# BEHAVIOR OF LIQUID STORAGE TANKS UNDER EARTHQUAKE LOADING

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### SUMMARY

The behavior and analysis of ground-supported cylindrical liquid storage tanks subjected to seismic loading is discussed. Specifically, deformations in the lower part of the cylindrical shell, buckling and axisymmetric "elephant's foot bulge" are studied. Significance of vertical along with horizontal ground motion in formation of deformations is investigated. A 3-dimensional finite element model based on equation of motion is constructed for the analysis of the liquid-tank system. Mixed Lagrangian-Eulerian element mesh is used to model large-amplitude sloshing on the free surface. Models of tanks with varying dimensions were subjected to horizontal and/or vertical ground-motion histories.

# INTRODUCTION

Cylindrical oil and water storage tanks have suffered damages, some completely failed losing their contents, in past earthquakes in various parts of the world. In some cases, devastating fires have resulted as the released oil has ignited. Today, liquefied natural gas is also stored in above-ground storage tanks. Should these tanks fail in earthquakes, the resulting large poisonous vapor clouds might ignite.

Damages to cylindrical liquid-storage tanks have usually been in one of the following forms: 1) damage to the roof and upper parts of the shell due to sloshing liquid, 2) damage to piping and equipment due to relative displacements, and 3) buckling or "elephants's foot bulge" (EFB) in the lower part of the tank shell. EFB has often been thought to be an instability phenomenon which, like buckling, is due to overturning effects resulting from horizontal ground motions. Design standards for oil and water tanks, and most studies on tanks, consider only the effects of horizontal excitation. Recently it has been suggested that hoop tension in the cylinder is an important factor in formation of EFB (Refs. 1 and 2), and also that vertical excitation may have a significant effect (Ref. 3). There may be spectral amplification for the frequency of the lowest axisymmetric "breathing mode" of the liquid-tank system. As the safety factor against yielding in hoop tension is 1.5 for tanks that have been designed for primary hoop stress, a yielding zone might develop at the bottom of the tank due to vertical excitation alone.

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### FINITE ELEMENT MODEL

A 3-dimensional finite element model was constructed to analyze the liquid-shell system subjected to seismic loads. Basis for the discretization of the liquid is the equation of motion for "slightly compressible" (Ref. 4) viscous fluid, with velocity as the nodal parameter. An 8-nodal isoparametric element was used with trilinear interpolation for velocities and constant average pressure. Mixed Lagrangian-Eulerian mesh description (Refs. 5 and 6) was employed to incorporate the capability to model large-amplitude sloshing on the free surface. Nodes on the free surface move vertically following the free surface. An assumption is made that a liquid particle which is initially on the free surface stays on the free surface. The dependence of the vertical velocities of the free surface nodes on the particle velocities at these nodes can be approximated as

$$\hat{\mathbf{v}} = \hat{\mathbf{A}}(\hat{\mathbf{x}}) \quad \mathbf{v} \tag{1}$$

where  $\hat{v}$  = vector of velocities of free surface nodes (dimension = number of free surface nodes, n),

 $A(\hat{x})$  = coefficient matrix which depends on the vertical coordinates,  $\hat{x}$ , of the free surface nodes (dimension = n × 3n),

v = vector of particle velocities at free surface nodes (dimension = 3n).

As resolution of the liquid boundary layer on the liquid-shell interface is not of interest, a sliding boundary condition is used on the interface. This is modeled by coupling the liquid nodes to the shell nodes with sliding contact element. The shell is modeled with 4-nodal faceted shell elements (Ref. 7).

As the formulation for both the liquid and shell is based on the equation of motion for continuum, the space occupied by the liquid and shell can be viewed as one continuous domain where the equation of motion is discretized according to finite element procedures, and the different physical behavior is accounted for by using different constitutive relations for the liquid and the shell. Thus, the liquid-shell interaction is realized by the element assembly procedure. Resulting coefficient matrices in the direct time integration algorithm are symmetric banded matrices when the nonlinear convective term is treated explicitly, and thus standard equation solvers for symmetric systems can be used.

# TRANSIENT ANALYSIS

A predictor-corrector algorithm based on Newmark time-integration schemes was used for the transient analysis of the coupled liquid-shell system (Refs. 5 and 6). The pressure term (or equivalently, the condition of near incompressibility) is treated implicitly and the convective term explicitly. The convective term sets a time-step restriction for stability (Ref. 8), but in practice accuracy rather than stability criterion determines time-step size. Two additional issues related to stability of the time integration were discovered. 1) When total pressure is used in the calculation and liquid is modeled as inviscid, any cross section (in elevation) of the 3-dimensional element mesh for the liquid must be rectangular. Any skewness in these cross sections will introduce spurious horizontal "forces" to the liquid, resulting

in unphysical horizontal flows in the liquid, which grow if the liquid is inviscid, without bounds and finally make the solution unstable. 2) When 1-point quadrature is applied to the gravity term of the 8-nodal liquid element (1-point quadrature needs to be applied for the static equilibrium of the free surface to correspond to the physical horizontal equilibrium position of the free surface), there are spurious zero-energy modes of the free surface which make the time integration unstable. This problem can be solved by modifying the element model force vector (Ref. 6).

In essence, the solution advances from time step n to time step n+1 by first solving the matrix equation of motion for the coupled system:

$$(\underbrace{\mathbb{M}} + \gamma \Delta t \ \underbrace{\mathbb{C}} + \beta \Delta t^{2} \ \underbrace{\mathbb{K}_{t}}) \ \underbrace{\mathbb{K}_{t+1}}^{(i+1)} = \underbrace{\mathbb{M}} \ \underbrace{\tilde{\mathbb{K}}_{n+1}} + \beta \Delta t^{2} \ \underbrace{\mathbb{K}_{t}} \ \underbrace{\tilde{\mathbb{K}}_{n+1}}_{n+1}$$

$$+ \gamma \Delta t (\underbrace{\mathbb{E}_{n+1}} - \mathbb{N}^{(c)} (\underbrace{\tilde{\mathbb{K}}_{n+1}}) - \underbrace{\mathbb{N}^{(s)}} (\underbrace{\tilde{\mathbb{K}}_{n+1}}))$$
 (2)

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= \underset{\sim}{\mathbb{M}}(l) + \underset{\sim}{\mathbb{M}}(s),
where M
        \widetilde{\widetilde{M}}(\mathfrak{k})
                    = mass matrix for shell,
                       = mass matrix for liquid,
                        = Newmark parameters,
                        = time step,
= Q^{(l)} + Q^{(s)},
         Q
Q(!)
                        = liquid "viscosity and incompressibility matrix,"
         ç(s)
                        = shell damping matrix,
                        = tangent stiffness matrix for shell,
                        = vector of nodal velocities at time step (n+1) after (i+1)
                         iterations,
                        = vector of predictor values for nodal velocities at time
                        step n+1,

= F(1) + F(1) + F(1),

= vector for liquid including external forces and additional
         Fn+1
F(1)
~n+1
                          terms resulting from pressure term,
                        = external force vector for shell,
         \tilde{\mathbf{y}}^{(c)}(\tilde{\mathbf{y}}_{n+1}) = \text{liquid convective term},
         N(s)(\tilde{d}_{n+1}) = internal force vector for shell.
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After this the vertical velocities of the free surface nodes are solved using Eq. (1), and values of all variables updated (for details see Ref. 6).

### CASE STUDIES

Using the model, the possibility of inducing an EFB by vertical excitation was investigated. First, the eigenfrequency of the lowest axisymmetric mode of the liquid-tank system was determined, then an actual vertical earthquake ground-motion history was applied as excitation, with time scaled so that the period of maximum response of the input ground motion coincided with the period of the first axisymmetric mode. Also, cases with combinations of horizontal and vertical excitation were studied.

### CONCLUSION

Numerical studies with the finite element model of the liquid-shell system to study buckling and EFB phenomena are underway at the time this paper is written. Results will be presented at the conference. To simulate and analyze a failure of a tank with the associated large deformations, a shell model capable of treating material and geometric nonlinearity should be incorporated in future studies. Furthermore, it is evident that lift-off of tanks from their foundations is an important factor in the behavior of unanchored tanks. Capability to model this nonlinear phenomenon would also be desirable.

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### REFERENCES

- D. Gotoh, H. Ohya, and N. Kobayashi, "Study on the elephants's foot bulge of the cylindrical tank shell plate caused by an earthquake," JHPI, Vol. 18, No. 4, 1980.
- 2. M. Yamada, "Elephant's foot bulge of cylindrical steel tank shells," JHPI, Vol. 18, No. 6, 1980.
- 3. T. J. Marchaj, "Importance of vertical accelerations in the design of liquid containing tanks," Proc. of 2nd U.S. National Conference on Earthquake Engineering, Aug. 22-24, 1979.
- 4. W. K. Liu, "Development of finite element procedures for fluid-structure interaction," California Institute of Technology, Earthquake Engineering Research Laboratory, Report EERL 80-06, August, 1980.
- 5. T. J. R. Hughes, W. K. Liu, and T. K. Zimmermann, "Lagrangian-Eulerian finite element formulation for incompressible viscous flows," U.S.-Japan Conference on Interdisciplinary Finite Element Analysis, Cornell University, August, 1978.
- 6. J. K. Arros, "Dynamic analysis of liquid storage tanks subjected to seismic excitation," Ph.D. thesis, Civil Engineering Department, Stanford University, Stanford, California, 1983.
- 7. T. J. R. Hughes and W. K. Liu, "Nonlinear finite element analysis of shells: Part I. Three dimensional shells," Computer Methods in Applied Mechanics and Engineering, Vol. 26, 1981.
- 8. T. J. R. Hughes, W. K. Liu, and A. Brooks, "Review of finite element analysis of incompressible viscous flows by the penalty function formulation," Journal of Computational Physics, Vol. 30, 1979.