

# PERMANENT DISPLACEMENT ESTIMATION ON EMBANKMENT DAMS DUE TO EARTHQUAKE EXCITATIONS

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## SUMMARY

The fundamental criterion for stability analysis of an embankment dam is the factor of safety against sliding of a part along one of the dam slopes. Usually, the dam is treated as a safe structure if this factor of safety is greater than unit, or in other words, if the factor of safety is larger than one ( $F > 1.0$ ) the dam body experiences elastic deformations only. But, if the dam is subjected to a strong earthquake motion, the deformations of its body far extend the elasticity limit, so permanent displacements take place. In such a case the factor of safety, as stability criterion, becomes immaterial, thus the character and magnitude of the permanent displacements become measures for estimating the overall stability of the dam as a structure. It is important too, that an engineering judgement should be introduced in assessing the safety of any dam under consideration.

Having in mind all the above assumptions, an analytical procedure for permanent displacements estimation has been elaborated, based upon the physical characteristics of the built-in material, as well as the nature of the ground

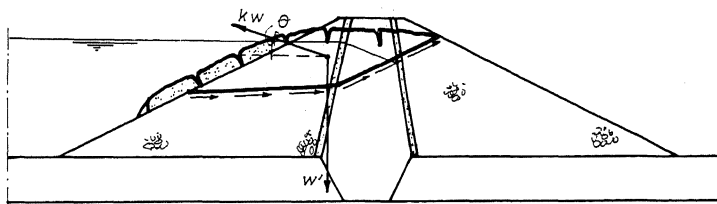


Fig. 1. Definition of a sliding wedge

motion and the response of the structure to that motion. Finally, the proposed procedure has been checked on a real case, where as a pilot structure was taken the Upper San Fernando Dam, damaged by the February 9, 1971 earthquake.

## INTRODUCTION

The idea for permanent displacement evaluation of an embankment dam due to earthquake action was originated first in 1959 by Ambrasseys (1)\*. Further development was made in 1965 by Ambrasseys and Sarma (2), and again in 1970/71 by Ambrasseys (3). Considerable contribution to the solution of this problem was given also in 1965 by Newmark (4) during his famous Rankine Lecture when he proposed very useful nomograms and formulae for practical use. Certain contribution in this field was added in 1973 and 1979 by the author of this paper (5,6). It is also worthwhile mentioning, the remarkable work in this field done by Seed (7) in 1966. The list of authors in this field does not end here.

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\* Numbers in brackets indicate listed literature.

Namely, considerable contribution for assessing the seismic stability of embankment dams has been given by several other investigators from different countries with developed earthquake engineering, but the limitation of space does not allow one to mention them even though they are worth mentioning.

In the frame of this study, an attempt is made to develop a new methodology for permanent displacement estimation of embankment dams due to earthquake excitations. The way for doing so, will be described in the following paragraphs.

#### BASIC CONCEPT OF THE ANALYSIS

The principal idea for permanent displacements evaluation of an earth-fill or rock-fill dam subjected to any earthquake ground motion is based on the following. Consider a sliding wedge which can move along a preselected sliding surface. If the dam is subjected to an arbitrary earthquake ground motion, inertia forces are generated into this wedge, the resultant of which acts at its center of gravity. When the resisting, due to friction and cohesion, forces along the sliding surface are greater than the resultant, due to gravity and inertia force the sliding body remains resting. But, in opposite, when the active forces exceed the intensity of the resisting ones, the sliding wedge starts moving until it reaches a permanent, nonrecoverable displacement (see Fig. 1). Considering the above postulate the mathematical formulation of the problem is the following:

(i) A so called critical acceleration coefficient " $k_c$ " is introduced. This is an acceleration amount introduced into the sliding wedge which causes limit equilibrium i.e. that is a case when the factor of safety against sliding becomes unit ( $F = 1.0$ ). Now, if the overall acceleration of the sliding wedge exceeds the critical one, the wedge starts moving and takes a permanent displacement.

(ii) In order to estimate the permanent displacements of a sliding wedge, the theory of rigid body dynamics should be introduced. In other words, according to the theory a rigid body subjected to an acceleration vector is experiencing movement along a slope if its inertia force is larger than the resisting force. The final amount of the permanent displacement can be determined by proper integration of the differential equation, which mathematically describes the motion. It should be pointed out again that motion occurs only if the resultant active force, acting downwards the slope, is greater than the resisting (frictional and cohesion) force.

(iii) The wedge input acceleration vector, as well as the critical acceleration coefficient are time dependent functions directly associated with the response characteristics of the dam structure. In order to define these characteristics in this study, the dam is treated as a lumped mass parameter system, experiencing shear vibration in the upstream-downstream direction of the river and pressure-tension vibrations in the vertical direction. The mathematical model of the structure is selected in such a manner that the lumping points coincide with the center of gravity of each sliding wedge. The adopted mathematical model is exposed to both horizontal and vertical ground acceleration time histories and response acceleration at each level is obtained. These response acceleration versus time functions are the inputs which may cause permanent displacement to any preselected sliding wedge. In the process of permanent displacement determination, the interaction effect from the sliding

wedge to the rest of the dam body is neglected. It is also assumed that the pore pressure forces are a portion of the gravity and the vertical inertia forces and it is determined by means of introducing a so called pore pressure coefficient  $\bar{B}$ .

From the above explanations it can be concluded that the proposed analysis consists of the following: (i) The mathematical model of the structure is adopted and acceleration response time histories at different levels are determined. (ii) The potential sliding wedges of the dam body are selected and the critical acceleration coefficients as time dependent functions are obtained. Now, by simple comparison of the total acceleration functions and the critical acceleration coefficients it can be assessed whether permanent displacements have to be expected.

#### CRITICAL ACCELERATION COEFFICIENTS OF THE SELECTED SLIDING WEDGE

Let us consider a typical sliding wedge in a manner described in Fig. 2, and assume that it represents a rigid body, which is able to slide along the slope. Now, if this rigid body is exposed to an acceleration input action, inertia forces will be generated inside the body, while resisting forces will be generated along the sliding surface, intending to restrict the body's motion. If the inertia forces exceed the magnitude of the resisting ones, motion takes place and the body receives a permanent displacement.

When the sliding body reaches the limit equilibrium, i.e. when inertia and gravity forces are equal to the resisting ones (the factor of safety  $F=1.0$ ) the corresponding acceleration is named as critical, while its ratio to the acceleration due to gravity is the so called critical acceleration coefficient.

Satisfying the equilibrium conditions as well as the internal relations of the reactive forces, after rearrangement the following system of algebraic equations can be obtained (see Fig. 2)

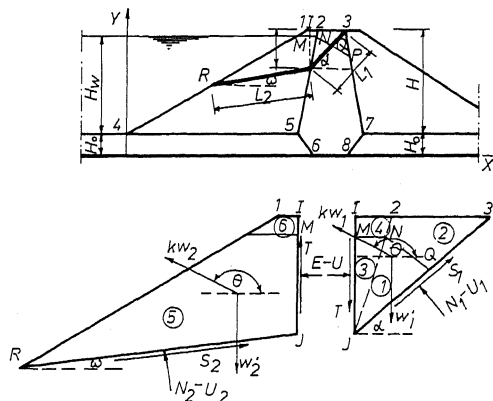


Fig. 2. Determination of the critical accelerations

$$\begin{aligned} -N_1 \sin \alpha + S_1 \cos \alpha + E + K_c W_1 (\cos \theta - \bar{B}_1 \tan \alpha \sin \theta) &= U^- - U_1 \sin \alpha \\ N_1 \cos \alpha + S_1 \sin \alpha - T + K_c W_1 (1 + \bar{B}_1) \sin \theta &= W_1^- + U_1 \cos \alpha \end{aligned}$$

$$\begin{aligned}
-N_2 \sin \omega + S_2 \cos \omega - E + K_c W_2 (\cos \theta - \bar{B}_2 \operatorname{tg} \omega \sin \theta) &= -\bar{U} - U_2 \sin \omega \\
N_2 \cos \omega + S_2 \sin \omega + T + K_c W_2 (1 + \bar{B}_2) \sin \theta &= \bar{W}_2 + U_2 \cos \omega \quad \dots (1) \\
S_1 &= N_1 \operatorname{tg} \phi_1 + K_c \bar{B}_1 W_1 \operatorname{tg} \phi_1 \sin \theta / \cos \alpha + (C_1 L_1 - U_1 \operatorname{tg} \phi_1) \\
S_2 &= N_2 \operatorname{tg} \phi_2 + K_c \bar{B}_2 W_2 \operatorname{tg} \phi_2 \sin \theta / \cos \omega + (C_2 L_2 - U_2 \operatorname{tg} \phi_2) \\
T &= E \cdot r \operatorname{tg} \phi_2 + r (C_2 L_3 - U \operatorname{tg} \phi_2)
\end{aligned}$$

where:

$$\begin{aligned}
\bar{U}_1 &= U_1 - \bar{B}_1 K_c W_1 \sin(\pi - \theta) / \cos \alpha; \quad U_1 = \bar{B}_1 W_1 / \cos \alpha \\
\bar{U}_2 &= U_2 - \bar{B}_2 K_c W_2 \sin(\pi - \theta) / \cos \omega; \quad U_2 = \bar{B}_2 W_2 / \cos \omega \quad \dots (2) \\
\bar{U} &= U_2 \cdot L_3 / L_2 \\
W_1 &= A_1 \gamma_{1p} + A_2 \gamma_1 + A_3 \gamma_{2p} + A_4 \gamma_2; \quad \bar{W}_1 = A_1 \gamma_{10} + A_2 \gamma_1 + A_3 \gamma_{20} + A_4 \gamma_2 \\
W_2 &= A_5 \gamma_{2p} + A_6 \gamma_2; \quad \bar{W}_2 = A_5 \gamma_{20} + A_6 \gamma_2
\end{aligned}$$

These notations represent the following parameters: The  $\gamma_p$ 's are saturated unit weights of the built-in material, while the  $\gamma_0$ 's - submerged unit weights.  $\bar{B}$  - is the so called pore pressure coefficient and "r" is the internal friction mobilization coefficient. "C" and " $\phi$ " are cohesion and internal friction angle, respectively. The  $A$ 's are the corresponding areas of the dam cross-section (see Fig. 2).

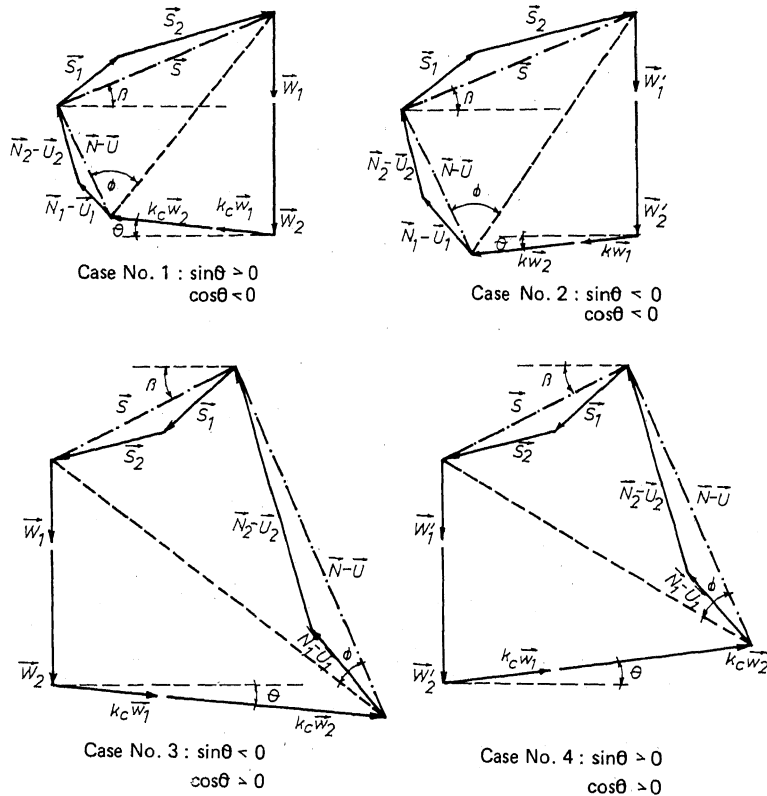


Fig. 3. Cases of critical acceleration coefficients

The system of algebraic equations (1) enables us to determine the seven unknown quantities, i.e.,  $N_1$ ,  $S_1$ ,  $N_2$ ,  $S_2$ ,  $E$ ,  $T$  and  $K_c$ , among which the last one appears as the most important for permanent displacements evaluation. This system also satisfies the limit equilibrium condition of a sliding wedge and allows to create closed polygon of forces like those shown in Fig. 3.

#### PERMANENT DISPLACEMENT EVALUATION OF THE POTENTIAL SLIDING WEDGES

In order to evaluate the permanent displacement of each potential sliding wedge due to an earthquake ground motion, it is, necessary, first, to obtain the acceleration response vectors, and then the corresponding critical acceleration coefficients which as vector coincide with the response acceleration. The way of determining the critical acceleration coefficients was explained in the previous paragraph. The four different cases of these coefficients described in Fig. 3 clearly show that the following kinematic mechanism of a sliding wedge can be obtained (see Fig. 4).

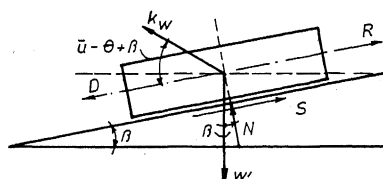


Fig. 4. Kinematic mechanism of a sliding wedge

From the above figure, the following forces can be distinguished:

- Active Force:  $D = (W_1 + W_2) \sin \beta + k(W_1 + W_2) \cos(\pi - \theta + \beta)$
  - Resisting Force:  $R = [(W_1 + W_2) \cos \beta - K(W_1 + W_2) \sin(\pi - \theta - \beta) - U] \operatorname{tg} \bar{\phi}$
  - Driving Force:  $\bar{D} = D - R$
- $$\bar{D} = (W_1 + W_2) (\sin \beta - \cos \beta \operatorname{tg} \bar{\phi}) + K(W_1 + W_2) [\cos(\pi - \theta + \beta) + \sin(\pi - \theta + \beta) \operatorname{tg} \bar{\phi}] + U \operatorname{tg} \bar{\phi} \quad \dots (3)$$

If  $\bar{D} < 0$  : motion does not exist

If  $\bar{D} = 0$  : it is limit equilibrium ( $F = 1.0$ ;  $K = K_c$ )

If  $\bar{D} > 0$  : there is motion

The only unknown parameter in equation (4) is the pore pressure force  $U$ . It can be obtained in the following manner:

$$S = (N - U) \operatorname{tg} \bar{\phi} / F \quad \dots (4)$$

or  $U = N - F S \operatorname{tg} \bar{\phi}$

For limit equilibrium conditions ( $F = 1.0$ ) the above equation becomes:

$$U = N - S \operatorname{tg} \bar{\phi} \quad \dots (5)$$

From Fig. 4 it is clear that

$$N = (W_1 + W_2) \cos \beta - K_c (W_1 + W_2) \sin(\pi - \theta + \beta) \quad \dots (6)$$

$$S = (W_1 + W_2) \sin \beta + K_c (W_1 + W_2) \cos(\pi - \theta + \beta) \quad \dots (7)$$

Introducing now expression (6) and (7) into equation (5), the pore pressure force can be obtained.

$$U = (W_1 + W_2) (\cos \beta - \sin \beta \operatorname{tg} \bar{\phi}) - K_c (W_1 + W_2) [\sin(\pi - \theta + \beta) + \cos(\pi - \theta + \beta) \operatorname{tg} \bar{\phi}] \quad \dots (8)$$

Taking into consideration the above expression, the driving force (3) becomes:

$$\begin{aligned} \bar{D} = & (W_1 + W_2) (\sin \beta - \cos \beta \tan \bar{\phi}) + K_c (W_1 + W_2) [\cos(\pi - \theta + \beta) + \sin(\pi - \theta + \beta) \tan \bar{\phi}] + \\ & + (W_1 + W_2) (\cos \beta \tan \bar{\phi} - \sin \beta) - K_c (W_1 + W_2) [\sin(\pi - \theta + \beta) \tan \bar{\phi} + \cos(\pi - \theta + \beta)] \\ \bar{D} = & (W_1 + W_2) (K - K_c) [\cos(\pi - \theta + \beta) + \sin(\pi - \theta + \beta) \tan \bar{\phi}] \quad \dots (9) \\ \bar{D} = & -(W_1 + W_2) (K - K_c) \cos(\theta - \beta + \bar{\phi}) / \cos \bar{\phi} \quad \dots (10) \end{aligned}$$

Taking into consideration the character of the acceleration response function, all the above parameters are time dependent functions, i.e.,

$$K = K(t); \quad K_c = K_c(t); \quad \beta = \beta(t); \quad \theta = \theta(t); \quad \bar{\phi} = \bar{\phi}(t) \quad \dots (11)$$

Due to the above fact the following differential equation of motion for any sliding wedge (rigid body motion) can be obtained

$$\begin{aligned} m d(t) &= \bar{D}(t) \quad ; \quad m = (W_1 + W_2) / g \\ d(t) &= -g [K(t) - K_c(t)] \cos[\theta(t) - \beta(t) - \bar{\phi}(t)] / \cos \bar{\phi}(t) \quad \dots (12) \end{aligned}$$

$$\begin{aligned} \text{or } d(t) &= Z(t); \quad Z(t) = -g [K(t) - K_c(t)] \cos[\theta(t) - \beta(t) - \bar{\phi}(t)] / \cos \bar{\phi}(t) \\ \tan \theta(t) &= \ddot{V}(t) / \ddot{U}(t) \quad \dots (13) \end{aligned}$$

where:  $\ddot{U}(t)$  - Horizontal Response Acceleration

$\ddot{V}(t)$  - Vertical Response Acceleration

$\beta(t)$  and  $\bar{\phi}(t)$  can be determined from Fig. 3.

Numerically the equation of motion (12) can be integrated by means of adopting any available integration technique. The simplest way is the use of the so called trapezoidal rule of integration. As a result of the integration, velocity and displacement of a particular sliding wedge can be derived.

#### APPLICATION OF THE PROCEDURE TO A PRACTICLE EXAMPLE

Following the analytical procedure described in the previous paragraphs suitable computer programs have been prepared covering all the aspects for permanent displacement estimation of earth-fill and rock-fill dams. As a pilot example the Upper San Fernando Earth-Fill Dam was chosen. The reason of doing

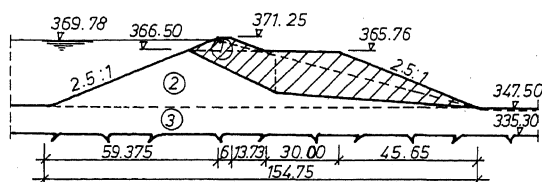


Fig. 5. Cross section of Uper San Fernando Dam

Table No. 1. Values used in the analysis

Material	Dry Unit Weight (kN/m <sup>3</sup> )	Subm. Unit Weight (kN/m <sup>3</sup> )	Satur. Unit Weight (kN/m <sup>3</sup> )	Porosity Coefficient (percent)	Cohesion (kN/m <sup>2</sup> )	Friction Angle (degrees)	Shear Wave Velocity (m/sec)
① Rolled Fill	20.00	12.50	22.50	25.0 %	126.80	25°	180.00
② Hydraulic Fill	17.00	10.25	20.25	32.5 %	0.00	34°	180.00
③ Alluvial Deposit	17.60	10.90	20.90	33.0 %	0.00	38°	230.00

so is because this is a rare dam which has experienced real strong earthquake. Namely, the dam was affected by the February 9, 1971 San Fernando Earthquake, being located in the closest epicentral area. Due to this earthquake the dam suffered significant damages similar to those described and analysed in the above procedure. On the other hand, there was not noticable liquefaction of the built-in material, a particular phenomenon which was not investigated in the frames of this study. The most significant consequences of the San Fernando Earthquake to the Upper San Fernando Dam can be briefly described in the following manner:

Severe longitudinal cracks were clearly evident running along the full length of the dam body on the upstream slope. At the time of the earthquake the water level in the reservoir was above these cracks so that they became visible after the reservoir had been drawn down (Seed H.B. et al, 1973). These cracks resulted from a general downstream movement and settlement of the top portion of the dam with respect to its foundation. Careful survey showed that the crest of the dam at its central portion moved downstream about 1.50 meters (five feet) and settled vertically about 0.90 meters (three feet).

The field observations at the Upper San Fernando Dam suggest that the movements appeared to be concentrated in few well defined slip surfaces going deep into the dam body. One of the possible slip surfaces is indicated in Fig. 5 forming a typical sliding wedge, which's permanent displacement was investigated. Fig. 5 shows the cross-section of the Upper San Fernando Dam at its highest portion, while in Table 1 are presented the characteristics of the built-in materials, which were used in the analysis. Applying the procedure described above and using the already developed computer programs for that purpose, permanent displacement of the indicated sliding wedge were obtained. As an input the San Fernando February 9, 1971 earthquake, recorded at Holyday Inn, 8244 Orion Blvd., Los Angeles, both the strongest horizontal and the vertical components were used, scaled to a maximum peak acceleration of about 0.50 g. This ground acceleration value is considerably lower compared to that used by Seed and his coinvestigators in investigating the causes of slides both at Upper and Lower San Fernando Dams (8). Namely, the maximum ground acceleration value assessed by Seed was 0.60 g.

The peak ground acceleration of 0.50g was assessed according to the results of the studies "Strong Motion Instrumental Data on The San Fernando Earthquake of February 9, 1971 (9), and "A Selection of Important Strong Motion Earthquake Records" (10), as well as after several detailed consultations and verbal communications between professor G.W. Housner and the author during his stay at the California Institute of Technology.

The results obtained in this study lead to the following general conclusion:

Varying the pore pressure coefficients in the range between  $B = 0.5$  and  $\bar{B} = 0.7$  only for the material 2 (Fig. 5) which is still far from the stage of liquefaction, the permanent displacement varies in a narrow range of 1.50 - 1.60 meters. From the other hand the maximum measured displacement on the dam after the San Fernando earthquake was in order of 1.50 meters. This fact clearly proves that the proposed procedure could be a good tool for permanent displacement determination on embankment dams and leads to a satisfying estimation of the resistance of the dam against earthquake actions. In Fig. 6 typical driving velocity and displacement time histories are presented.

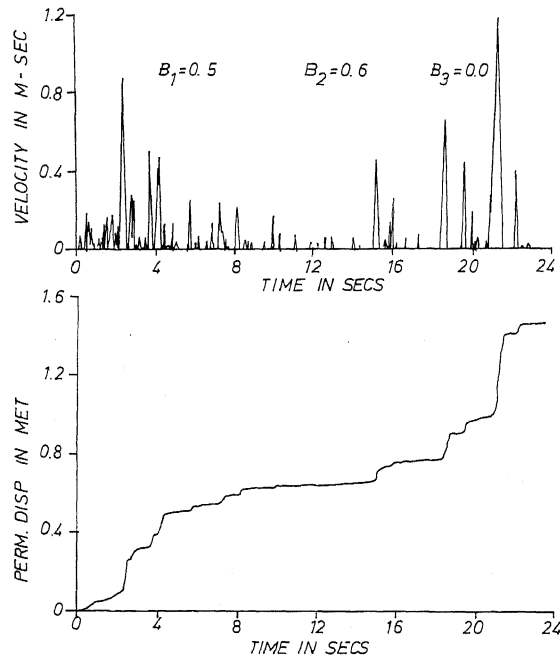


Fig. 6. Driving velocity and displacement diagrams

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