INFLUENCE OF RESERVOIR AND DAM ON THE HYDRODYNAMIC PRESSURE

D.K. PAUL[(I)]

SUMMARY

The dam-reservoir system is considered as two field coupled problem. An Eulerian-Lagrangian (EL) formulation is adopted where the motion of the reservoir water is described by displacement potential variable in an Eulerian system and the motion of the structure is described by displacement in a Lagrangian system. In this paper, the influence of shape of dam on the hydrodynamic pressure distribution along the height of the dam at the time of maximum hydrodynamic force, and the influence of shape of reservoir on the response of dam and hydrodynamic force are investigated.

INTRODUCTION

The hydrodynamic force plays an important role in the design of a major dam against seismic forces. The various factors which influence the hydrodynamic forces are the compressibility of the water, flexibility of the dam, shape of the reservoir, shape of the dam, silt suspension and silt deposit at the upstream face of the dam. In this investigation, influence of shape of reservoir and dam on the respone of dam and the hydrodynamic pressure have been investigated.

A field partitioned solution scheme, which eliminates the inefficiencies due to overall unsymmetric matrices is adopted here (Refs. 1,2). In this procedure the concrete dam and the reservoir water are considered as two separate fields which are continuously interacting with each other at the contact surface. In the finite element analysis of the reservoir, a displacement potential formulation in Eulerian system is adopted, in which the water is considered inviscid, irrotational and compressible. For the concrete dam 2D-plane stress finite elements have been used to discretise the dam. The transient dynamic response of dam-reservoir system is carried out in time domain.

DISPLACEMENT POTENTIAL FORMULATION FOR WATER

In this formulation the displacement in water is replaced by a displacement potential which is scalar quantity (Ref. 3). The displacement potential which conveniently defines the displacement as:

$$\psi_{,i} = -\rho_f \, u_i \qquad (1)$$

where, ψ is the displacement potential, u_i is the displacement in the i^{th} coordinate axis, and ρ_f is the density of fluid (water).

The momentum equation for fluid is expressed as:

$$\rho_f \, (D \, \dot{u}_i / Dt) = \sigma_{ij,j} - \rho_f \, g_i \qquad (2)$$

(I) Reader, Earthquake Engineering Department, University of Roorkee, Roorkee, 247667, INDIA.

where, $D\dot{u}_i/Dt = \ddot{u}_i + \dot{u}_{i,j}\ \dot{u}_j \simeq \ddot{u}_i$ (3)

For small water motion, the term $\dot{u}_{i,j}\ \dot{u}_j$ in (3) can be neglected as compared to $\ddot{u}_i$; g_i is the appropriate gravity component and σ_{ij} are the stress component. If the stresses and the pressures are the excess above hydrostatic pressure then body forces can be discounted. For small displacements the constitutive law for stresses in inviscid water are defined as:

$$\sigma_{ij} = -P(\rho_f)\,\delta_{ij} \tag{4}$$

in which, P is the pressure and δ_{ij} Kroneker's delta. Substitution of (3) and (4) in (2), gives:

$$\rho_f\,\ddot{u}_i = -P_{,i} \tag{5}$$

Substitution of (1) in (5), gives:

$$\ddot{\psi}_{,i} = p_{,i} \tag{6}$$

Integration of (6) and discounting the arbitrary constants yields:

$$\ddot{\psi} = p \tag{7}$$

The continuity condition is expressed as:

$$\varepsilon_v = u_{i,i} = -p/K \tag{8}$$

where, ε_v is the volumetric strain, K is the bulk modulus of water. Eliminating p, u and ρ_f from (1), (6) and (8), we obtain,

$$\nabla^2\psi = \frac{1}{c^2}\ddot{\psi} \tag{9}$$

where $c = \sqrt{K/\rho_f}$ = acoustic velocity of water

Boundary Conditions

The various types of boundary conditions are prescribed as follows:

(i) The prescribed pressure on the free surface with small amplitude waves can be expressed as:

$$p = \rho_f\, gy \tag{10}$$

using (1) and (7), (10) can be expressed as :

$$\partial\psi/\partial y = \ddot{\psi}/g \tag{11}$$

with no surface waves, p = 0 is prescribed.

(ii) On the solid boundaries where the normal component of displacement u_n is prescribed,

$$\partial\psi/\partial n = -\rho_f\ u_n \tag{12}$$

On a rigid boundary $u_n = 0$, therefore, $\partial\psi/\partial n = 0$. In case of base excitation, u_n is composed of the translation at the base and the relative displacement of the structure with respect to the base.

(iii) At the radiating boundaries, the Sommerfield condition is applied to suppress the reflecting waves (Ref. 4)

$$\partial\psi/\partial n = -\dot{\psi}/c \tag{13}$$

Finite Element Discretisation

The equation (9) together with the boundary conditions defines the system. The Galerkin method is used to derive the discretised equations (Ref. 5). An identical variable name for field and nodal quantities is used to express:

$$\psi = \underline{N}\underline{\psi} \; ; \; u = \underline{\bar{N}}\,\underline{\bar{u}} \tag{14}$$

where $\underline{N}$ and $\underline{\bar{N}}$ are the shape functions for water and solid respectively. Finally, following discretised euqations are obtained.

$$\underline{M}_f\ddot{\underline{\psi}} + \underline{C}_f\dot{\underline{\psi}} + \underline{K}_f\underline{\psi} = -\rho_f\,\underline{Q}^T\;(\underline{u} + \underline{d}) \tag{15}$$

where, $(M_f)_{ij} = \frac{1}{g}\int_{\Gamma_F} N_i\,N_j\,d\Gamma + \frac{1}{c^2}\int_{\Omega_f} N_i N_j\,d\Omega$

$$(K_f)_{ij} = \int_{\Omega_f}\left(\frac{\partial N_i}{\partial x}\,\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\,\frac{\partial N_j}{\partial y}\right)\,d\Omega \tag{16}$$

$$(C_f)_{ij} = \frac{1}{c}\int_{\Gamma_R} N_i\,N_j\,d\Gamma; \quad (Q)_{ij} = \int_{\Gamma_I}\bar{N}_i\,\underline{\hat{n}}\,N_j\,d\Gamma$$

Γ_F, Γ_I and Γ_R are the free surface, interaction and radiation boundaries; Ω_f is the fluid (water) domain ; $\underline{d}$ is the ground displacement, and $\underline{\hat{n}}$ unit normal veetor.

DYNAMIC EQUILIBRIUM EQUATIONS FOR SOLID

For dynamic equilibrium of a solid body in motion, the principle of Virtual Work can be used to write the following equations at any time irrespective of material behaviour.

$$\int_{\Omega_s}\partial\underline{\varepsilon}^T\underline{\sigma}\,d\Omega - \int_{\Omega_s}\delta\underline{u}^T\;(\underline{b} - \rho_s\ddot{\underline{u}} - \mu\dot{\underline{u}})d\Omega - \int_{\Gamma_t}\delta\underline{u}^T\,\underline{t}\,d\Gamma - \int_{\Gamma_I}\underline{u}^T\,\underline{p}\,d\Gamma \tag{17}$$

where $\delta\underline{u}$ is the vector of virtual displacements, $\delta\underline{\varepsilon}$ is the vector of associated virtual strains, $\underline{b}$ is the vector of applied body forces, $\underline{t}$ is the vector of surface traction on boundary Γ_t, p is the pressure from water at the interaction surface Γ_I; $\underline{\sigma}$ is the vector of stresses, ρ_s is the mass density of solid, μ is the damping parameter; Ω_s represents solid domain and the displacements $\underline{u}$ are specified on Γ_u.

Finite Element Discretisation

In displacement formulation, the displacements, strains and their virtual counterparts are given by the following relationships (using the same notations for field variables and nodal values).

$$\underline{u} = \bar{\underline{N}}\,\underline{u} \;;\quad \delta\underline{u} = \bar{\underline{N}}\delta\underline{u} \tag{18}$$

$$\underline{\varepsilon} = \underline{B}\,\underline{u} \;;\quad \delta\underline{\varepsilon} = \underline{B}\delta\underline{u}$$

where $\underline{u}$ is the vector of nodal displacements, $\delta\underline{u}$ is the vector of virtual nodal variables, $\bar{\underline{N}}$ is the matrix of global shape functions and $\underline{B}$ is the global strain-displacement matrix. Substitution of (18) in (17) and noting that the resulting equation is true for any set of virtual displacements $\delta\underline{u}$ then the following equations are obtained (If ground acceleration, $\ddot{d}$ is also considered)

$$\underline{M}_s\,\ddot{\underline{u}} + \underline{C}_s\,\dot{\underline{u}} + \underline{q}_s(\underline{u}) = \underline{f}_s + \underline{Q}\ddot{\underline{\Psi}} - \underline{M}_s\,\ddot{\underline{d}} \tag{19}$$

where,

$$\underline{M}_s = \int_{\Omega_s} \bar{\underline{N}}^T \rho_s \bar{\underline{N}}\; d\Omega \;;\quad \underline{C}_s = \int_{\Omega_s} \bar{\underline{N}}^T \mu \bar{\underline{N}}\; d\Omega ;\quad f_s = \int_{\Omega_s} \bar{\underline{N}}^T \underline{b}\; d\Omega + \int_{\Gamma_t} \bar{\underline{N}}^T \underline{t}\; d\Gamma \tag{20}$$

GOVERNING EQUATIONS OF COUPLED MOTION

The structure and water are together idealised as a 2D - system subjected to support excitation both in the horizontal and vertical directions and the equation of motion of individual fields can be expressed as above. If the displacement potential formulation is used for the water then coupled water-structure equations can be expressed in the following matrix form (for linear problem $\underline{q}_s(\underline{u}) = \underline{K}_s\,\underline{u}$).

$$\begin{bmatrix} \underline{M}_s & -\underline{Q} \\ \underline{0} & \underline{M}_f \end{bmatrix} \begin{Bmatrix} \ddot{\underline{u}} \\ \ddot{\underline{\Psi}} \end{Bmatrix} + \begin{bmatrix} \underline{C}_s & \underline{0} \\ \underline{0} & \underline{C}_f \end{bmatrix} \begin{Bmatrix} \dot{\underline{u}} \\ \dot{\underline{\Psi}} \end{Bmatrix} + \begin{bmatrix} \underline{K}_s & \underline{0} \\ \rho_f\, Q^T & \underline{K}_f \end{bmatrix} \begin{Bmatrix} \underline{u} \\ \underline{\Psi} \end{Bmatrix} = \begin{Bmatrix} \underline{f}_s & -\underline{M}_s\,\ddot{\underline{d}} \\ & -\rho_f\,\underline{Q}^T\,\underline{d} \end{Bmatrix} \tag{21}$$

SOLUTION SCHEME

It can be seen that the overall matrices of the combined water-structure system are unsymmetrical. The direct solution of coupled equations involve large unsymmetrical matrices requiring excessive computer storage and time. The adoption of partitioned solution schemes (eliminates most of the above inefficiences). An efficient computer code MIXDYN-FSI has been used (Ref. 6) which uses partitioned solution scheme together with Newmark's predictor-corrector implicit-explicit time integrating scheme (Refs. 7,8). Here both the reservoir and the structure are treated as separate fields coupled together through the contact boundaries. The reservoir and structure meshes may be further partitioned into implicit or explicit group of elements. Both the structure and water may be treated here as nonlinear continua. In this analysis, the water is considered linear, however, a nonlinear water model allowing for cavitation can also be considered (Ref. 9). Eight noded isoparametric element has been used to discretise both for the dam and the reservoir. A 2x2 integration has been used for stiffness matrix and 3x3 integration has been used for consistent mass matrix. In the analysis surface waves are neglected where as radiation boundaries in water are considered.

INFLUENCE OF DAM ON HYDRODYNAMIC PRESSURE

The hydrodynamic pressure distribution along the height of dam is obtained at a time when maximum hydrodynamic force is attained. The influence of dam on the hydrodynamic pressure distribution along the height under various conditions of water compressibility and dam flexibility is investigated.

The dams may be of different height, shape and properties. Three different dams are considered as shown in Fig. 1. The three dams considered are the Pine Flat, Koyna and Bhakra dams. The upstream slope of Pine Flat and Koyna dams are small as compared to the Bhakra dam. Figure 2 shows the normal pressure distribution curves for various dams when subjected to a Heaviside unit base excitation for the following conditions:

a. Dam and foundation are rigid, and the water is incompressible (Fig. 2a)

b. Dam and foundation are rigid, and the water is compressible (Fig. 2b)

c. Dam and foundation are flexible, and the water is compressible (Fig. 2c)

When the dams are considered rigid, the normalised pressure distribution for the Koyna and Pine Flat dams are nearly identical for both the incompressible and compressible water (see Figures 2a-b). For compressible water the pressure distribution curves are plotted when the hydrodynamic force on the dam is at its maximum. The curves for the Bhakra dam have a different distribution at the lower half of the dam and this may be attributed to the inclined upstream slope of the dam. Figure 2c shows the normalised pressure distribution curves when both the dam and foundation are flexible, and the water is considered compressible. The distribution curves are significantly different for the various dams considered.

Table 1 shows the ratios of peak hydrodynamic force to hydrostatic force on various dams under different conditions when subjected to a Heaviside unit base excitation. When the dams are rigid, the hydrodynamic force decreases with the increase in the ratio of fundamental time period of the reservoir to the dam. When the dam is considered flexible the pressure force does not show a definite trend. The effect of compressibility of water and flexibility of dam is to increase the peak hydrodynamic force significantly as can be seen from the table.

Table -1 Comparision of peak hydrodynamic force on various dams under different conditions when subjected to a Heaviside unit base excitation

Dam-reservoir System	Rigid dam		Flexible dam
	Water incompressible	Water compressible	Water compressible
Koyna dam system γ=0.566	1.070	1.550	2.180
Pine Flat dam system γ=0.919	1.055	1.530	1.890
Bhakra dam system γ=1.156	0.945	1.327	2.308

$\gamma = \frac{(T_f)\ \text{reservoir}}{(T_f)\ \text{dam}}$; The ratio of fundamental periods of reservoir to the dam

INFLUENCE OF BOTTOM SLOPE OF THE RESERVOIR

Three different shapes of reservoirs for the Bhakra dam-reservoir system have been considered as shown in Figures 3a-c and the corresponding responses when subjected to EL-Centre earthquake (N-S component) are shown in Figures 4a-c. When the bottom slope is changed from 1/6 to 1/3, the response of dam-reservoir system does not change appreciably. However, the response significantly changed when the reservoir is bounded as shown in the Figure 4c. Chopra has also observed that the pressure response is not sensitive to reservoir length as long as $L/H > 3$ (where L = Length of the reservoir and H = Height of dam)

CONCLUSIONS

Hydrodynamic pressure distribution curves are not sensitive for rigid dam with nearly vertical upstream face for both incompressible and compressible water. Rigid dam with inclined upstream slope changes the pressure distribution curves significantly. The pressure distribution curves for flexible dams at the time of peak hydrodynamic force is significantly different for the various dams. The response of dam-reservoir system does not change appreciably when the bottom slope of reservoir is changed from 1/6 to 1/3. However, the response significantly changes when the reservoir is bounded.

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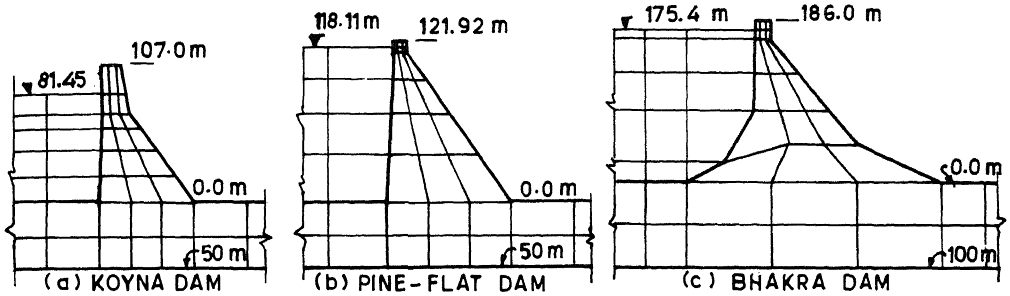

FIG. 1 _DAM-RESERVOIR SYSTEMS

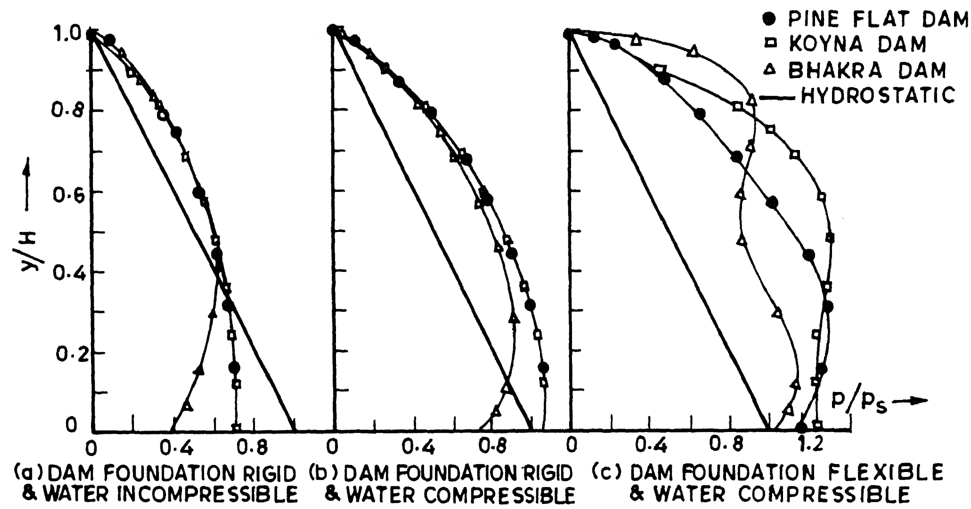

FIG. 2 _COMPARSION OF NORMALISED PRESSURE DISTRIBUTION CURVES FOR DIFFERENT DAMS UNDER VARIOUS CONDITIONS

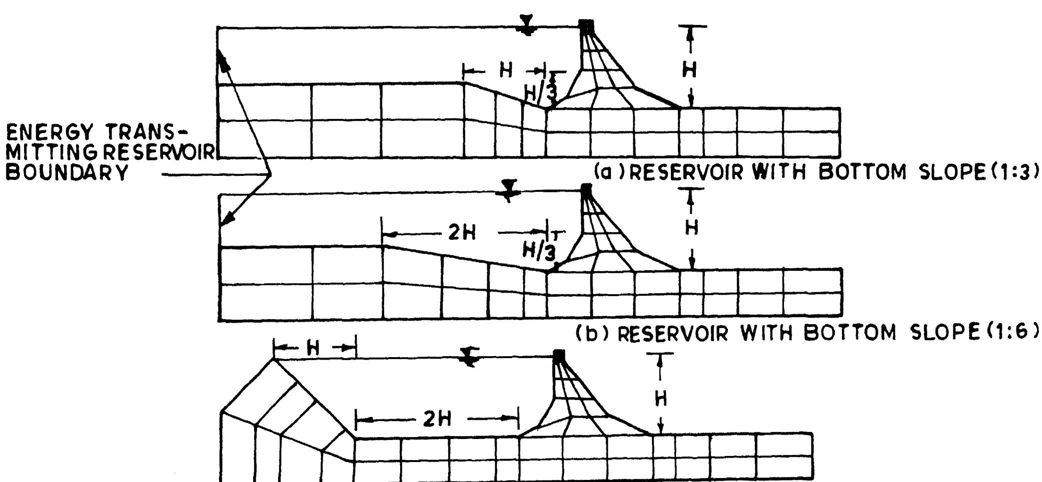

FIG. 3 _RESERVOIR WITH VARIOUS SHAPES

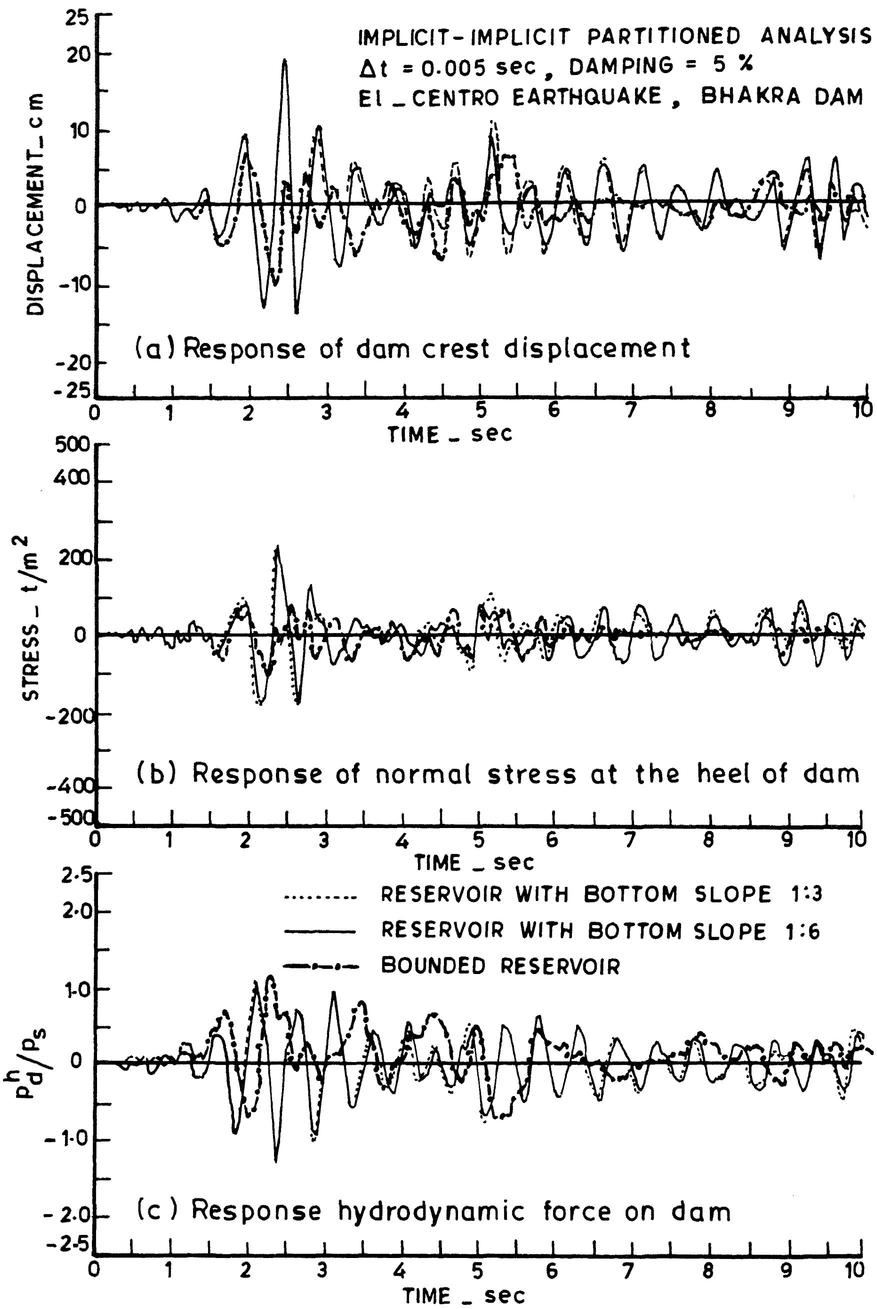

FIG. 4 _ INFLUENCE OF SHAPE OF RESERVOIR ON THE DAM-RESERVOIR RESPONSE