RANDOM VIBRATION OF HIGH-RISE CHIMNEY UNDER GRAVITY AND EARTHQUAKE LOADS

T. Y. Shih (I)
Y. C. Chen (II)

Presenting Author: T. Y. Shih

SUMMARY

This paper presents a stochastic analysis of high-rise chimney subjected to gravity as well as horizontal and vertical earthquake accelerations. Mean-square responses, including displacement, bending moment, and shear for a class of chimneys are determined. Both time histories of responses and spatial distribution of maximum responses along the elevation of chimney are presented. The implication of the significant analytical results on the design is discussed.

INTRODUCTION

Recent trend of building taller chimney to reduce air pollution has increased the importance of understanding the chimney's dynamic behavior for designing. Earthquake is a major load a chimney has to be able to withstand. Most investigations are restricted to the behavior of existing chimneys subjected to actual earthquake motions. Refs. 1-5 are some publications of these studies. A chimney is usually treated as an Euler-Bernoulli beam-column which may be modeled either as a continuous system or as a discrete lumped mass system. The input loads are either actual strong motion earthquake time history or response spectra curves developed by enveloping the maximum responses generated from real earthquakes. These deterministic studies have helped us to understand the seismic behavior of chimney better, and, hence aseismic chimney design can be improved.

Since earthquake motion is random in nature and chimney configuration is diverse, the usefulness of case studies is rather limited in scope. To supplement these shortcomings, a random vibration investigation is made. The earthquake acceleration is modeled as a modulated non-stationary random process. The equation of motion for the chimney is nondimensionalized for generality. In addition to horizontal earthquake, the effects of gravity and vertical earthquake are also investigated. The implication of these analytical results on the design is discussed.

FORMULATION OF THE PROBLEM

The equation of motion for the flexural vibration of a nonuniform column subjected to the simultaneous action of both gravity and earthquake loads is given by the following equation

---

(I) Engineer, NUTECH Engineers, Inc., San Jose, CA 95119

(II) Consultant, NUTECH Engineers, Inc., San Jose, CA 95119

$$\frac{\partial^2}{\partial x^2}\left[EI(x) \frac{\partial^2 y(x,t)}{\partial x^2} \right] + m(g + \ddot{v}_g) \frac{\partial}{\partial x}\left[\bar{A}(x) \frac{\partial y(x,t)}{\partial x}\right] + c \frac{\partial y(x,t)}{\partial t}$$
$$+ mA(x) \frac{\partial^2 y(x,t)}{\partial t^2} = - mA(x)\ddot{u}_g \tag{1}$$

with the following boundary conditions

$$y(0,t) = \frac{\partial y(0,t)}{\partial x} = \frac{\partial^2 y(h,t)}{\partial x^2} = \frac{\partial^3 y(h,t)}{\partial x^3} = 0 \tag{2}$$

In Eq.(1), $\bar{A}(x)$ is used to denote the following integral

$$\bar{A}(x) = \int_x^h A(u)\, du \tag{3}$$

$EI(x)$ is flexural rigidity; $y(x,t)$ is lateral displacement; m is mass density; $A(x)$ is cross sectional area; c is damping coefficient; g is acceleration due to gravity; h is height of the chimney; u_g and v_g are horizontal and vertical earthquake accelerations, respectively.

Consider the class of chimneys in which both outer radius and wall thickness linearly decrease with the increase of chimney elevation, namely

$$R_0(x) = R_b \left[1 - (1 - c_1)\frac{x}{h}\right] , \quad T(x) = T_b \left[1 - (1 - c_2)\frac{x}{h}\right] \tag{4}$$

where $c_1 = R_t/R_b$, $c_2 = T_t/T_b$. $R_0(x)$, R_b, R_t are outer radius of the chimney at any elevation, at the bottom, and at the top, respectively. $T(x)$, T_b, T_t are wall thickness of the chimney at any elevation, at the bottom, and at the top, respectively. It can be shown that classical normal modes for Eq.(1) exists, which, in the chimney case, can be determined from the following eigenvalue problem

$$\frac{1}{A_*(x_*)} \left\{ \frac{d^2}{dx_*^2} \left[I_*(x_*) f_*''(x_*) \right] + R_G \bar{A}_*(x_*) f_*''(x_*) \right\} - R_G f_*'(x_*)$$

$$= \lambda_* f_*(x_*) \tag{5a}$$

$$f_*(0) = f_*'(0) = f_*''(1) = f_*'''(1) = 0 \tag{5b}$$

in which $x_* = x/h$, $A_*(x_*) = A(x)/(T_b R_b)$, $R_G = (mgh^3)/(ER_b^2)$, $\lambda_* = \omega_*^2 = (mh^4\omega^2)/(ER_b^2)$, $\bar{A}_*(x_*) = \bar{A}(x)/(T_b R_b h)$, $I_*(x_*) = I(x)/(R_b^3 T_b)$.

By using normal mode expansion, the equation of motion for the chimney is reduced to the following uncoupled equations

$$Q_j''(\tau) + 2\,\beta_j^*\omega_j^*\,\beta\,Q_j'(\tau) + (\omega_j^*)^2\,(1-\theta_j)\,Q_j(\tau) - R_{v,j}\,V_g''(\tau)Q_j(\tau)$$
$$= -R_{h,j}\,U_g''(\tau) \qquad\qquad j = 1, 2, 3, \dots . \tag{6}$$

in which

$$\beta_j^* = \frac{I_{2,j}}{I_{2,1}}\left(\frac{I_{3,1}\,I_{1,1}}{I_{3,j}\,I_{1,j}}\right)^{\frac{1}{2}},\quad \omega_j^* = \left(\frac{I_{1,1}\,I_{3,j}}{I_{1,j}\,I_{3,1}}\right)^{\frac{1}{2}},$$

$$\theta_j = \frac{I_{4,j}}{I_{3,j}}\,R_G\,,\quad R_{v,j} = \frac{I_{4,j}}{I_{1,j}}\,,\quad R_{h,j} = \frac{I_{5,j}}{I_{1,j}}\,.$$

where these parameters are written in terms of the following integrals

$$I_{1,j} = \int_0^1 A_*(x_*)\,f_j^2(x_*)\,dx_*,\quad I_{2,j} = \int_0^1 f_j^2(x_*)\,dx_*,$$
$$I_{3,j} = \int_0^1 f_j(x_*)\,\frac{d^2}{dx_*^2}\left[\,I_*(x_*)\,f_j''(x_*)\,\right]dx_*\,,$$
$$I_{4,j} = \int_0^1 \bar{A}_*(x_*)\left[\,f_j'(x_*)\,\right]^2 dx_*\,,\quad I_{5,j} = \int_0^1 A_*(x_*)\,f_j(x_*)\,dx_*$$

In the above equations, the following dimensionless variables are introduced: $\tau = \omega t$, $U_g = u_g/h$, $V_g = v_g/h$, $Q_j = q_j/h$; q_j is the jth generalized coordinate.

A modulated nonstationary stochastic model is employed to account for the nonstationary nature of an earthquake; namely, the earthquake acceleration is modeled as a product of a stationary random process and a deterministic time function called envelope function. The envelope function is used to reflect the initial build-up phase and final die-down phase of a realistic earthquake acceleration. For simplicity, the stationary random process is idealized as a Gaussian white noise process. However, the use of white noise is unnecessary. In fact, the solution procedure presented here can be applied to any spectrum of meromorphic function type. Let the envelope function be denoted by $e(\tau)$ and the auto-correlation functions for the white noise processes of horizontal and vertical accelerations be given, respectively, by

$$R_{W_1W_1}(s) = 2\,D_{11}\,\delta(s)\quad,\ R_{W_2W_2}(s) = 2\,D_{22}\,\delta(s) \tag{7}$$

For a point source earthquake, it can be shown that the cross-correlation function of $W_1(\tau)$ and $W_2(\tau)$ is given by

$$R_{W_1 W_2}(s) = 2\,(D_{11}\, D_{22})^{\frac{1}{2}}\, \delta(s) \tag{8}$$

SOLUTION PROCEDURE

By introducing the state variables of responses

$$Z_{1j}(\tau) = Q_j(\tau), \quad Z_{2j}(\tau) = Q_j'(\tau) \tag{9}$$

the j-mode generalized equation of motion given in Eq.(6) can be converted to the stochastic differential equation given below

$$d\,\underline{Z}_j = \underline{m}_j(\,\underline{Z}_j,\, \tau\,)\, d\tau + \underline{G}_j(\,\underline{Z}_j,\, \tau\,)\, d\underline{B}(\tau) \tag{10}$$

in which

$$\underline{Z}_j^T = \lfloor\, Z_{j1} \quad Z_{j2} \,\rfloor$$

$$\underline{m}_j^T(\underline{Z}_j,\tau) = \lfloor\, Z_{j2} \quad -2\,\beta_j^*\,\omega_j^*\,\beta\, Z_{j2} - (\omega_j^*)^2\,(\,1-\theta_j\,)\, Z_{j1} \,\rfloor$$

$$\underline{G}_j(\,\underline{Z}_j,\tau) = \begin{bmatrix} 0 & 0 \\ -R_{h,j}\, e(\tau) & R_{v,j}\, e(\tau)\, Z_{j1} \end{bmatrix}$$

$$\underline{B}^T(\tau) = \lfloor\, B_1(\tau) \quad B_2(\tau) \,\rfloor$$

where $B_1(\tau)$, $B_2(\tau)$ are Brownian motion processes corresponding to the white noise processes given in Eq.(7).

Let the first-order and second-order moments of responses be denoted by

$$m_i = E\,[\, Z_{ji}\,], \quad m_{ik} = E\,[\, Z_{ji} Z_{jk}\,] \tag{11}$$

It can be shown that these two moments of responses are governed by the following equations

$$m_1' = m_2 \tag{12a}$$

$$m_2' = -2\,\beta_j^*\,\omega_j^*\,\beta\, m_2 - (\omega_j^*)^2\,(\,1-\theta_j\,)\, m_1 \tag{12b}$$

$$m_{11}' = 2\, m_{12} \tag{12c}$$

$$m_{12} = m_{22} - 2\,\beta_j^*\,\omega_j^*\,\beta\, m_{12} - (\omega_j^*)^2\,(\,1-\theta_j\,)\, m_{11} \tag{12d}$$

$$m_{22}' = -4\,\beta_j^*\,\omega_j^*\,\beta\,m_{22} - 2\,(\omega_j^*)^2\,(\,1-\theta_j\,)\,m_{12} + 2\,R_{h,j}^2\,e^2(\tau)D_{11}$$
$$+\,2\,R_{v,j}^2\,e^2(\tau)D_{22}\,m_{11} - 4\,R_{h,j}\,R_{v,j}\,e^2(\tau)D_{12}\,m_1 \qquad (12e)$$

When there is no initial disturbances, first-order moments are identically zeros; it is then only necessary to solve the latter three equations simultaneously.

For chimney analysis, natural frequencies are usually well seperated, hence the contribution of modal correlation terms in combining modes is negligible. Thus, considering the first n modes, the responses of chimney are given by

$$E\,[\,R_*^2\,(x_*,\tau)\,] = \sum_{j=1}^{n} F_j^2\,(x_*)\,E\,[\,Q_j^2(\tau)\,] \qquad (13)$$

in which $R_*(x_*,\tau)$ denotes response quantities of either displacement (Y_*), shear (S_*), or bending moment (M_*). The function $F_j(x_*)$ is given, for each individual response, by

$$F_j(x_*) = f_j(x_*) \qquad \text{for displacement} \qquad (14a)$$
$$F_j(x_*) = -\frac{d}{dx_*}\,[\,I_*(x_*)\,f_j''(x_*)\,] \qquad \text{for shear} \qquad (14b)$$
$$F_j(x_*) = I_*(x_*)\,f_j''(x_*) \qquad \text{for bending moment} \qquad (14c)$$

To find the maximum spatial distribution of responses, the following approximation is used

$$\left|E\,[\,R_*^2\,(x_*,\tau)\,]\right|_{\text{Max.,all }\tau} \cong \sum_{j=1}^{n} F_j^2(x_*)\left|E\,[\,Q_j^2(\tau)\,]\right|_{\text{Max.}} \qquad (15)$$

NUMERICAL ANALYSIS AND RESULTS

Numerical studies are made for a class of chimneys having the parameters $c_1 = 0.45$, and $c_2 = 0.5$. The envelope function of earthquake used is

$$e(\tau) = (6.75)^{\frac{1}{2}}\,(\,e^{-0.25\,\tau} - e^{-0.75\,\tau}\,) \qquad (16)$$

which, together with a simulated sample function of acceleration, is shown in Fig.1.

The effects axial loading has on the natural frequency of the chimney is presented in Fig.2. As seen in this figure, axial load has less effect on the higher modes.

A damping ratio of $\beta = 0.03$ will be used throughout the following illustrations where "H", "V", "G" are used to denote the contribution of horizontal earthquake ($D_{11} = 0.01$), vetrical earthquake ($D_{22} = 0.8\ D_{11}$), and gravity ($R_G = 1.2$), respectively. Fig.3 presents the time-history of mean-square displacement of chimney tip. Fig. 4 is another illustration of mean-square displacement of chimney tip, which indicates that only first mode contribution is significant. Fig.5 presents the standard deviation of the contribution to total displacement due to the first three modes. The spatial distribution of maximum mean-square displacement is shown in Fig.6. Figs.7 and 8 present the spatial distribution of maximum mean-square shear force. The spatial distribution of maximum mean-square bending moment are presented in Figs.9 and 10.

CONCLUSIONS

1. The gravity force may change the modal frequencies of the chimney significantly. Its effect decreases for higher modes. However, the gravity force has less obvious influence on mode shapes. When the parameter R_G is small, changes in mode shapes are negligible.
2. The contribution of the higher modes to total mean-square displacement is negligible. However, higher modes do play an important role in contributing to mean-square shear and bending moment.
3. The effect gravity has on the chimney response is uniquely determined by the R_G value. A good design would be to make R_G small enough to reduce the effects of gravity.
4. The vertical earthquake acts as a parametric excitation which introduces additional responses to a large extent on a lateral displacement level. Therefore, a chimney with a stiffer design or a larger damping can significantly reduce the destructive power of the vertical earthquake motion.

REFERENCES

1. Abu-Saba, E.G., " Vibration of Chimney Supported on Elevated Concrete Slab ", J. of the Structural Division, A.S.C.E., Vol.97, No.ST2, February, 1971.
2. Arya, A.S. , Paul, D.K., " Earthquake Response of Tall Chimneys ", Proc. Sixth World Conf. Earthq. Engrg., New Delhi, India, 1977.
3. Rosenfeld, H., " Consideration of the Rotation of the Foundation in the Analysis of a Steel Chimney 130 M. High, Subject to Seismic Movement", Proc. Third World Conf. Earthq. Engrg., Auckland and Wellington, New Zealand, 1965.
4. Rumman, W.S., " Earthquake Forces in Reinforced Concrete Chimneys", J. of the Structural Division, A.S.C.E., Vol.93, No.ST6, December, 1967.
5. Shiau, L-C , Yang, H.T.Y., " Elastic-Plastic Seismic Response of Chimney ", J. of the Structural Division, A.S.C.E., Vol.106, No.ST4, April, 1980.

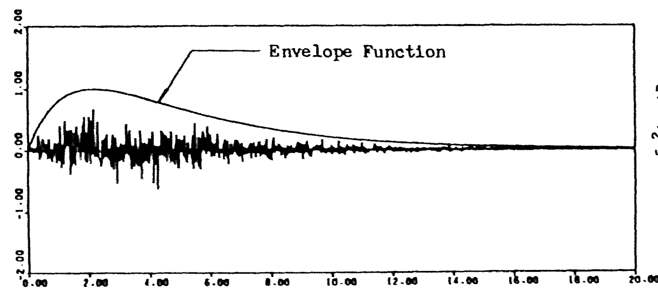

Fig.1 Sample Acceleration Time History And Envelope Function

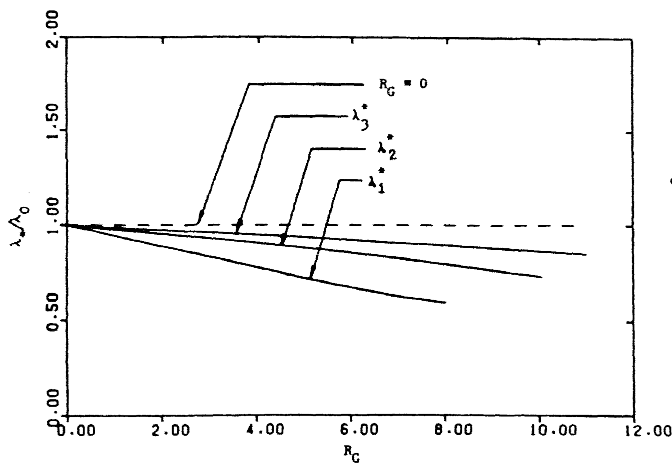

Fig.2 Reduction of Eigenvalue due to Gravity

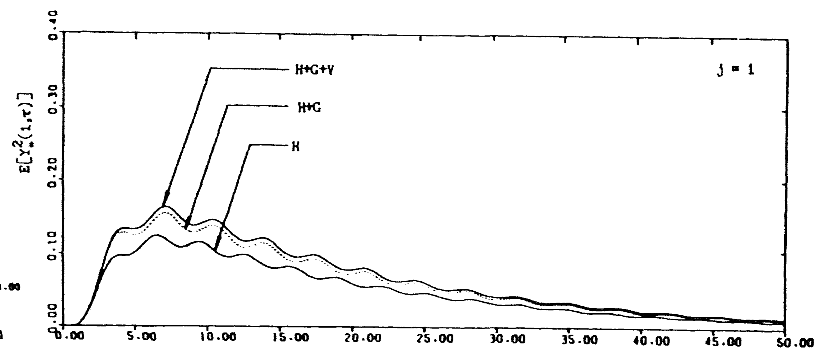

Fig.3 Mean-square Displacement Time History of Chimney Tip (I)

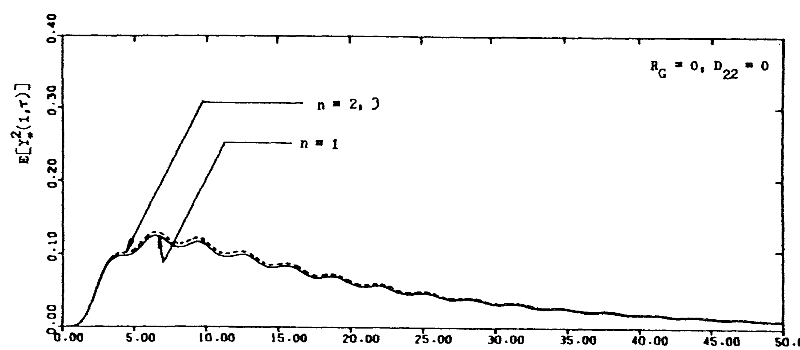

Fig.4 Mean-square Displacement Time History of Chimney Tip (II)

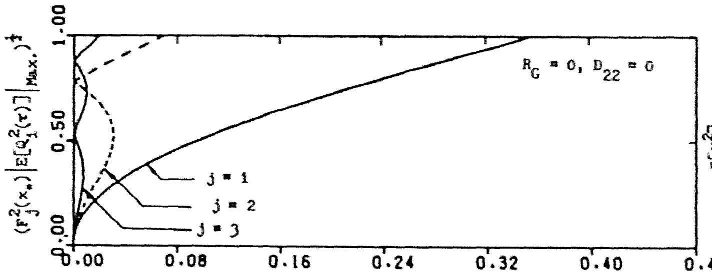

Fig.5 Standard Deviation of the Contribution to Displacement due to first three modes

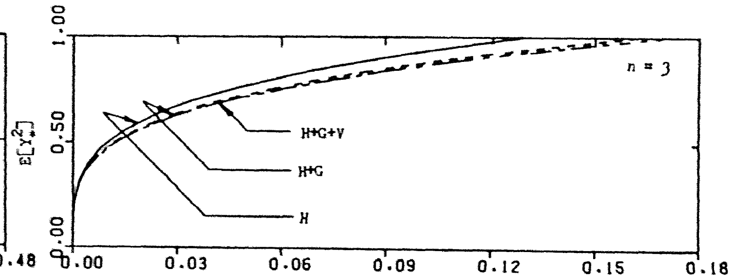

Fig.6 Maximum Mean-square Displacement

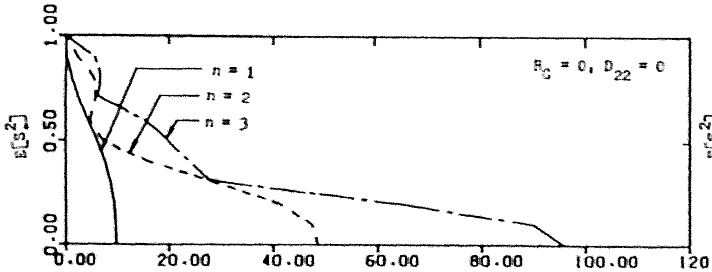

Fig.7 Maximum Mean-square Shear (I)

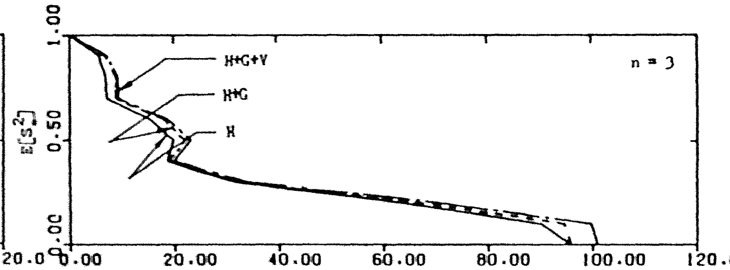

Fig.8 Maximum Mean-square Shear (II)

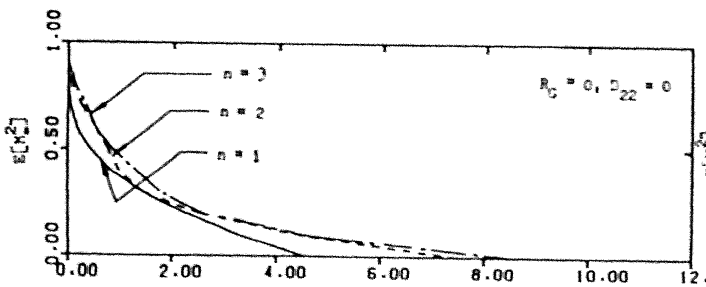

Fig.9 Maximum Mean-square Moment (I)

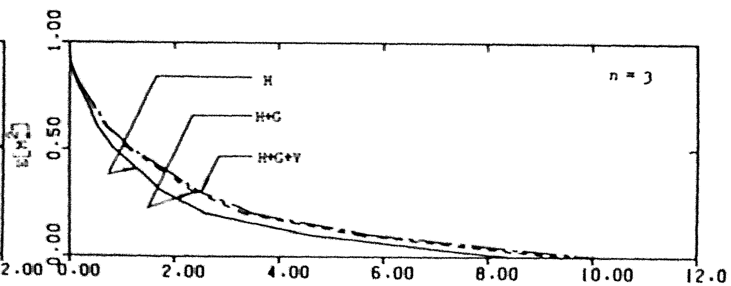

Fig.10 Maximum Mean-square Moment (II)