EARTHQUAKE RESPONSE OF LIQUID STORAGE THIN SPHERICAL TANKS

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SUMMARY

This paper provides the formulation of the response analysis of interaction system of elastic spherical tank, inner liquid and supporting structures subjected to horizontal earthquake, and then examines the response behaviors of this system numerically. The inner liquid is treated as the combination of hemi-spherical part and the rest which can be divided into some thin circular disks. Representing the spatial modes of the spherical tank in a feasible form as finite series of Legendre bi-polynomials, which satisfy boundary conditions and orthogonality condition, analytical solutions of the interaction system are given. Thence, prosecuting the numerical calculation of these solution, some characteristics of the response behavior of the system are discussed.

INTRODUCTION

In order to confirm the safety and strength of the large liquid storage tank subjected to dynamic loading, the response behaviors of the whole system which consists of inner liquid, tank and supporting structures should be analyzed in consideration of the interactions among them. Regarding this topic, the exact expression of the solution does not exist. From this point of view, it can be said that the literature available in this field is not adequate. Most studies which have been already published introduce the assumption of rigid tank [1],[2] and engineers, therefore, have been frustrated by the lack of useful solution on this topic.

Mathematically analytical procedure as interaction system is undertaken here, introducing some modelization of inner liquid. The inner liquid is treated as the combination of hemi-spherical portion and the rest which is divided into some thin circular disks. Representing the spatial modes of the the spherical tank in feasible forms as finite series of Legendre bi-polynomials and applying Lagrange's principle, kinetic equation of the system is obtained mathematically. Modes used herein, which satisfy boundary conditions and orthogonality condition, are in very close agreement with exact natural modes of the tank. Coefficient matrices of the kinetic equation and vector of modal participation factor are amenable to numerical calculation quite easily.

MODELING

Because of thin shell characteristics, it is not advisable to connect the tank to supporting columns directly. Installing, therefore, the rigid ring around the equator of the tank, columns are assumed to be connected to tank through this ring as shown in Fig.1. Supporting columns are replaced with spring-lumped mass system of 1 DOF, setting the position of lumped mass at βL($0<\beta<1$) and equating the relative displacement at top of columns, $u_L(t)$, with the displacement of ring. Inner liquid is assumed as inviscid ideal potential fluid.

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MODES OF SPHERICAL TANK

With the assumption of flexural free vibration, the governing equations of spherical shell based on linear elastic theory are derived as follows [3],

$$K\cdot H_2H_2(w) - a\cdot H_2(F) + ma^4\ddot{w} = 0 \qquad \cdots\cdots\cdots \qquad (1\text{-a})$$

$$aH_2H_1(F) - (1-\nu)K\cdot H_2H_2(w) + Eha^2H_2(w) = 0 \qquad \cdots\cdots\cdots \qquad (1\text{-b})$$

where w, F, a, h and K denote deflection, stress function, radius, thickness and plate rigidity, respectively, and

$$H_2(\ ) = (\ )^{\circ\circ} + \cot\phi\ (\ )^{\circ} + 2(\ ),\ H_1(\ ) = H_2(\ ) - (1+\nu)(\ ),\ (\ )^{\circ} = \frac{\partial}{\partial\phi}(\ )$$

Exact and approximate solutions of eqs.(1) have been already obtained by the author et al [3],[4]. But, those solutions are not suitable for response analysis, since they are represented as the sum of four Legendre bi-functions of real fractional or complex order. Modal analysis entails integrals of the product of deflection with respect to spatial variables, and those integrals with regard to exact solution can not be reduced to some functions of closed form. Response analysis proposed herein introduces the approximate deflection modes given by the author [5]. Integrals with regard to these approximate modes can be expressed in functions of closed form. The close resemblance of the approximate modes with those obtained by exact solution has been shown.

Deflection modes and related stress function are represented as follows,

$$w(\phi,\theta,t) = \sum_j^{M} x_j(t)w_j(\phi)\cos\theta, \quad w_j(\phi) = h\sum_{i=1}^{M+3} G_{ij}\ P_i^1(\cos\phi) \qquad \cdots\ (2\text{-a})$$

$$F(\phi,\theta,t) = \sum_j^{M} x_j(t)F_j(\phi)\cos\theta, \quad F_j(\phi) = \frac{K}{a}h\sum_{i=1}^{M+3} D_iG_{ij}P_i^1(\cos\phi) \qquad \cdots\ (2\text{-b})$$

where $D_i = [(1-\nu)\{2-i(i+1)\} - \kappa]/\{1-\nu-i(i+1)\},\quad \kappa = 12(1-\nu^2)a^2/h^2$

Because rigid ring is installed at $\phi = 90^{\circ}$, boundary conditions are $w = w^{\circ} = u = v = 0$. Deflection modes given in eq.(2) do not contain fundamental mode(rocking type mode). The following mode, therefore, will be added as fundamental mode, despite this does not satisfy boundary conditions and orthogonality conditions.

$$w_0(\phi) = hC_0P_\mu^1(\cos\phi), \quad \mu = 1.90 \quad : \quad \Omega\ (\equiv \sqrt{ma^2/(Ehg)}\ \omega) = 0.5620$$

w_0 will be used in this paper, expanding in a series of Legendre bi-polynamials. As the natural frequency in correspondence with this mode, the frequency based on exact solution is used in the response analysis as given.

VELOCITY POTENTIAL OF INNER LIQUID

Because of the assumption of inviscid ideal fluid, velocity potential can be defined for inner liquid. Velocity potential of inner liquid is assumed to be composed of three components:(1) compatible one to tank deformation;(2) sloshing one irrelevant with tank deformation;(3) complemental one to (1), as follows.

$$\Psi = \Psi_1 + \Psi_2 + \Psi_3 \qquad \cdots\cdots\cdots\cdots\cdots \qquad (3)$$

It is assumed that Ψ_i $(i=1,2,3)$ satisfies continuity condition, $\nabla^2\Psi_i = 0$ in V and the following boundary conditions. Notations are shown in Fig.2.

$$-\frac{\partial}{\partial r}\Psi_1(r,\phi,\theta,t) = -\dot{w} \text{ on } S, \quad -\frac{\partial}{\partial r}\Psi_2(r,\phi,\theta,t) = 0 \text{ on } S$$

$$\frac{\partial^2}{\partial t^2}\Psi_2(r,\phi,\theta,t) + g\frac{\partial}{\partial z}\Psi_2(r,\phi,\theta,t) = 0 \text{ on } S_f$$

$$-\frac{\partial}{\partial r}\Psi_3(r,\phi,\theta,t) = 0 \text{ on } S, \quad \frac{\partial}{\partial z}\{\Psi_1(r,\phi,\theta,t) + \Psi_3(r,\phi,\theta,t)\} = 0 \text{ on } S_f \qquad \cdots\cdots\cdots\cdots (4)$$

<u>Determination of $\Psi_1(r,\phi,\theta,t)$</u>

Corresponding to the j-th deflection mode of tank, ${}_j\Psi_1$ is determined as follows,

$${}_j\Psi_1 = h\, a \sum_i^{M+3} \frac{1}{i} G_{ij} \left(\frac{r}{a}\right)^i P_i^1(\cos\phi)\, \dot{x}_j(t)\cos\theta \qquad \cdots\cdots\cdots\cdots (5)$$

<u>Determination of $\Psi_2(r,\phi,\theta,t)$</u>

In order to determine Ψ_2 , inner liquid will be divided into two portions:(1) hemi-spherical portion;(2) assemble of thin circular disks. That is, $\Psi_2 = \Psi_2^c + \Psi_2^s$. Ψ_2^s is defined for hemi-spherical portion of the liquid, satisfying the following field equation and boundary conditions,

$$\nabla^2 \Psi_2^s = 0 \text{ in } V, \quad -\frac{\partial}{\partial r}\Psi_2^s = 0 \text{ on } S, \quad -\frac{\partial}{\partial z}\Psi_2^s = f(r,\phi,\theta,t)\big|_{\phi=\pi/2} \text{ on } S_c$$

where $f(r,\phi,\theta,t)$ is treated as a known function. Then, Ψ_2^s is expressed in the following form.

$$\Psi_2^s(r,\phi,\theta,t) = \int_S \Psi_2^s(\bar{r},\bar{\phi},\bar{\theta},t)\,C\,dS - \int_{S_c} N(\bar{r},\tfrac{\pi}{2},\bar{\theta};r,\phi,\theta)\,f(\bar{r},\tfrac{\pi}{2},\bar{\theta},t)\,dS_f \quad \cdots (6)$$

Where $\bar{r}$, $\bar{\phi}$ and $\bar{\theta}$ denote position on S and S_c, and function N is determined from Neumann function of complete sphere. Introducing such a assumption that all quantities relevant to this problem can be expressed in the following form

$$\{f, \Psi_2^s, N\}(r,\phi,\theta,t) = \{f, \Psi_2^s, N\}(r,\phi)\dot{y}(t)\cos\theta$$

and noting them at $\theta = 0$, the following expression can be derived finally from eq.(6).

$$\Psi_2^s(r,\phi) = -\sum_{n=1}^{\infty} \frac{1}{n(n+1)} P_n(0)P_n(\cos\phi)G_n(r)\,h \qquad \cdots\cdots\cdots\cdots (7)$$

Where

$$G_n(r) \equiv \int_0^r \left(\frac{\bar{r}}{r}\right)^{n+1} f(\bar{r},\tfrac{\pi}{2})d\bar{r} + \int_r^a \left(\frac{r}{\bar{r}}\right)^n f(\bar{r},\tfrac{\pi}{2})d\bar{r} + \frac{n+1}{n}\int_0^a \frac{r^n \bar{r}^n}{a^{2n+1}} f(\bar{r},\tfrac{\pi}{2})\bar{r}d\bar{r}$$

Ψ_2^c is determined by the similar manner as in [2]. The rest of the liquid, the upper portion on the hemi-spherical one, is sliced into thin circular disk elements of n layers, and each element is denoted by circular coordinate. Velocity potential of the i-th layer, ${}^i\Psi_2^c$, satisfies the following field and boundary equations

$$\nabla^2\, {}^i\Psi_2^c = 0 \text{ in } V, \quad \frac{\partial}{\partial r}\, {}^i\Psi_2^c\big|_{r=R_i} = 0, \quad -\frac{\partial}{\partial z}\, {}^i\Psi_2^c = \begin{cases} h^u(r,\theta,t)\cdot h & (z=l_i) \\ h^l(r,\theta,t)\cdot h & (z=0) \end{cases}$$

where l_i is the thickness of i-th element. The velocity potential, thence, for s-th sloshing mode is expressed as

$$ {}^{i}_{s}\Psi^{c}_{2} = \dot{y}_s(t) J_1(\varepsilon_s \frac{r}{R_i}) h\{ {}_sA_i \cosh({}_sk_i z) + {}_sB_i \sinh({}_sk_i z)\} \cos\theta \quad \cdots\cdots \quad (8) $$

where ${}_sk_i = \varepsilon_s / R_i$: ε_s being the s-th real positive root of $J_1'(\varepsilon) = 0$. Representing hydrodynamic pressure p ($= \rho_L \cdot \dot{\Psi}$) and velocity of the i-th disk toward upper direction as

$$ p^{\overset{u}{l}}(r,\theta,t) = \ddot{y}_s(t)\, {}_sP_i^{\overset{u}{l}} J_1(\varepsilon_s \frac{r}{R_i}) \cos\theta, \quad h^{\overset{u}{l}}(r,\theta,t) = \dot{y}_s(t)\, {}_sV_i^{\overset{u}{l}} J_1(\varepsilon_s \frac{r}{R_i}) \cos\theta $$

respectively, the transfer matrices through element and node are derived as ,

$$ \begin{Bmatrix} {}_sP_n^l \\ {}_sV_n^l \end{Bmatrix} = A \begin{Bmatrix} {}_sP_1^u \\ {}_sV_1^u \end{Bmatrix}, \qquad A = A_n B_{n-1} A_{n-1} \cdots\cdots\cdots B_1 A_1 $$

$$ \begin{Bmatrix} {}_sP_i^l \\ {}_sV_i^l \end{Bmatrix} = A_i \begin{Bmatrix} {}_sP_i^u \\ {}_sV_i^u \end{Bmatrix}, \quad A_i = \begin{bmatrix} \cosh({}_sk_i l_i), & (\rho_L/{}_sk_i)\sinh({}_sk_i l_i) \\ ({}_sk_i/\rho_L)\sinh({}_sk_i l_i), & \cosh({}_sk_i l_i) \end{bmatrix} $$

$$ \begin{Bmatrix} {}_sP_{i+1}^u \\ {}_sV_{i+1}^u \end{Bmatrix} = B_i \begin{Bmatrix} {}_sP_i^l \\ {}_sV_i^l \end{Bmatrix}, \quad B_i = \begin{bmatrix} (R_{i+1}/R_i), & 0 \\ 0 & , (R_{i+1}/R_i)^2 \end{bmatrix} $$

Two unknown constants, ${}_sA_i$ and ${}_sB_i$, are determined from the conditions of continuity of velocity and hydrodynamic pressure between hemi-spherical portion and upper portion. The velocity continuity condition yields

$$ {}_s\Psi_2^s(r,\phi,\theta,t) = -\dot{y}_s(t)\, h\, {}_sV_n^l \sum_{p=1}^{\infty} \frac{1}{p(p+1)} P_p^1(0) P_p^1(\cos\phi)\, {}_sG_p(r) \cos\theta $$

for the s-th sloshing mode. From the hydrodynamic pressure continuity condition, we get

$$ ({}_sP_n^l / {}_sV_n^l) = -[\rho_L \sum_{p=1}^{\infty} \{1/p(p+1)\}\{P_p^1(0)\}^2\, {}_sG_p(r)\,] / J_1(\varepsilon_s \frac{r}{a}) \equiv C \quad \cdots \; (9) $$

The ratio C which should be constant irrelevantly to position in the exact theory will be evaluated from $C = \sum_{i=1}^{m} C_i / m$ in this paper, where C_i is the ratio at $r = r_i$. If ${}_sV_1{}^u$ is included in $y_s(t)$, ${}_sP_1{}^u$ is calculated from eq.(9) and ${}_sA_i$ and ${}_sB_i$ can be determined. The natural circular frequency of the sloshing can be calculated from $\omega_s = \sqrt{-\rho_L g / {}_sP_1{}^u}$

Determination of $\Psi_3(r,\phi,\theta,t)$

Also, Ψ_3 is divided into two portions as $\Psi_3 = \Psi_3{}^c + \Psi_3{}^s$. $\Psi_3{}^c$, corresponding to the j-th mode of tank, is given in the following form

$$ {}_j\Psi_3^c(r,z,\theta,t) = \dot{x}_j(t)\, h \sum_s^{\infty} D_{sj} J_1(\varepsilon_s \frac{r}{R}) \{ {}_sA^j \cosh({}_sk z) + {}_sB^j \sinh({}_sk z)\} \cos\theta $$

Unknown coefficients D_{sj} are determined from boundary condition eq.(4), utilizing collocation method. The expression similar to $\Psi_2{}^s$ is applicable to $\Psi_3{}^s$. Including ${}_jV_1{}^u$ into D_{sj} , the pressure continuity condition yields

$$\sum_s D_{sj}[a_{11}J_1(\varepsilon_s \frac{r}{a}) + \rho_L a_{21} \sum_{p=1}^{\infty} \frac{1}{p(p+1)} \{P_p^1(0)\}^2 {}_sG_p(r)]\, {}_sP_1^u$$

$$= - \sum_s D_{sj}[\rho_L a_{22} \sum_{p=1}^{\infty} \frac{1}{p(p+1)} \{P_p^1(0)\}^2 {}_sG_p(r) + a_{12}J_1(\varepsilon_s \frac{r}{a})] \quad \cdot\cdot\ (10)$$

for ${}_jP_1{}^u$, where a_{ij} is element of transfer matrix **A** . Because this equation can be solved by introduction of collocation method, ${}_sA_i^j$ and ${}_sB_i^j$ can be determined.

DERIVATION OF KINETIC EQUATION

Applying Lagrange's principle, kinetic equation for interaction system is derived. Set the relative deflection of tank as eq.(2-a) and absolute displacement components as

$$w_s(\phi,\theta,t) = w(\phi,\theta,t) - \{u_g(t) + u_L(t)\}\sin\phi\cdot\cos\theta$$
$$v_s(\phi,\theta,t) = v(\phi,\theta,t) - \{u_g(t) + u_L(t)\}\sin\theta$$
$$u_s(\phi,\theta,t) = u(\phi,\theta,t) + \{u_g(t) + u_L(t)\}\cos\phi\cdot\cos\theta$$

Note that velocity potential in eq.(3) does not contain the effect of ground displacement.

Kinetic Energy of the System

Kinetic energy of the system : $K = K_s + K_l + K_c$

where

$$K_s = \frac{1}{2}m \int_{S_a} \{\dot{w}_s^2 + \dot{u}_s^2 + \dot{v}_s^2\} dS_a \quad (u,v \to 0), \quad K_c = \frac{1}{2}m_c\{\dot{u}_g + \alpha\dot{u}_L\}^2$$

$$K_l = \frac{1}{2}\rho_L \int_V [\{-\frac{\partial}{\partial x}\Psi + \dot{u}_g + \dot{u}_L\}^2 + \{-\frac{\partial}{\partial y}\Psi\}^2 + \{-\frac{\partial}{\partial z}\Psi\}^2] dV$$

Potential Energy of the System

Potential energy of the system : $P = \overline{P}_s + P_s + P_l + P_c$

where

$$\overline{P}_s = -2m(\ddot{u}_g + \ddot{u}_L)\sum_j x_j(t) \int_{Sa} w_j(\phi)\sin\phi\cos^2\theta dSa, \quad P_c = \frac{1}{2}k_c u_L^2$$

$$P_s = \frac{1}{2}\sum_j {}_jP_s x_j^2(t) \;, \quad {}_jP_s\text{: coef. of } x_j(t) \text{ of } \int_{Sa}[eq.(1\text{-}a)]w_j(\phi)\cos\theta dSa$$

$$P_l = \frac{1}{2}\rho_L g \int_{S_f}\{-\frac{\partial}{\partial z}\Psi\}^2 dS_f + \frac{1}{2}\rho_L g\int_S\{-\frac{\partial}{\partial r}\Psi\}^2\cos\phi dS \quad (\dot{x},\dot{y} \to x,y)$$

Kinetic Equation of the System

$$[\tilde{M} + \tilde{M}_a]\{\ddot{Q}\} + [\tilde{K} + \tilde{K}_a]\{Q\} = -\{m_0 + m_{0a}\}\ddot{u}_g \quad \cdots\cdots\cdots\cdot \quad (11)$$

where

$$\{Q\} = \{x_1, x_2, \cdots, x_M \vdots u_L \vdots y_1, y_2, \cdots, y_N\}^T$$

$$\tilde{M} = \begin{bmatrix} C_1(i,i) & 0 \\ i=1\sim M & \\ 0 & 0 \end{bmatrix}, \qquad \tilde{K} = \begin{bmatrix} C_5(i,i) & 0 \\ i=1\sim M & \\ 0 & 0 \end{bmatrix}$$

$$\tilde{M}_a = \left[\begin{array}{c|c|c} C_2(i,j) \; {i \atop j}=1\sim M & C_8(1) \cdots C_8(M) & C_3(p,q) \; p=1\sim M, \; q=1\sim N \\ \hline \{C_{15}\}^T & C_{10} & \{C_9\}^T \\ \hline [C_3]^T & C_9(1) \cdots C_9(N) & C_4(s,t) \; {s \atop t}=1\sim N \end{array}\right], \quad \tilde{K}_a = \left[\begin{array}{c|c|c} C_6(i,j) \; {i \atop j}=1\sim M & 0 & 0 \\ \hline 0 & C_{11} & 0 \\ \hline 0 & 0 & C_7(s,t) \; {s \atop t}=1\sim N \end{array}\right]$$

$$\{m_0\} = \{\, C_{13}(1), C_{13}(2), \cdots, C_{13}(M) \mid 0 \mid 0 \,\}^T$$

$$\{m_{0a}\} = \{\, C_{14}(1), C_{14}(2), \cdots, C_{14}(M) \mid C_{12} \mid C_9(1), C_9(2), \cdots, C_9(N) \,\}^T$$

$$C_1(j) = m\int_{Sa} W_j^2 \, dSa, \qquad C_5(j) = {}_jP_s \text{ of potential energy } P_s$$

$$C_2(j,l) = \frac{1}{2}\rho_L \int_S [\{{}_l\Psi_1 + {}_l\Psi_3\}W_j + \{{}_j\Psi_1 + {}_j\Psi_3\}W_l]dS$$

$$C_3(j,i) = \frac{1}{2}\rho_L [\int_S {}_i\Psi_2 W_j dS + \int_{Sf} \{{}_j\Psi_1 + {}_j\Psi_3\}\frac{\partial}{\partial z}\, {}_i\Psi_2 \, dS_f]$$

$$C_4(i,l) = \frac{1}{2}\rho_L \int_{Sf} \{{}_l\Psi_2 \cdot \frac{\partial}{\partial z}\, {}_i\Psi_2 + {}_i\Psi_2 \cdot \frac{\partial}{\partial z}\, {}_l\Psi_2\} \, dS_f$$

$$C_6(j,l) = \rho_L g \int_S W_j W_l \cos\phi \, dS, \qquad C_7(i,l) = \rho_L g \int_{Sf} \frac{\partial}{\partial z}\, {}_l\Psi_2 \cdot \frac{\partial}{\partial z}\, {}_i\Psi_2 \, dS_f$$

$$C_8(j) = -\,[\, 3m \int_{Sa} W_j \sin\phi\cos\theta dSa + \rho_L \int_V \frac{\partial}{\partial x}\{{}_j\Psi_1 + {}_j\Psi_3\} \, dV \,]$$

$$C_9(i) = -\,\rho_L \int_V \frac{\partial}{\partial x}\, {}_i\Psi_2 \, dV, \qquad C_{10} = M_s + M_L + \alpha^2 m_c, \quad C_{11} = k_c$$

$$C_{12} = M_s + M_L + \alpha m_c, \qquad C_{13}(j) = -\,3m \int_{Sa} W_j \sin\phi \cos\theta \, dSa$$

$$C_{14}(j) = -\,\rho_L \int_V \frac{\partial}{\partial x}\{{}_j\Psi_1 + {}_j\Psi_3\}dV, \quad C_{15}(j) = C_8(j) + 2m \int_{Sa} W_j \sin\phi\cos\theta dSa$$

$$W_j \equiv w_j(\phi)\cos\theta, \quad M_s \equiv m\int_{Sa} dSa, \quad M_L \equiv \rho_L g \int_V dV$$

k_c = total spring coefficient of columns, α = shown in Fig.1

Most integrals are represented in functions of closed form.

NUMERICAL EXAMPLE

Approximate mode shapes of tank in single or interaction system with or without columns are shown in Fig.3. The fundamental mode, which is taken into consideration in interaction analysis, is not depicted in case of single tank. Because nondimensionalized treatment of quantities is not substantial in interaction system, the following quantities are set.

Steel tank of $a = 10^m$ with $a/h = 500$; Water; 14 Steel columns of $L = 15^m$, dia. $= 70^{cm}$ and thickness $= 2^{cm}$, $\alpha = \beta = 0.5$

In free vibration, effects of deformation of tank and columns on periods of sloshing are scarcely recognized as shown in Figs.4. That is, periods of sloshing can be calculated without consideration of interaction system. Generalized periods of sloshing are depicted in Fig.5. Figs.6, which show the effect of inner liquid on the periods of tank, indicate that elongation of periods is remarkable in relatively lower modes and modal coupling appears in case of deep liquid with sloshing. Increasing of deflection, however, is not induced, because eigen value becomes only imaginal . In usual response analysis, it is desirable to treat inner liquid as free and fixed mass. Conversion charts are given in Figs.7. Examples of maximum hydrodynamic pressure caused by El Centro'40 EW (converted as max.accel.=300 gal) are shown in Figs.8, where PS, PG and PT are pressure caused by sloshing, that caused by ground and column motions and total pressure as interaction system, respectively, being plotted at each point without regard to time.

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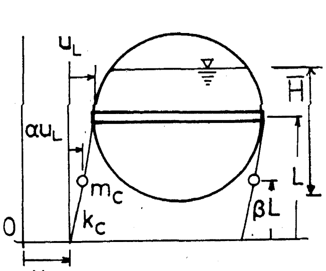


Fig.1 Analysis Model

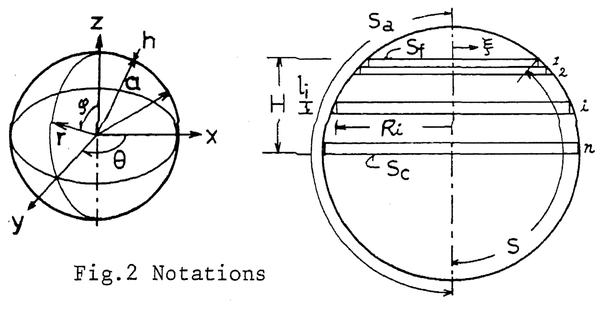


Fig.2 Notations

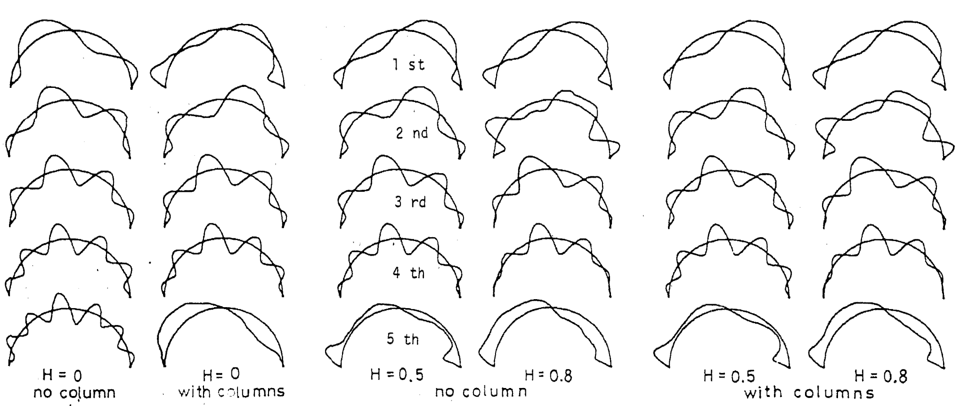


Fig.3 Mode Shapes of Tank

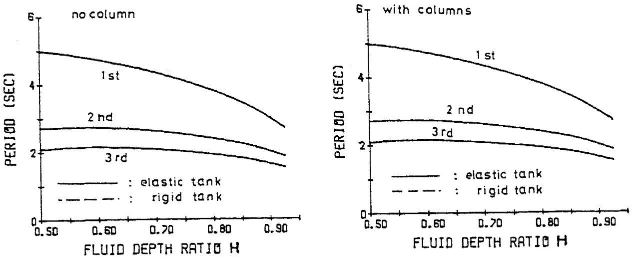


Fig.4 Comparison of Sloshing Period of Flexible and Rigid Tank

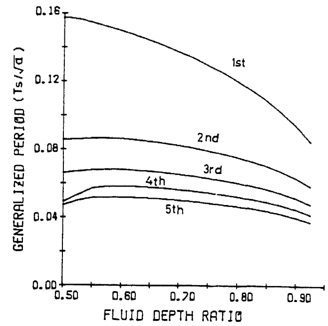


Fig.5 Generalized Sloshing Period

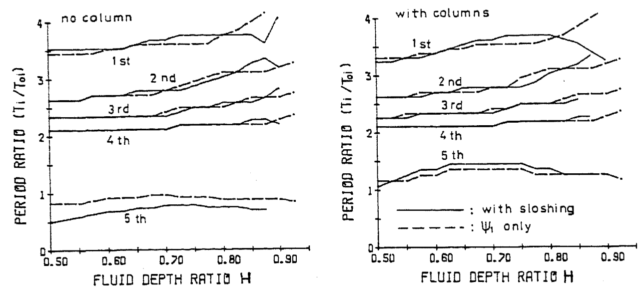
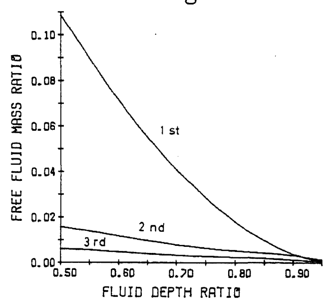
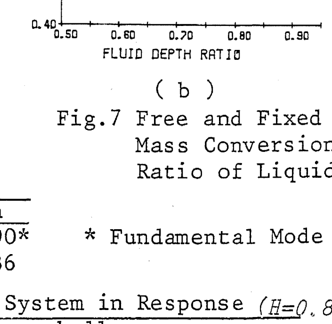


Fig.6 Elongation of Tank Period due to Liquid: *in case of sloshing considered and of* Ψ_1 *only considered.*



(a)



(b)

Fig.7 Free and Fixed Mass Conversion Ratio of Liquid

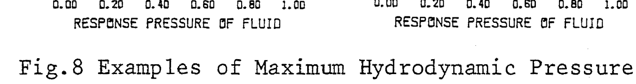


Fig.8 Examples of Maximum Hydrodynamic Pressure Distribution caused by a Earthquake

Table 1. Period of Empty Tank: $a=10^m, a/h=500$

T_{oi} (sec)	1st	2nd	3rd	4th	5th
No Column	0.01306	0.01256	0.01243	0.01236	0.02190*
With Columns	0.01330	0.01259	0.01244	0.01236	0.01036

* Fundamental Mode

Table 2. Modal Participation Ratio of Interaction System in Response $(H=0.8)$

	sloshing			col.	shell				
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
for slosh.	0.8085	0.1968	0.1336	1.0000	0.0005	0.0001	0.0004	0.0001	0.0050
for shell	0.0056	0.0005	0.0003	0.2087	0.4198	0.0235	0.4654	0.4297	1.0000