EARTHQUAKE RESPONSE OF SHORT-LENGTH GRAVITY DAMS

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SUMMARY

This paper presents an analysis of the earthquake response of short-length gravity dams. The three-dimensional effects on the dam response are investigated. The analytical procedure used accounts for dam-reservoir interaction, water compressibility and flexibility of reservoir boundaries. Crest acceleration and displacement responses to Pacoima Dam Record are obtained for a range of dam geometries. It is shown that the response decreases significantly as the length to height ratio of dam decreases. Furthermore, the cross-stream component of ground motion appears to have little contribution to the response. Recommendations are made about the range of validity of the two-dimensional approximation.

INTRODUCTION

The possible failure due to earthquakes of dams retaining large quantities of water presents a hazard for life and property in seismically active regions. In addition, any structural damage to the dams themselves may result in a considerable economic loss. This has led to increased attention concerning the dynamic behavior of dams during the past decade.

In analyzing the response of concrete gravity dams to earthquake ground motions, most work to date has considered the dam to be infinitely long, an assumption which simplifies the problem to one in two-dimensions. This would be expected to be satisfactory for a dam whose length is relatively large compared to its height. Intuitively, any method of analysis based on that assumption cannot be used reliably for dams having relatively small length to height ratios. This conclusion is supported by the results of a vibration experiment (Ref. 1) performed on a model wall retaining a body of water. Furthermore, antisymmetric mode shapes and corresponding natural frequencies were clearly identified in the results of a full-scale test (Ref. 2) performed on Pine Flat dam, a short-length concrete gravity dam. Accordingly, a three-dimensional analysis is strongly needed, particularly since a large number of gravity dams in the United States (Ref. 3) have length to height ratios that are too small to justify the two-dimensional assumption.

PROBLEM FORMULATION

Assumptions and Simplifications

In view of the fact that short-length gravity dams are large three-

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dimensional structures of complicated geometry, an analytical evaluation of their dynamic behavior is extremely difficult. A three-dimensional finite element treatment of the dam, formulated by direct application of the three-dimensional elasticity theory, involves a large number of degrees of freedom and is very costly. As a result, some assumptions regarding the geometry of the dam-reservoir system are introduced in order to simplify the analysis. These are:

1. The dam is founded in a rectangular canyon, has a rectangular cross-section uniform along its length, and rests on a rigid foundation.
2. The concrete material of the dam is homogeneous, isotropic and linearly elastic.
3. The out-of-plane deformation of the dam is modeled by a thick plate theory which neglects any bending effects.
4. The reservoir is of constant depth and has parallel sides extending to infinity in a direction normal to the dam face.

A more realistic, but also more complex and expensive analysis would assume a triangular dam cross-section and consider both shear and bending effects on dam deformation. However, since the main objective of the present analysis is to investigate the three-dimensional effects on the dam response, it is believed that the simplified model is quite adequate for this task. Once these effects are proven to be significant, the more realistic model may be used to obtain a more precise representation of the dam behavior.

Equations of Motion

The dam-reservoir system under consideration is shown in Fig. 1. The dam is of length, B, height, D, and constant thickness, h. The reservoir is of width, B, and is partly filled with water to a height, H.

For small amplitude irrotational motion of homogeneous, inviscid and linearly compressible water, the hydrodynamic pressure $p(x,y,z;t)$ generated in the reservoir by motions applied to its boundaries, is governed by the three-dimensional wave equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad (1)$$

in which c is the velocity of sound in water.

Four cases of reservoir boundary motions are of interest: 1) deformational motion of dam, with no ground motion; 2) longitudinal; 3) transverse; and 4) vertical components of ground motion, with the dam assumed rigid in the last three cases. Neglecting the effects of waves at the free surface of the water, and assuming harmonic motions of frequency ω, analytical solutions of Eq. 1 were obtained (Ref. 4) for the amplitudes $\bar{p}_u(x,y,z)$, $\bar{p}_x(x,y,z)$, $\bar{p}_y(y,z)$ and $\bar{p}_z(y,z)$ of the harmonic pressure resulting in these four cases, respectively. When the flexibility of the reservoir floor and sides is included, the pressure in all four cases was found to be frequency-dependent and complex-valued.

The differential equation governing the lateral vibration of the dam is given by:

$$\rho_d h \ddot{u}(y,z;t) - K G h \nabla^2 u(y,z;t) = f(y,z;t) \quad (2)$$

in which u(y,z;t) is the horizontal out-of-plane displacement at a point (y,z) on the dam face at time t; ρ_d and G are the mass density and shear modulus of the dam material, respectively; K = constant relating the average shear stress and strain; ∇^2 is Laplace operator in two-dimensions; and f(y,z;t) is the total, hydrodynamic plus inertia, force per unit area acting on the dam.

RESPONSE TO HARMONIC GROUND MOTION

For a dam forced into vibration by a harmonic ground motion of frequency ω, u(y,z;t) and f(y,z;t) are harmonic with the same frequency, and Eq. 2 can be written in the form:

$$-\rho_d h \omega^2 \bar{u}(y,z) - K G h \nabla^2 \bar{u}(y,z) = \bar{f}(y,z) \quad (3)$$

in which $\bar{u}(y,z)$ and $\bar{f}(y,z)$ are the amplitudes of u(y,z;t) and f(y,z;t), respectively. Depending on the direction of ground motion, $\bar{f}(y,z)$ can be expressed as:

$$\left.\begin{aligned} \bar{f}(y,z) &= -\bar{p}_u(0,y,z) - \bar{p}_x(0,y,z) - \rho_w h \bar{a}_x && \text{; longitudinal ground motion} \\ &= -\bar{p}_u(0,y,z) - \bar{p}_y(y,z) && \text{; transverse ground motion} \\ &= -\bar{p}_u(0,y,z) - \bar{p}_z(y,z) && \text{; vertical ground motion} \end{aligned}\right\} \quad (4)$$

where ρ_w is the mass density of water and $\bar{a}_x$ is the amplitude of the longitudinal or x-component of ground motion.

An approximate solution of Eq. 3 is obtained through the use of the Assumed Modes method. The dam displacement is expressed as a linear combination of N admissible functions, $\phi_n(y,z)$, n = 1,2,...,N, satisfying the essential boundary conditions of the dam. Thus, one can write:

$$\bar{u}(y,z) = \sum_{n=1}^{N} \bar{e}_n \cdot \phi_n(y,z) \quad (5)$$

in which $\bar{e}_n$ are amplitudes of generalized coordinates. Substituting into Eq.3, multiplying by the admissible functions, and integrating over the dam face, the following matrix equation of motion (Ref. 4) is obtained:

$$\left[-\omega^2 \left[[M] + [\bar{M}]\right] + i\omega[C] + [K]\right] \{\bar{E}\} = \{\bar{F}\} \quad (6)$$

where [M], [C] and [K] are the mass, damping and stiffness matrices of the dam, $[\bar{M}]$ is the added mass matrix, resulting from the interactive pressure $\bar{p}_u$, $\{\bar{E}\} = \{\bar{e}_1, \bar{e}_2, \ldots, \bar{e}_N\}^T$, and $\{\bar{F}\}$ is the load vector, given by:

$$\left.\begin{aligned} \{\bar{F}\} &= -\{\bar{F}^x\} - \{\bar{F}^I\} && \text{; longitudinal ground motion} \\ &= -\{\bar{F}^y\} && \text{; transverse ground motion} \\ &= -\{\bar{F}^z\} && \text{; vertical ground motion} \end{aligned}\right\} \quad (7)$$

where $\{\bar{F}^x\}$, $\{\bar{F}^y\}$ and $\{\bar{F}^z\}$ are the hydrodynamic load vectors, resulting from

$\bar{p}_x$, $\bar{p}_y$, and $\bar{p}_z$, respectively, and $\{\bar{F}^I\}$ is the inertia load vector, resulting from the rigid body motion in the x-direction.

The solution of Eq.6, for all values of the excitation frequency ω, yields the frequency dependent vector $\{\bar{E}\}$ of the amplitudes of the generalized coordinates. These, upon substitution into Eq.5, yield the frequency domain displacement response of the dam. The corresponding acceleration response is then obtained as:

$$\ddot{\bar{u}}(y,z) = -\omega^2 \bar{u}(y,z) \quad (8)$$

Numerical Example

The method of analysis described above is applied to dams having a thickness to height ratio, h/D = 0.4, and made of concrete whose properties are: unit weight, γ_d = 155 pcf (24.3 kN/m^3), and shear modulus G = 2.14 x 10^6 psi (14.74 x 10^6 kPa), which corresponds to a modulus of elasticity and Poisson's ratio of 5X10^6 psi (34.45X10^6 kPa) and 0.17, respectively. The unit weight of water, γ_w = 62.4 pcf (9.81 kN/m^3), and the velocity of sound in water, c = 4720 ft/s (1440 m/s). A value of 0.81 is chosen for K. The following four admissible functions are used:

$$\phi_n(y,z) = \sin(k\pi\frac{y}{B}) \cdot \sin\left[\frac{(2\ell-1)}{2}\pi\frac{z}{D}\right] \; ; \quad k,\ell = 1,2 \quad (9)$$

where n = k+2ℓ − 2. These functions are precisely the first four mode shapes of vibration of a dam with an empty reservoir. The damping matrix is taken as [C] = α[M] + β[K], where α and β are constants chosen such that the fraction of critical damping ζ in the first two modes be equal. A value of ζ = 3% is chosen, based on the results of vibration test (Ref. 2). The reservoir is assumed to be totally full.

Eq. 6 is solved for dams with various B/D ratios, and the frequency dependent vectors of amplitudes of generalized coordinates are obtained for all three components of harmonic ground motion. These are used, along with Eqs. 5 and 8, to obtain the frequency domain responses of the dams. Relative crest displacement and acceleration responses are evaluated at mid-length of dam, in the case of longitudinal and vertical ground motions, and at quarter-length, in the case of transverse ground motion. The displacement results are normalized by multiplying by the square of ω_{01}^r, the fundamental frequency of the full reservoir, and dividing by $\bar{a}$, the amplitude of the ground acceleration. The acceleration results are normalized by $\bar{a}$. When plotted as functions of the normalized excitation frequency $\bar{\omega} = \omega/\omega_{01}^r$, as shown in Figs. 2 and 3, these normalized responses become applicable to dams of any height.

Examination of these two figures shows that the change in B/D affects significantly the frequency domain response of dams, both in magnitude and locations of resonant peaks, which in turn would affect the response to earthquake ground motion. It is also observed that the transverse component of ground motion contributes a small amount to the dam response, as compared to the longitudinal component contribution. The vertical component contributes a fair amount.

RESPONSE TO EARTHQUAKE GROUND MOTION

The frequency domain responses, obtained by the analysis presented in the previous section, are used to evaluate the time domain responses of the dam to arbitrary earthquake ground motions. The Fourier Integral technique is used, in which the Fourier transform of the ground acceleration is first obtained as follows:

$$\bar{a}(\omega) = \int_{-\infty}^{\infty} a(t) \cdot \exp(-i\omega t)dt \qquad (10)$$

The dam displacement and acceleration responses are then given by the following inverse Fourier transforms:

$$u(y,z;t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{u}(y,z) \cdot \bar{a}(\omega) \cdot \exp(i\omega t)d\omega \qquad (11)$$

$$\ddot{u}(y,z;t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{\ddot{u}}(y,z) \cdot \bar{a}(\omega) \cdot \exp(i\omega t)d\omega \qquad (12)$$

where $\bar{u}(y,z)$ and $\bar{\ddot{u}}(y,z)$ are the frequency dependent dam responses to harmonic ground acceleration of unit amplitude.

Numerical Example

To study the three-dimensional effects on the earthquake response of short-length gravity dams, the technique presented above is used to evaluate crest displacement and acceleration responses of several dams, of equal heights and different lengths, to existing earthquake ground motions. The dams considered have equal heights, D = 300 ft (91.44m), and length to height ratios, B/D, ranging from 1.0 to 5.0. The properties of concrete and water are as chosen in the previous example. The N-S, E-W and vertical components of the ground acceleration recorded at the abutment of Pacoima dam, during the San Fernando earthquake of February 9, 1971 are assumed to be applied to the dam-reservoir systems in the longitudinal, transverse and vertical directions, respectively. For efficiency of computation, the forward transform of the ground accelerations, Eq. 10, and the inverse transforms, Eqs. 11 and 12, are performed using a special Fast Fourier Transform algorithm (Ref. 5), especially suited for structural dynamics. The resulting time histories of the relative crest displacement and the absolute crest acceleration are presented in Figs. 4 and 5, respectively.

Analyzing these results, it is observed that:

1. For longitudinal ground motion, a decrease in B/D from 5.0 to 2.0, and from 5.0 to 1.0, reduces the peak crest acceleration by 20% and 42%, respectively, and reduces the peak crest displacement by 47% and 82%, respectively. A decrease in B/D, from 5.0 to 2.0, reduces the peak acceleration response to transverse and vertical motions by 51% and 43%, respectively, and reduces the peak displacement by 71% and 61%.

2. Depending on B/D, the vertical ground motion produces peak acceleration and displacement which are 20→35% and 40→55%, respectively, of what the longitudinal ground motion produces.

3. The transverse ground motion produces, at quarter-span of the dam, peak acceleration and displacement which are less than 8% of what the longitudinal motion produces at mid-span.

Although the previous observations are based on results of responses of dams having a particular height to a single earthquake, it is believed that the trends in the response would generally be the same for other earthquakes.

CONCLUSIONS

Based on the results presented above, the following conclusions are arrived at:

1. The two-dimensional solution, currently used for the analysis of gravity dams, could greatly overestimate the earthquake response of a dam whose length is less than four to five times its height. In such cases, a three-dimensional analysis must be used, and this could result in substantial savings in the dam cost.
2. The antisymmetrical modes have little effect on the dam response, and the contribution of the transverse component of ground motion can be neglected without introducing a considerable error.
3. The level of importance of the vertical component of ground motion is comparable to that of the longitudinal component, and should be included in the analysis of dam responses to earthquake ground motion.

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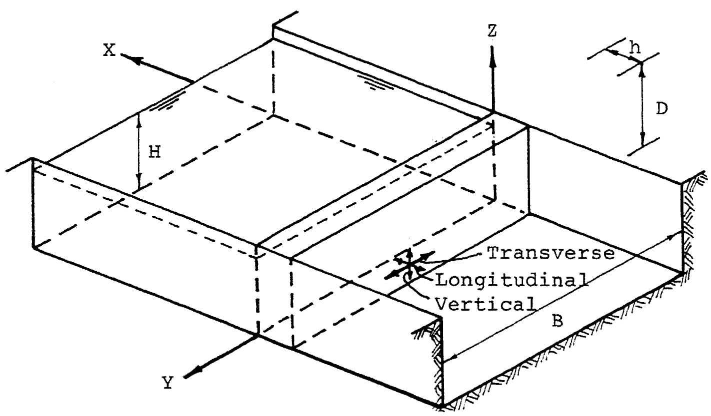

FIG. 1 - Dam-Reservoir Geometry

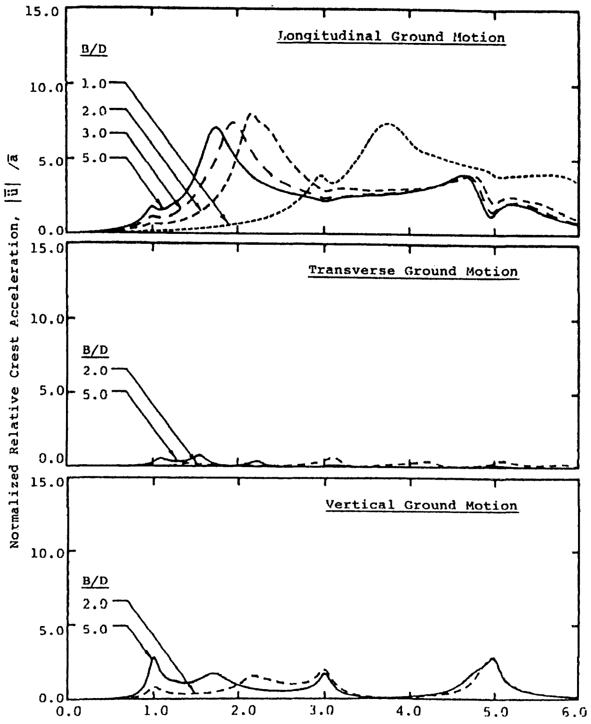

FIG. 3 - Relative Crest Acceleration Response

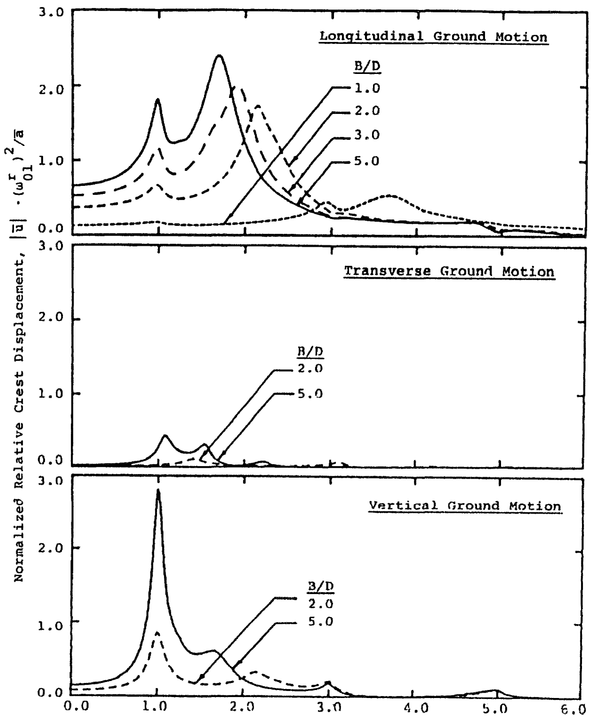

FIG. 2 - Relative Crest Displacement Response

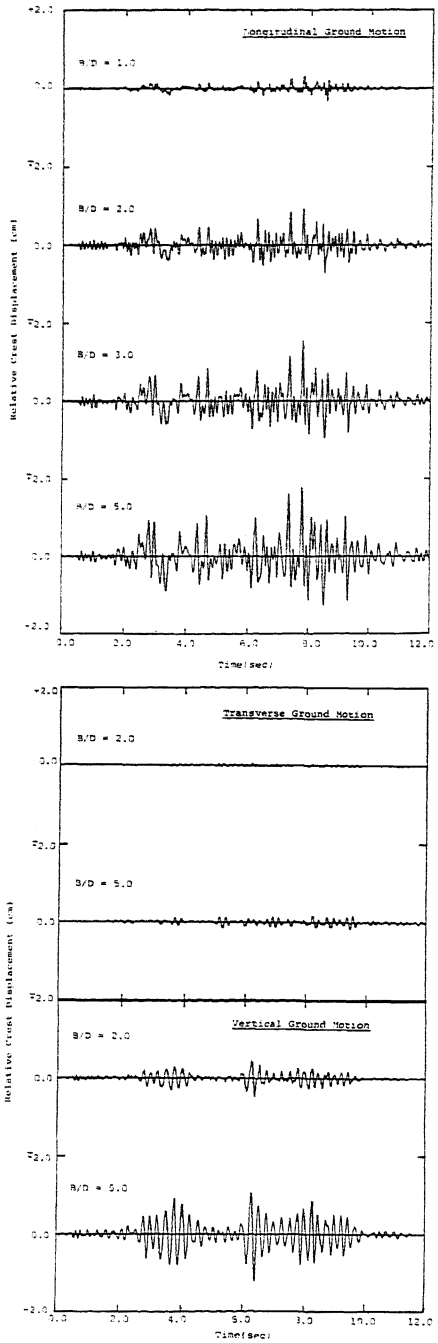

FIG. 4 - Relative Crest Displacement Response

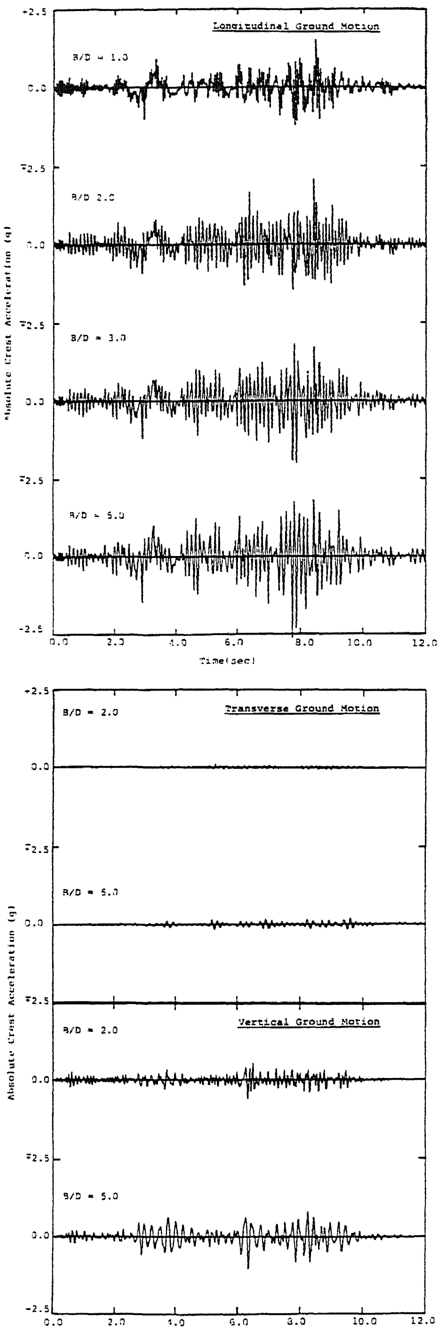

FIG. 5 - Absolute Crest Acceleration Response