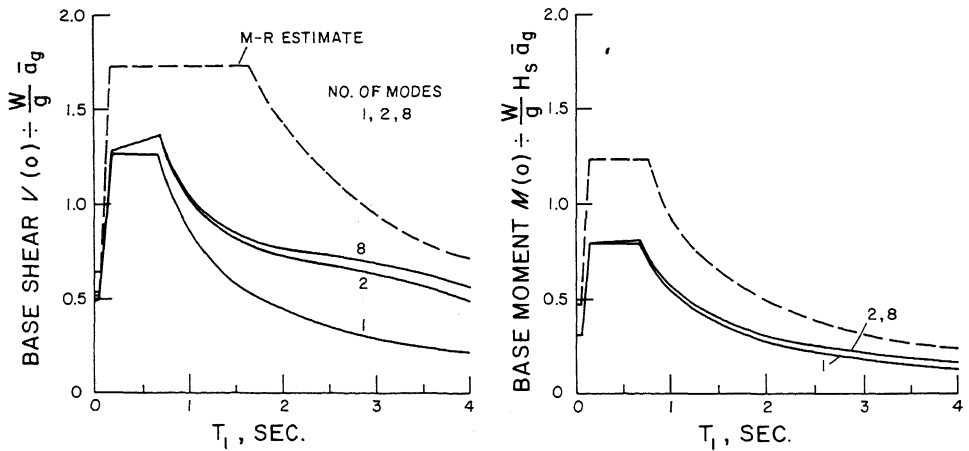


Fig. 15 Comparison of Base Shears and Moments in Tapered Towers from Modal Analysis and M-R Procedure.