DYNAMIC RESPONSE OF VERTICALLY EXCITED LIQUID STORAGE TANKS

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SUMMARY

The highlights of an investigation into the response of circular cylindrical liquid storage tanks to a vertical component of ground shaking are presented, including a simple, approximate procedure for evaluating the principal effects. It is shown that the hydrodynamic effects for a flexible tank may be substantially larger than those induced in a rigid tank of the same dimensions, and for an intense excitation, they may be of the same order of magnitude as the hydrostatic effects. The analysis presented makes no provision for the influence of soil-structure interaction, which is expected to be important in this problem.

INTRODUCTION

Previous studies of the seismic response of liquid storage tanks (e.g., Refs. 1, 2) were concerned almost exclusively with the effects of the horizontal component of ground shaking. The first comprehensive analysis of the effects of the vertical component of excitation appear to have been made in a dissertation by Kumar (Ref. 3), and it is the objective of this paper to highlight the more important results of this study. Special effort is made to emphasize the practical significance of the information obtained and to present this material in physically motivated terms, with a minimum reference to the underlying mathematics. The relevant mathematical details are available in the original reference and will be amplified elsewhere.

SYSTEM, COORDINATES AND ASSUMPTIONS

A ground-supported, upright circular cylindrical tank of radius, a, height, H, and constant wall thickness, h, is considered. The tank is presumed to be fixed to a rigid base and fully filled with liquid of mass density ρ_ℓ. The upper surface of the liquid is assumed to be free. The mass density of the tank wall is denoted by ρ, and the modulus of elasticity and Poisson's ratio for the tank material are denoted by E and ν, respectively. The excitation is a vertical component of ground shaking with acceleration $\ddot{y}(t)$, where t denotes time.

With the response being axisymmetric, a point in the system may be defined by the radial distance, r, and the longitudinal or axial distance, x. The origin of the coordinate system is taken at the center of the circular base. The radial displacement of the tank wall, w, is presumed to be independent of the longitudinal or axial motion. The positive directions of w and $\ddot{y}$ are taken in the positive directions of the corresponding coordinates axes.

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SUMMARY OF PRINCIPAL EFFECTS

Let p(x,t) be the hydrodynamic pressure exerted by the liquid on the tank wall, positive when outward. For the conditions considered, this pressure is uniformly distributed in the circumferential direction and varies in the axial direction as well as with respect to time. For a rigid tank, it is given by

$$p(x,t) = \left[1 - \frac{x}{H}\right] \rho_\ell H\, \ddot{y}(t) \qquad (1)$$

It should be clear that the timewise variation of the pressure in this case is the same as that of the ground acceleration, and its peak value increases linearly from zero at the free surface to $\rho_\ell H \ddot{y}_o$ at the base. The symbol $\ddot{y}_o$ in the latter expression represents the maximum value of the ground acceleration.

Tank flexibility affects the magnitude as well as the temporal and spatial variations of the hydrodynamic wall pressure and of the resulting forces in the tank. However, as is true of the impulsive components of the hydrodynamic effects induced in laterally excited tanks (Ref. 1), a reasonable, generally conservative, approximation to the effects of the vertical component of ground shaking may be obtained by assuming that tank flexibility affects only the magnitude and the timewise variation of p(x,t) but not its heightwise distribution.

The hydrodynamic wall pressure under this assumption may be determined from Eq. 1 merely by replacing $\ddot{y}(t)$ with $A_1(t)$. The latter quantity represents the pseudoacceleration of a similarly excited single-degree-of-freedom oscillator which has a natural frequency equal to that of the fundamental axisymmetric, breathing mode of vibration of the tank-liquid system, f_1, and the percentage of critical damping for that mode, ζ_1. The instantaneous value of the pressure is then given by

$$p(x,t) = \left[1 - \frac{x}{H}\right] \rho_\ell H\, A_1(t) \qquad (2)$$

and its maximum value at any point may be obtained from Eq. 2 by replacing $A_1(t)$ with its maximum or spectral value, A_1. The latter value may be determined from the response spectrum for the prescribed ground motion using the values of f_1 and ζ_1 referred to above. Inasmuch as A_1 may be significantly greater than $\ddot{y}_o$, it should be clear that the maximum value of the hydrodynamic wall pressure in a flexible tank may be significantly greater than that in the corresponding rigid tank.

With the magnitude and distribution of the maximum hydrodynamic wall pressure established, the maximum values of the displacements of the tank wall and of its internal forces may be determined by a static analysis. Since the distributions of the hydrodynamic and hydrostatic pressures are the same for the assumption made, the effects of the two pressures will be proportional, the proportionality ratio being $A_1/\ddot{y}_o$. Although the contribution of the radial inertia of the flexible tank wall is not considered explicitly in this approach, it is shown later that it is provided for implicitly.

The remainder of this paper is devoted to the computation of the fundamental natural frequency of axisymmetric vibration, f_1, and to a discussion of the origin, rationale and approximation of Eq. 2. Some limited information also is given concerning the factors that affect the fundamental mode of vibration of the system and the corresponding wall pressure.

FREE VIBRATION OF SYSTEM

The response considered in this section is for a tank-liquid system with a stationary base undergoing axisymmetric harmonic motion with a radial displacement

$$w(x,t) = \psi(x)\sin(\omega t + \delta) \tag{3}$$

In this expression, $\psi(x)$ represents the heightwise variation of a natural mode: $\omega = 2\pi f$ represents the associated circular frequency of vibration; f represents the corresponding frequency in cycles per second; and δ denotes an arbitrary phase angle.

For the solutions reported herein, $\psi(x)$ was expressed in the form

$$\psi(x) = \sum_{n=1}^{N} c_n X_n(x) \tag{4}$$

in which X_n is the nth axisymmetric, breathing mode of vibration of the tank without the liquid; c_n is the corresponding participation factor; and N is an integer equal to the number of empty tank modes considered. It can be shown that these modes are identical to those of a vertical strip of the tank wall vibrating as a beam in flexure. The nth circular natural frequency of the empty tank mode, $\tilde{\omega}_n$, and the corresponding frequency of the beam, $\bar{\omega}_n$, are naturally different, but they are interrelated by the equation

$$\tilde{\omega}_n = \sqrt{\omega_o^2 + \bar{\omega}_n^2} \tag{5}$$

in which ω_o represents the fundamental breathing frequency of a ring with the cross sectional dimensions of the tank wall, and it is given by

$$\omega_o = 2\pi f_o = \frac{1}{a}\sqrt{\frac{E}{\rho}} \tag{6}$$

The participation factors, c_n, in Eq. 4 are determined by satisfying the differential equation governing the radial motion of the tank wall, along with Laplace's equation for the contained liquid. In the solution of the latter equation, the following boundary conditions are satisfied: (a) the vertical velocity of the liquid is zero at $x = 0$; (b) the hydrodynamic pressure is zero at $x = H$; and (c) the radial velocities of the liquid and the tank are the same along the vertical, curved boundary. On satisfying these conditions, and making use of the orthogonality of the functions X_n and of the cosine functions that arise in the solution of Laplace's equation, one obtains a characteristic value problem of the form

$$[A]\{c\} = \lambda [B]\{c\} \tag{7}$$

in which $\{c\}$ is the vector of the coefficients c_n; $[A]$ is a diagonal matrix of the empty tank frequencies, $\tilde{\omega}_n$; $[B]$ is a square, generally full, matrix; and λ is a characteristic value related to the square of the desired natural frequency, ω.

Numerical solutions have been obtained for two groups of tanks fully filled with water: (1) concrete tanks with $a/h = 100$, $\nu = 0.17$ and $\rho_\ell/\rho = 0.40$; and (2) steel tanks with $a/h = 1000$, $\nu = 0.3$ and $\rho_\ell/\rho = 0.127$. Several different values of H/a in the range between 0.1 and 5 been considered The number of terms required to ensure convergence of the results was found to increase with increasing values of H/a and a/h. The number of terms employed for the solutions reported herein ranged from 10 for concrete tanks with $H/a < 0.5$ to 30 for steel tanks with $H/a > 3$.

The results for the fundamental natural frequency are listed in Table 1, where they are compared with those obtained from the following specialized form of a more general expression given in Ref. 3:

$$\frac{\omega_1}{\omega_o} = \frac{f_1}{f_o} = \left[1 + \frac{\rho_\ell}{\rho}\frac{a}{h}\frac{1}{\lambda}\frac{I_o(\lambda)}{I_1(\lambda)}\right]^{-\frac{1}{2}} \tag{8}$$

In this expression, $\lambda = (\pi/2)/(H/a)$, and I_o and I_1 are modified Bessel functions of the zero and first order, respectively.

Table 1. Fundamental natural frequencies of axisymmetric, breathing modes of vibration

$\frac{H}{a}$	Values of $\omega_1/\omega_o = f_1/f_o$			
	Concrete Tanks		Steel Tanks	
	Exact Solution	Membrane Solution	Exact Solution	Membrane Solution
0.30	0.4167	0.324	0.2034	0.189
0.50	0.2828	0.246	0.1476	0.141
0.75	0.2086	0.190	0.1112	0.108
1.0	0.1650	0.153	0.0889	0.0867
1.5	0.1151	0.109	0.0633	0.0617
2.0	0.0878	0.0844	0.0482	0.0475
3.0	0.0590	0.0575	0.0327	0.0323
5.0	0.0355	0.0349	0.01985	0.0196

Equation 8 is based on a theory that considers the tank to behave as a membrane. As a result, the corresponding modes of vibration violate the condition of zero radial displacement at the base, and the frequency values are lower than the exact. However, the agreement of the two sets of results is reasonable, particularly for steel tanks of realistic aspect ratios.

In part (a) of Fig. 1 are shown the heightwise distributions of the displacements of the tank wall for the fundamental mode of vibration, and in part (b) are shown the corresponding distributions of the hydrodynamic pressures. A wide range of H/a values is considered, including the unrealistically low value of 0.1. For simplicity, the results are normalized such that the peak ordinate of each diagram is unity. The same results, in consistent units, are available in Ref. 3.

The displacement modes in Fig. 1 depend importantly on the value of H/a involved; they range from the mode of a cantilever flexural beam for extremely broad tanks, to one that increases almost linearly from top to bottom for tall tanks. For steel tanks of realistic proportions, however, the differences in the modal displacements are not particularly significant. The modal pressures are generally less sensitive to variations in the value of H/a than are the displacements, particularly for steel tanks with $H/a \geq 0.5$, for which the pressure distribution may be approximated by a sine curve that increases from zero at the top to a maximum at the base. Incidentally, the latter distribution is the same as that predicted by the membrane theory.

FORCED VIBRATION OF SYSTEM

Let $\psi_k(x)$ denote the heightwise variation of the radial component of the wall displacement for the kth axisymmetric, breathing mode of vibration of the tank-liquid system, and ω_k denote the corresponding circular natural frequency. The displacement, $w(x,t)$, induced by the vertical component of ground shaking may then be expressed in either of the following forms:

$$w(x,t) = \sum_{k=1}^{N} \alpha_k \psi_k(x) D_k(t) = \sum_{k=1}^{N} \frac{\alpha_k}{\omega_k^2} \psi_k(x) A_k(t) \qquad (9)$$

in which α_k are dimensionless participation factors that may be computed in terms of $\psi_k(x)$, ω_k and either of the two square matrices in Eq. 7; $D_k(t)$ represents the deformation of a simple oscillator whose natural frequency and damping are the same as those of the kth mode of vibration of the tank-liquid system and is excited by the same ground motion as the tank; and $A_k(t) = \omega_k^2 D_k(t)$ is the corresponding pseudoacceleration of the oscillator. Numerical values of α_k are available in Ref. 3. The maximum values of the component terms in Eq. 9 may be determined by replacing $D_k(t)$ and $A_k(t)$ with their maximum or spectral values, D_k and A_k; and the maximum values of the total displacements may be approximated by application of any of the common rules for combining the component or modal maxima.

With the displacement of the tank wall established, the circumferential force in the tank wall may be evaluated readily, and the bending moments and transverse shear may be determined by differentiation. The convergence of the differentiated expressions is likely to be slow, however, and an alternative approach involving the use of the concept of an equivalent external static pressure is preferable. This approach also is helpful in assessing the accuracy of the simple approximation proposed earlier in this paper.

Let $q = q(x,t)$ be the intensity of the equivalent static radial pressure which, at any time, produces the same effects in the tank as those actually induced by the ground shaking. This pressure can be shown to be given by

$$q(x,t) = \sum_{k=1}^{N} \alpha_k \left[\chi_k(x) + \varepsilon \psi_k(x) \right] \rho_\ell H A_k(t) \qquad (10)$$

in which $\chi_k(x)$ represents the heightwise distribution of the hydrodynamic wall pressure corresponding to the kth mode of axisymmetric vibration of the tank-liquid system, and $\varepsilon = (\rho/\rho_\ell)(h/H)$. The first series of terms on the right-hand member of this equation represents the contribution of the hydrodynamic wall pressure, and the second series represents the contribution of the inertia forces of the tank wall. More specifically, $\chi_k(x)\,\rho_\ell H$ represents the hydrodynamic wall pressure when the tank-liquid system is vibrating in its kth axisymmetric natural mode with a unit peak acceleration, and $\rho h\psi_k(x)$ represents (see Eq. 9) the corresponding radial inertia of the tank wall. As before, the maximum values of the modal components of $q(x,t)$ may be determined by replacing $A_k(t)$ with A_k, and the maximum value of $q(x,t)$ at any point may be determined by an appropriate combination of the corresponding modal maxima. The effects of this pressure may then be determined from a static analysis of the tank.

It is important to note that if all the pseudoacceleration functions, $A_k(t)$, on the right hand member of Eq. 10 are taken equal to the ground acceleration, $\ddot{y}(t)$, the resulting expression must represent the hydrodynamic wall pressure exerted in a very-high-frequency, rigid tank. Since this pressure also is defined by Eq. 1, the participation factors, α_k, may be interpreted as the coefficients in an expansion of this pressure in terms of the hydrodynamic modal pressures and the associated radial inertia of the tank wall. It follows further that the unit linear function, $1 - x/H$, can also be expressed in the form

$$1 - \frac{x}{H} = \sum_{k=1}^{N} \alpha_k \left[\chi_k(x) + \varepsilon\,\psi_k(x) \right] \tag{11}$$

In the approximate procedure proposed earlier in this paper, the functions $A_k(t)$ for the higher modes of vibration are effectively taken equal to that for the fundamental mode. For an earthquake ground motion, the fundamental natural frequency of axisymmetric vibration of the tank-liquid system is likely to fall in the amplified, constant acceleration region of the response spectrum. Under these conditions, the spectral value of $A_k(t)$ for the higher modes of vibration will be equal to or less than that for the fundamental mode, and the procedure will lead to conservative results. Even when the fundamental frequency falls in the low-frequency or medium-frequency regions of the design spectrum, for which the values of A_k for the higher modes may be substantially greater than for the fundamental mode, the resulting errors are likely to be small for the following reasons: (1) the fundamental mode is generally the dominant contributor to the response; and (2) the peak values of the resulting displacements and tank forces are insensitive to the detailed distribution of the wall pressure.

As an illustration, the results of an analysis are presented for a model of a prestressed concrete tank with a = 120 ft, H = 90 ft, h = 1.2 ft, E = 4,000 ksi and ν = 0.17. The tank is presumed to be fully filled with water ($\rho_\ell/\rho = 0.4$) and to be excited by the vertical component of a design earthquake ground motion with a peak acceleration $\ddot{y}_o$. The characteristics of the excitation are presumed to be such that the spectral value of the pseudo-acceleration for systems with 2 percent of critical damping is $A = 3.7\,\ddot{y}_o$ over a range of natural frequencies from 2 cps to 8 cps.

The reference ring frequency in this case is ω_o = 92.66 rad/sec or f_o = 14.75 cps; and the first three natural frequencies of axisymmetric vibration of the tank-liquid system are $f_1 = 0.2086\,f_o = 3.08$ cps, $f_2 = 0.3886\,f_o = 5.73$ cps, $f_3 = 0.5158\,f_o = 7.60$ cps. Determined from Table 1 and the additional data in Ref. 3, these frequencies fall in the constant acceleration region of the design spectrum. Hence, $A_1 = A_2 = A_3 = 3.7\,\ddot{y}_o$.

In Fig. 2 are shown approximations to the maximum values of the equivalent static pressure, q_{max}, and of the radial displacement of the tank wall, w_{max}. All displacements are normalized with respect to

$$(w_{st})_o = \frac{\rho_\ell H \ddot{y}_o}{Eh} a^2 \tag{12}$$

which represents the static displacement of the reference ring due to a radial pressure equal to the peak hydrodynamic pressure induced by the ground shaking in a rigid tank. The results were computed on two different bases: (1) considering the contribution of the fundamental mode of vibration only; and (2) considering the contributions of the first three modes and combining the modal maxima either by the root-mean-squares (RMS) rule or by a modified RMS rule, in which the maximum contribution of the fundamental mode is combined with the RMS value of the higher modal maxima. Also shown is the linear pressure distribution predicted by the proposed approximate procedure.

The maximum radial displacement of the tank wall for the latter pressure can be shown to equal $2.84(w_{st})_o$, a result that coincides with the value computed by the exact procedure using the modified RMS rule.

The absolute maximum wall pressure, $\tilde{q}_{max}$, in this example is approximately three times as large as the peak hydrodynamic pressure induced in a rigid tank of the same dimensions, and for $\ddot{y}_o$ of the order of one-third the gravitational acceleration, it is about the same as the maximum hydrostatic pressure. It is important to note, however, that no provision has been made in the analysis presented for the effects of soil-structure interaction and the associated radiation damping in the supporting medium. Because the radiation damping capacity for vertically excited foundations is quite large, the interaction effects are expected to be important in this problem.

CONCLUSION

With the proposed approximate procedure, the response of a circular cylindrical liquid storage tank to a vertical component of ground shaking may be evaluated readily and with good accuracy.

REFERENCES

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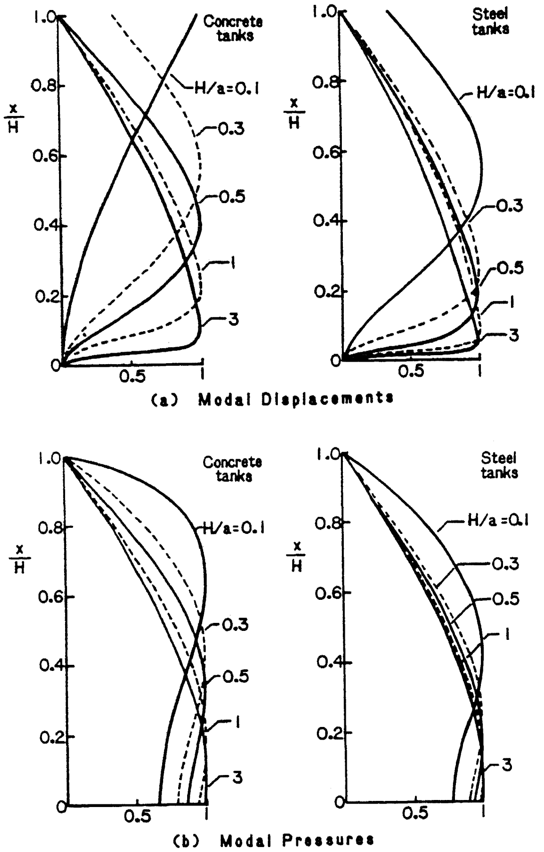

Fig. 1 Fundamental Modes of Axisymmetric Vibration of Liquid-Filled Tanks

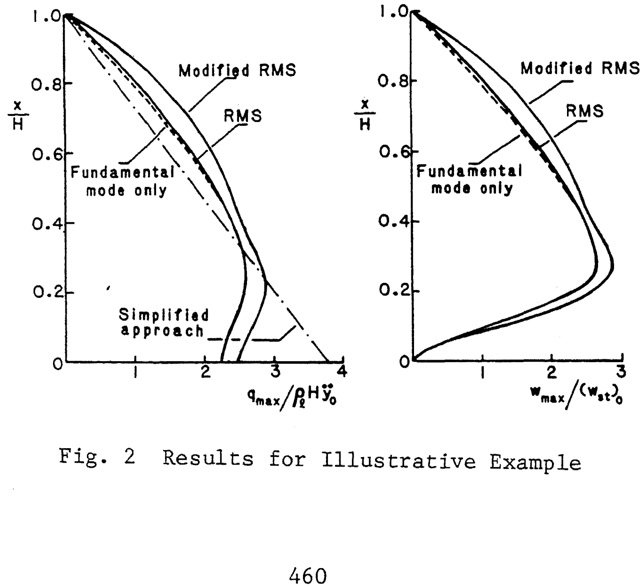

Fig. 2 Results for Illustrative Example