DYNAMIC BEHAVIOR OF CYLINDRICAL LIQUID STORAGE TANKS UNDER VERTICAL EARTHQUAKE EXCITATION

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SUMMARY

A method for analyzing the earthquake response of cylindrical liquid storage tanks under the action of vertical acceleration is described. The method is based on superposition of the free axisymmetrical vibrational modes obtained numerically by the finite element method. The validity of these modes has been checked analytically and the formulation of the load vector has been examined by a static analysis. Experimental confirmation of the preceding analyses is underway.

INTRODUCTION

Earthquake motion is three dimensional in nature and recent observations of recorded ground motion showed that maximum amplitude of the vertical component of ground acceleration can exceed peak horizontal amplitude especially near the center of the earthquake. Because of the inherent stiffness of typical structures in the vertical direction, the effect of the vertical component of ground acceleration has been often ignored. However, in a liquid-filled tank, vertical acceleration can be transmitted to a horizontal hydrodynamic loading. As a result, tank wall undergoes radial deformations in addition to axial displacements.

A research project is underway to assess, both theoretically and experimentally, the relative importance of vertical excitations in the design of tanks. The study (Fig. 1) includes a theoretical treatment of the free and forced vibrations of the coupled liquid-shell system both analytically and numerically; vibration tests of reduced-scale models subjected to both harmonic and random vertical excitations; and development of design charts which take into account the effect of vertical ground acceleration. Much of the efforts spent up to the writing of this paper has been directed to the development of a theoretical model for the evaluation of the dynamic characteristics such as natural frequencies, mode shapes of vibration and modal participation factors under excitation parallel to the tank axis. The experimental phase has just started along with the theoretical evaluation of tank response under vertical seismic excitations.

TANK GEOMETRY, COORDINATE SYSTEM AND ASSUMPTIONS

The tank under consideration is shown in Fig. (2). It is a ground-supported, circular cylindrical, thin-walled liquid container of radius R, length L, and thickness h. The tank is partly filled with liquid to a

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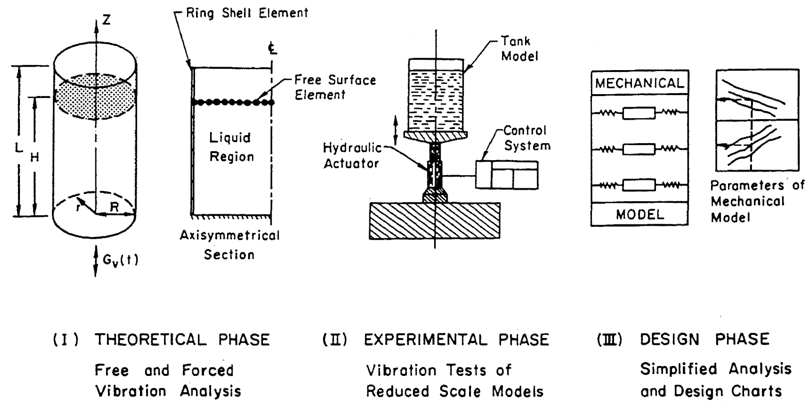

Fig. (1). Outline of Overall Study.

height H. A cylindrical coordinate system is used with the center of the base being the origin. The radial and axial displacement components of a point on the shell middle surface are denoted by w and u, respectively. Throughout the investigation, the liquid is assumed to be homogeneous, inviscid and incompressible and the shell is assumed to be elastic. The cylindrical wall is rigidly connected to a thin bottom plate of thickness h_b. In the anchored configuration, the shell is fastened to a concrete ring wall foundation whereas, in the unanchored configuration, the base plate rests directly on a compacted soil.

HYDRODYNAMIC PRESSURE

The hydrodynamic pressure exerted on the wall of a flexible tank due to a vertical ground excitation $G_v(t)$ can be obtained by superposition of three

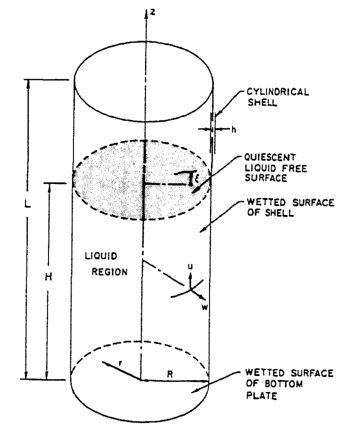

Fig. (2). Coordinate System.

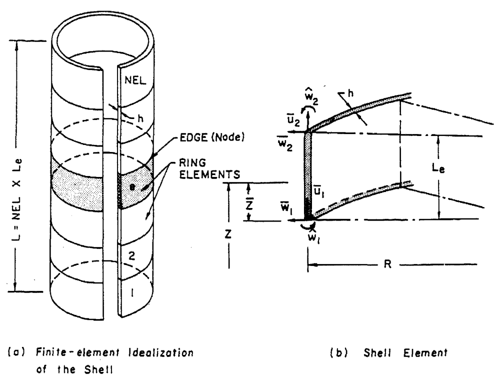

Fig. (3). Finite Element Model.

pressure components: the long period component contributed by fluid sloshing; the impulsive pressure component which varies in synchronism with the vertical ground acceleration; and the short period component contributed by the vibrations of the tank wall. Each of these pressures can be obtained from a velocity potential function which satisfies the Laplace equation and the appropriate boundary conditions. For example, the short period pressure component can be expressed as

$$p_d(R,z,t) = -\rho_\ell \frac{\partial \phi}{\partial t}(R,z,t)$$
$$= -\frac{2\rho_\ell}{H} \sum_{n=1}^{\infty} \frac{\int_0^H \ddot{w}(\eta,t)\cos(\alpha_n\eta)\,d\eta}{\alpha_n\, I_0'(\alpha_n R)} I_0(\alpha_n R)\cos(\alpha_n z) \qquad (1)$$

where $\alpha_n = (2n-1)\pi/2H$; ρ_ℓ is the mass density of the liquid and I_0 is the modified Bessel function.

A NUMERICAL APPROACH TO THE AXISYMMETRICAL FREE VIBRATION

For the problem under consideration, it has been advantageous to combine analytical with numerical methods rather than solving the whole problem numerically. The series solution for the hydrodynamic pressure is used, and therefore, only the shell needs to be modeled by ring-shaped finite elements (Fig. 3). The strain energy expression of the shell is written in terms of the assemblage stiffness matrix and the assemblage nodal displacement vector as

$$U(t) = \frac{1}{2}\{q\}^T [K_s] \{q\} \qquad (2)$$

whereas the kinetic energy of the shell is given by

$$T(t) = \frac{1}{2}\{\dot{q}\}^T [M_s] \{\dot{q}\} \qquad (3)$$

The short-period fluid pressure is taken into consideration by introducing an additional mass matrix in the matrix equation of motion of the shell. The work done by such pressure through an arbitrary virtual displacement, δw, is expressed as

$$\delta W = 2\pi R \int_0^H p_d(R,z,t)\,\delta w\,dz \qquad (4)$$

The elements of the added mass matrix can be computed by expressing Eq. (4) in terms of the displacement vector $\{q\}$. Thus

$$\delta W = -\{\delta q\}^T \sum_{i=1}^{\infty} b_i \{F^{(i)}\}\{F^{(i)}\}^T \{\ddot{q}\} = -\{\delta q\}^T [DM]\{\ddot{q}\} \qquad (5)$$

where

$$b_i = \frac{4\pi R\,\rho_\ell\, I_0(\alpha_i R)}{H\,\alpha_i\, I_0'(\alpha_i R)} \qquad (6)$$

The series in Eq. (5) converges very rapidly and only the first few terms are needed for adequate representation of the infinite series. The matrix equation for the free axisymmetrical undamped vibrations of the tank wall becomes

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\} \tag{7}$$

where $[M] = [M_s] + [DM]$; and $[K] = [K_s]$.

AN ANALYTICAL APPROACH TO THE AXISYMMETRICAL FREE VIBRATION

Two partial differential equations govern shell motion; one of the second order governing the dynamic equilibrium in the axial direction, and the other of the fourth order governing the dynamic equilibrium in the radial direction. These equations take the form

$$\frac{\partial^2 u}{\partial z^2} + \frac{\nu}{R}\frac{\partial w}{\partial z} - \frac{\rho_s h}{D}\frac{\partial^2 u}{\partial t^2} = 0 \qquad 0 < z < L \tag{8}$$

$$K\frac{\partial^4 w}{\partial z^4} + \frac{Eh}{(1-\nu^2)}\frac{w}{R^2} + \frac{\nu Eh}{(1-\nu^2)R}\frac{\partial u}{\partial z} + \rho_s h\frac{\partial^2 w}{\partial t^2} = \begin{cases} p(z,t) & 0 < z < H \\ 0 & H < z < L \end{cases} \tag{9}$$

where E is the modulus of elasticity of the tank material; ν is Poisson's ratio; $p(z,t)$ is the pressure exerted by the liquid on the tank wall at any time t; and D and K are the extensional and bending rigidities of the shell, respectively.

The complete solution for the axial and radial displacements of the shell can be expressed as

$$\begin{aligned} u(z,t) &= \left[\sum_{i=1}^{6} A_i\, G_i(z) + \sum_{n=1}^{\infty} P_n \sin(\alpha_n z)\right] \\ w(z,t) &= \left[\sum_{i=1}^{6} A_i\, F_i(z) + \sum_{n=1}^{\infty} Q_n \cos(\alpha_n z)\right] \end{aligned} \tag{10}$$

where $F_i(z)$ and $G_i(z)$ are real valued functions representing the homogeneous solution and P_n and Q_n are bounded coefficients. Algebraic manipulations of Eqs. (8), (9) and (10) yields the following expression for Q_n in terms of the unknown coefficients A_i

$$Q_n = \lambda_n \sum_{i=1}^{6} A_i\, T_{in} \tag{11}$$

where λ_n are factors dependent on the properties of the liquid-shell system and its frequencies of vibration and

$$T_{in} = \int_0^H F_i(x)\cos(\alpha_n x)\,dx \tag{12}$$

For a partly-filled tank, two sets of solutions are obtained; one for the wet part of the shell and the second for the dry part. Each of these sets contains six unknown constants, A_i, and hence the two sets together include 12 unknown coefficients. Enforcing the boundary conditions at the base and top of the shell and the compatibility equations at the junction of the lower and upper parts of the shell yields twelve simultaneous algebraic equations. The frequency equation is obtained by setting the determinant of the coefficient matrix equal to zero.

COMPUTER IMPLEMENTATION

Two computer programs have been developed on the VAX 780 at UCI to compute the natural frequencies of the liquid-shell system and the corresponding mode shapes. In the numerical approach, the shell nodal displacements (eigenvectors) are a direct result of the eigenvalue problem, and these are used to solve for the shell force resultants and for the hydrodynamic pressure distribution along the wall of the tank. In the analytical approach, an iterative procedure is adopted. An approximate value for the natural frequency is first assumed and the determinant of the coefficient matrix (12 x 12 in case of partly filled tanks and 6 x 6 in case of completely filled or empty tanks) is calculated. If the value of the determinant is not zero, the entire process is repeated until such condition is met; and this provides the natural frequencies of the system.

PRACTICAL APPLICATION

Recent developments in seismic codes for liquid storage tanks have recognized the effect of wall flexibility on the response of these structures to lateral ground acceleration. However, most of these codes do neglect the effect of vertical ground motion. A simple and closed form equation for the fundamental natural frequency of the system, which can be attractive for inclusion in a simplified code analysis, has been obtained. For practical tank dimensions, the fundamental mode shape is approximated by a cosine function leading to the following closed form formula for the fundamental natural frequency

$$\omega^2 = \frac{Eh/R^2(1 - \nu^2)}{(2H\,\rho_\ell\, I_0(\alpha_1 R)/\pi\ 'I_0(\alpha_1 R))} \qquad (13)$$

A comparison between the frequencies calculated using the analytical and numerical methods and those calculated from Eq. (13) shows that the frequencies computed from the simplified equation agree favorably with those obtained by both computer programs. Once the theoretical and experimental phases are complete, attention will be directed to developing additional simplified formulae which provide shell stresses for use in the design process.

TANK RESPONSE TO VERTICAL EXCITATION

Earthquake response of tanks is greatly influenced by the conditions at the tank base. For an anchored tank, the matrix equation which governs the response to vertical excitations can be written as

$$[M]\ \{\ddot{q}\} + [C]\ \{\dot{q}\} + [K]\ \{q\} = \{P_{eff}\} = -\{F\}\ \ddot{G}_v(t) \qquad (14)$$

where [C] is the damping matrix; and $\{P_{eff}\}$ is the effective earthquake load vector resulting from a given vertical ground motion $G_v(t)$. The external forces acting on the shell due to vertical ground motion include the distributed inertia force of the shell and the hydrodynamic pressure on the tank wall assuming it to be rigid. This pressure takes the familiar form

$$p_g(r,z,t) = -\rho_\ell(z - H)\ \ddot{G}_v(t) \qquad (15)$$

The virtual work done by these external loads can be expressed as

$$\delta W = 2\pi R \int_0^L \{F_g\}^T \{\delta d\}\, dz + 2\pi R \int_0^H p_g(r,z,t)\, \delta w\, dz$$
$$= - \ddot{G}_v(t) \{\delta q\}^T (\{\bar{F}\} + \{\bar{\bar{F}}\}) = - \ddot{G}_v(t) \{\delta q\}^T \{F\} \qquad (16)$$

ILLUSTRATIVE NUMERICAL EXAMPLES

The preceding analyses are applied to several tanks having different proportionality and properties; off these tanks, two are considered herein

(a) Tank 'T': R = 24.0 ft (7.32 m), L = 72.0 ft (21.95 m);

(b) Tank 'B': R = 60.0 ft (18.29 m), L = 40.0 ft (12.19 m);

and both have a uniform wall thickness of h = 1.0 inch (2.54 cm). Both tanks are made of steel and are filled with water.

Free Vibration Analysis: Inspection of the natural frequencies and mode shapes obtained from the finite element model indicates excellent agreement with those obtained from the analytical solution. It has been clear that, for empty tall tanks, axial deformation governs the dynamic behavior whereas radial expansion dominates for short tanks. For completely full tanks, the added liquid mass is much larger than that of the shell, and therefore, the frequencies are reduced appreciably than those of empty tanks. Figure (4)

Table 1. Comparison of Natural Frequencies of Full Tanks (Hz)

Mode No.	Tall Tank 'T'		Broad Tank 'B'	
	Numerical	Analytical	Numerical	Analytical
1	6.86	6.75	6.40	6.27
2	18.26	17.99	11.97	11.77
3	26.16	25.79	15.34	15.13

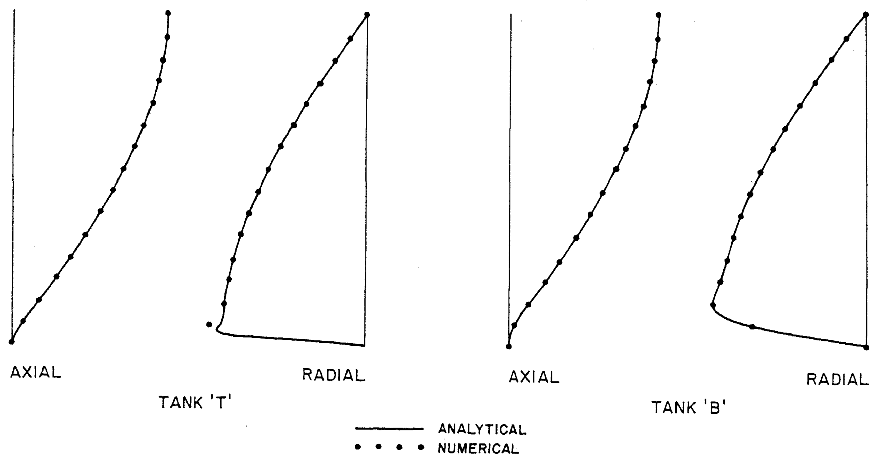

Fig. (4). Fundamental Mode Shape of Full Tanks.

shows the fundamental mode shape of the completely-filled tanks while Fig. (5) shows shell stress distributions associated with that mode. Figure (6) presents the hydrodynamic pressure distribution associated with the two lowest modes of vibration of tank 'T'.

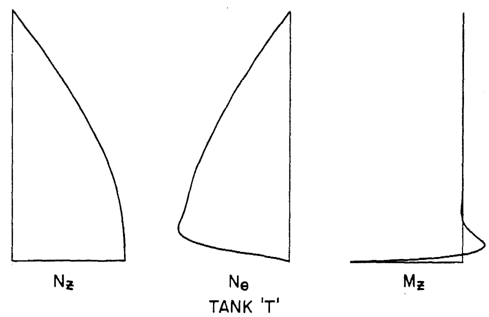

Fig. (5). Shell Stress Distributions.

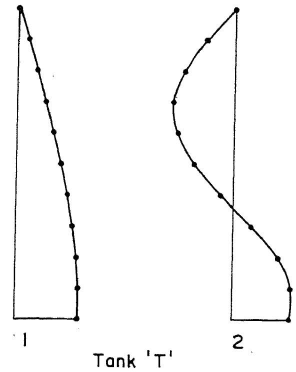

Fig. (6). Hydrodynamic Pressure.

Reduction in liquid level inside the tank increases the natural frequencies of the system as shown in Fig. (7) where the variation of the fundamental natural frequency of tank 'T' and tank 'B' with liquid height is plotted. The variation of the fundamental mode shape with liquid depth inside the broad tank is presented in Fig. (8).

Forced Vibration Analysis: To check the formulation of the load vector, static response of the two tanks was computed. The finite element results were obtained using 30 elements with a finer mesh near the bottom of the tank to capture the rapid variation in displacements and stresses. A comparison between the numerical and the closed form solutions for tank displacements and stresses under hydrostatic pressure has confirmed the accuracy of the load

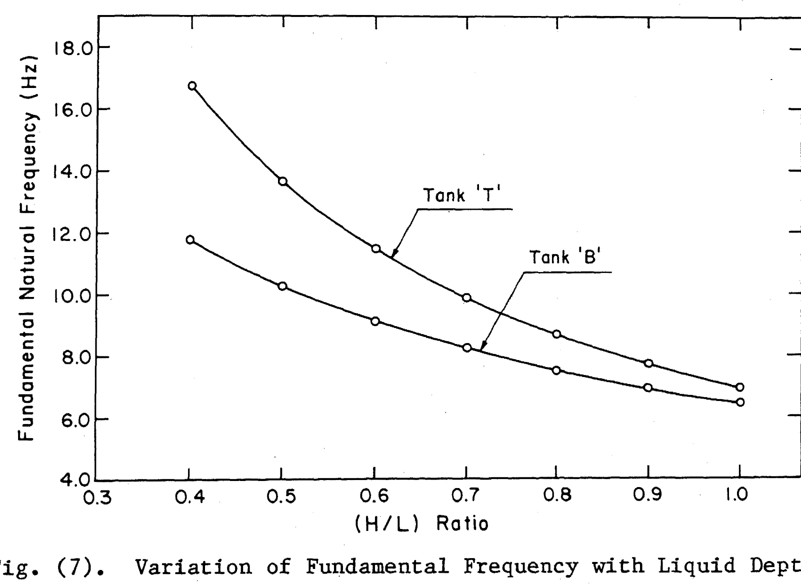

Fig. (7). Variation of Fundamental Frequency with Liquid Depth.

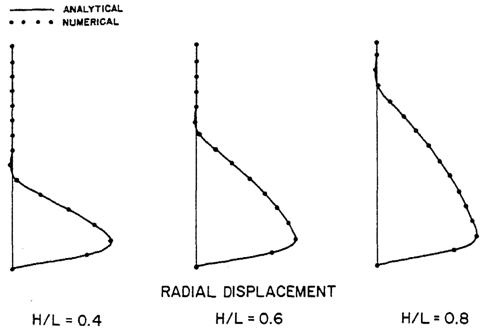

Fig. (8). Fundamental Mode Shape of a Partly-Filled Broad Tank.

vector. Two types of excitations have been used in the forced vibration analysis: impulse functions and actual ground motion records. Preliminary results of the earthquake response analysis have shown that vertical acceleration can play a significant role in tank response to seismic excitations. It should be noted that the relative importance of such acceleration depends on the kinematic conditions at the tank base.

CONCLUSION

A method for seismic analysis of tanks under vertical excitations is described; it is based on superposition of the free axisymmetrical vibration modes obtained analytically and numerically. Inspection of the numerically computed natural frequencies and modes shows excellent agreement with those obtained analytically. A closed form formula for the fundamental natural frequency is obtained. The remainder of the research project is underway with a scheduled completion date of July 1984.

REFERENCES

1. Haroun, M. A., and Tayel, M. A., "Analytical Evaluation of Axisymmetrical Dynamic Characteristics of Cylindrical Tanks," Proceedings of the Fourth EMD/ASCE Specialty Conference, West LaFayette, Indiana, May 1983.
2. Haroun, M. A., and Tayel, M. A., "Numerical Investigation of Axisymmetrical Vibrations of Partly-Filled Cylindrical Tanks," Proc. of the International Symposium on Lifeline Earthquake Eng., PVP Conf., ASME, Portland, June 1983.
3. Kumar, A., "Studies of Dynamic and Static Response of Cylindrical Liquid-Storage Tanks," Ph.D. thesis, Rice University, Houston, Texas, 1981.
4. Marchaj, T. J., "Importance of Vertical Acceleration in the Design of Liquid Containing Tanks," Proc. of the 2nd U. S. National Conference on Earthquake Eng., August 1979, Stanford, California.

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