DYNAMIC RESPONSE OF PIPELINES TO MOVING LOADS

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SUMMARY

Dynamic response of buried pipelines (modeled as cylindrical shells) to moving (seismic) disturbances has been studied in this paper. The pipe is assumed to be embedded in an infinite homogeneous isotropic elastic medium. Numerical results are presented for the maximum dynamic axial and hoop stresses induced in the pipe when the disturbance is a plane longitudinal wave propagating at different angles to the axis of the pipe. It is found that maximum amplifications occur when the ground is soft and they are significantly influenced by the angle of incidence and the frequency of the incident wave.

INTRODUCTION

Three dimensional dynamic response of buried pipelines is a subject of considerable interest to earthquake engineers. Buried pipelines and underground tunnels can be modeled as cylindrical shells. The problem of practical interest is the estimation of peak displacements and stress induced in the pipe or tunnel wall. Recently (Refs. 1-4) axisymmetric (longitudinal) modes of vibration of cylindrical shells have been studied and it is found that the stresses and displacements induced in the pipe wall attain very large values when the frequency of the disturbance is low and when the surrounding soil is soft.

In this paper the general nonaxisymmetric (including bending) modes of vibration of the pipeline are considered. First the problem of moving loads applied on the pipe wall is analyzed. Once the responses of these applied loads are known, the motion due to any incident seismic waves can be solved. As an illustration we consider the effect of a plane longitudinal wave propagating at an arbitrary angle to the axis of the pipeline.

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GOVERNING EQUATIONS

The pipeline will be modeled as a cylindrical shell, infinitely long and continuous, and elastically homogeneous and isotropic. It is assumed that the surrounding ground is elastic, isotropic, and homogeneous. In our earlier work (Refs. 3 and 4) it was found that the response of the shell was not greatly modified if it was allowed not to be perfectly bonded with the surrounding soil. So it will be assumed here that the shell is perfectly bonded with its surrounding.

If $\underset{\sim}{u}^{(i)}$ (r, θ, x, t) denotes the incident displacement field and $\underset{\sim}{u}^{(s)}$ the change in this due to the presence of the pipeline, then the total displacement field at any point in the ground is given by

$$\underset{\sim}{u} = \underset{\sim}{u}^{(i)} + \underset{\sim}{u}^{(s)} \tag{1}$$

The cylindrical polar coordinates r, θ, x have been defined in Figure 1.

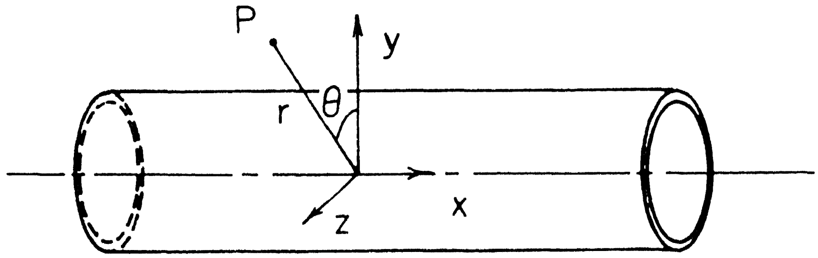

Fig. 1. Geometry of the pipeline.

Note that both $\underset{\sim}{u}^{(i)}$ and $\underset{\sim}{u}^{(s)}$ will satisfy the elastic wave equation

$$\tau^2 \underset{\sim}{\nabla}\, \underset{\sim}{\nabla} \cdot \underset{\sim}{u} - \underset{\sim}{\nabla} \wedge \underset{\sim}{\nabla} \wedge \underset{\sim}{u} = - k_2^2\, \underset{\sim}{u} \tag{2}$$

where $\tau = C_1/C_2$, $k_2 = \omega/C_2$, C_1 , C_2 being the longitudinal and shear wave speeds in the ground. Harmonic time dependence of the form $e^{-i\omega t}$ has been assumed.

Denoting by $\underset{\sim}{U}$ the displacement of a point on the shell middle surface it can be shown that (Ref. 5) the radial, transverse, and axial components, W, V, U, respectively, of $\underset{\sim}{U}$ have the Fourier expansions

$$W = \sum_{n=0}^{\infty} W_n^{1} \cos n\theta + \sum_{n=1}^{\infty} W_n^{2} \sin n\theta$$

$$V = \sum_{n=1}^{\infty} V_n^{1} \sin n\theta + \sum_{n=0}^{\infty} V_n^{2} \cos n\theta \qquad (3)$$

$$U = \sum_{n=0}^{\infty} U_n^{1} \cos n\theta + \sum_{n=1}^{\infty} U_n^{2} \sin n\theta$$

If it is assumed that the incident disturbance is a propagating wave along the axis of the pipeline (x - axis) then the Fourier coefficients W_n^{α}, V_n^{α}, and U_n^{α} $(\alpha = 1,2)$ are found by solving the matrix equation.

$$[A - MT - \Omega^2 I]\, S^{\alpha} = - M\, T\, U^{\alpha (i)} + \tilde{T}^{\alpha(i)} \qquad (4)$$

where the matrices A and MT are defined in Ref. 5.

In writing (4) it has been assumed that the x-independence of $\underset{\sim}{U}$ is of the form $e^{i\xi x}$. Thus the apparent speed of propagation of the disturbance along the pipeline is $C = \omega/\xi$, $2\pi/\xi$ being its wavelength. Various quantities appearing above are $M = \mu/G$, $\Omega^2 = \tau^2\varepsilon^2 Mm\rho^*$, $\rho^* = \rho_s/\rho$, $m = h/R$, $\varepsilon = k_2 R/\tau$. Note that h = thickness of the pipewall; R = mean radius; μ = shear modulus of the ground; G = shear modulus of pipe; ρ_s = density of pipe; ρ = density of ground.

$W_n^{\alpha(i)}(R)$, etc., are the nth Fourier coefficients of the incident disturbance evalauted at r = R. Similarly, $\tau_{rr(n)}^{\alpha(i)}(R)$, etc. are the nth Fourier coefficients of the traction components due to the incident field evaluated at r = R. In deriving (4) Flugge's bending theory has been used. The matrix MT represents the effective stiffnesses of the springs replacing the ground. These stiffnesses are functions of ω, ξ, C_1 and C_2 and n. They incorporate the radiation damping and the inertial effect of ground motion. Full expressions for the elements of T can be found in Ref. 5. It is seen from Eq. (4) that the nth mode of the dynamic displacement, S^{α} is caused by the distributed loading p^{α} given by the right hand side of the equation. Once S^{α} are known for each n these are then substituted in Eq. (3) to form the modal sums for the determination of W, V, and U. The induced axial and hoop stresses in the pipe are then found from the equations

$$N_{xx} = i\xi E_p U - \frac{D}{R}\xi^2 W + \frac{E_p \nu}{R}\left(W + \frac{\partial V}{\partial \theta}\right) \qquad (5)$$

$$N_{\theta\theta} = \frac{1}{R}\left(E_p + \frac{D}{R^2}\right)\left(\frac{\partial v}{\partial \theta} + W\right) - \frac{D}{R^3}\frac{\partial}{\partial\theta}\left(V - \frac{\partial W}{\partial \theta}\right)$$

$$+ i\xi E_p \nu U$$

where

$$E_p = \frac{Eh}{1-\nu^2}, \quad D = E_p \frac{h^2}{12}, \quad \nu = \text{Poisson's ratio of pipe}$$

E = Young's modulus of pipe.

SOLUTION

To solve for the response of the pipe due to a seismic disturbance it would then be convenient to find S^α for the following three types of loading:

$$\text{(1)}\ p^\alpha_{(1)} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}, \quad \text{(ii)}\ p^\alpha_{(2)} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}, \quad \text{(iii)}\ p^\alpha_{(3)} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$(\alpha = 1,2)$$

Denoting these loads by $p^\alpha_{(i)}$ $(i = 1,2,3)$ and the corresponding responses by $S^\alpha_{(i)}$, the nth modal response S^α due to an incident field $\underset{\sim}{u}^{(i)}$ is then found to be

$$S^\alpha = \sum_{j=1}^{3} \left(-M \sum_{k=1}^{3} T_{jk}\, U_k^{\alpha(i)} + \tilde{T}_j^{\alpha(i)}\right) S^\alpha_{(j)} \tag{6}$$

Here $U_j^{\alpha(i)}$ and $T_j^{\alpha(i)}$ represent the nth modes of the jth components of the incident field $\underset{\sim}{u}^{(i)}$ and the associated traction $\underset{\sim}{\tilde{T}}^{(i)}$, respectively, both evaluated at $r = R$.

Incident Plane Longitudinal Wave

Suppose that the incident disturbance is a plane longitudinal wave given by

$$\underset{\sim}{u}^{(i)} = \underset{\sim}{\nabla}\phi^{(i)} \tag{7}$$

where

$$\phi^{(i)} = u_0^{(i)}\, e^{ik_1(x\cos\theta_0 + y\sin\theta_0)}/ik_1$$

$$k_1 = \omega/C_1$$

This represents a wave with propagation vector making an angle θ_0 with the x-axis, wavelength $\xi = k_1 \cos\theta_0$, and apparent wave speed $C = \frac{C_1}{\cos\theta_0}$ along the pipe.

Numerical results for the maximum normalized axial and hoop stresses, NXX and NHH, respectively, are presented graphically in the following. Here

$$NXX = \frac{N_{xx}/h}{i\, n_0^{(i)} \rho C_1^2 \varepsilon/R}, \quad NHH = \frac{N_{\theta\theta}/h}{i n_0^{(i)} \rho C_1^2 \varepsilon/R}$$

NUMERICAL RESULTS AND DISCUSSION

Figures 2 and 3 show the dependence of maximum NXX and NHH on the angle of incidence θ_0 for different frequencies. These are for a concrete cylinder of thickness ratio m = 0.05 lying in a soft soil for which

$$\rho^* = 0.84, \ \sigma = 0.45, \ \nu = 0.2, \ M = 0.04.$$

σ is the Poisson's ratio of the soil.

It is seen that at low frequencies maximum axial stress decreases with increasing angles of incidence. Furthermore, at low frequencies axial stress increases with increasing frequency. Note that at higher frequencies the maximum axial stress increases first with increasing angle of incidence then decreases.

The variation of maximum hoop stress with the angle of incidence and frequency is quite different from that of the axial stress, except that it also increases with frequency at low frequencies. It is interesting to note that the hoop stress has a minimum at some angle of incidence that depends on the frequency; however, at high frequencies, it increases with increasing incident angles. The maximum hoop stress is found to be generally higher than maximum axial stress at all angles of incidence and frequencies. This suggests that the beam models used in estimating the stresses in buried pipelines would not properly predict the failure modes of buried pipelines.

Changes in the maximum axial stress with frequency for three different material properties are shown in Figure 4. The first two cases are

I. Steel cylinder in rock ($\rho^* = 3.0$, $\sigma = 0.25$, $M = 0.3$, $\nu = 0.25$).

II. Concrete cylinder in hard soil ($\rho^* = 0.84$, $\sigma = 0.25$, $M = 0.45$, $\nu = 0.2$).

The third case is the one described above. It is seen that very large axial stresses are induced in a concrete pipe in soft soil at high frequencies. Also it is noted that larger stresses are induced for smaller M. This is further illustrated in Figure 5 where M is varied keeping the other values fixed as in case II.

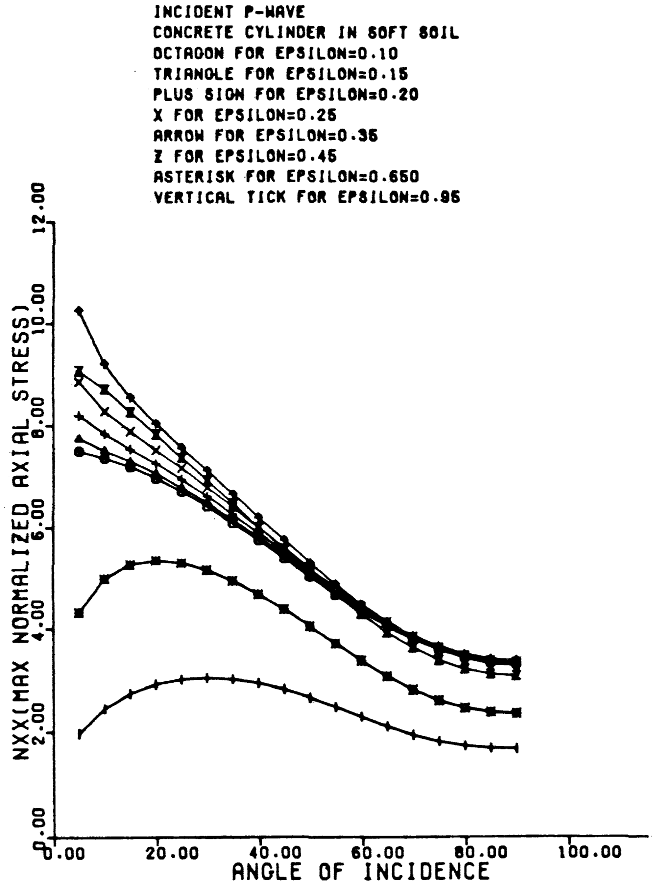

Fig. 2. Maximum axial stress in the pipeline for different angles of incidence.

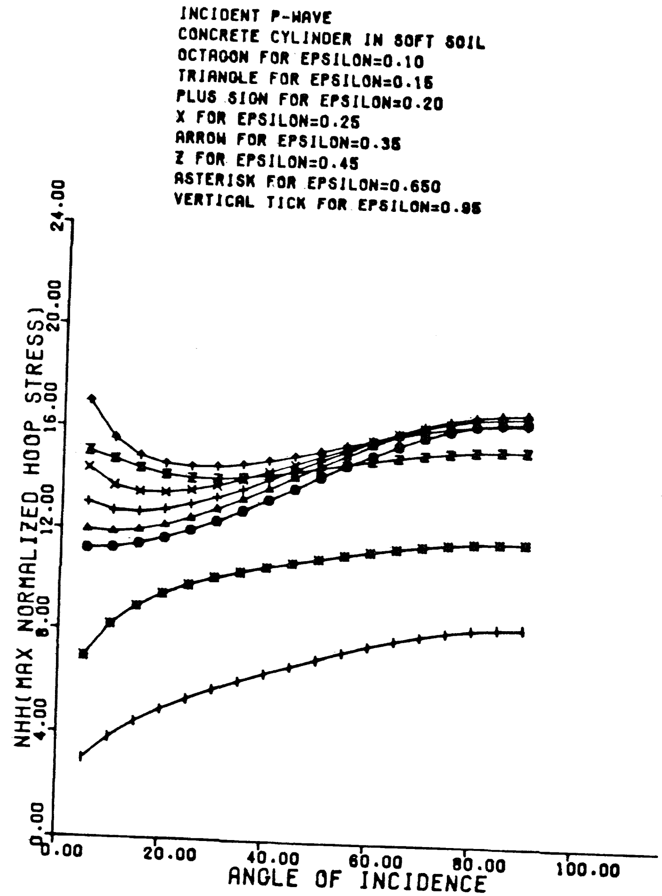

Fig. 3. Maximum hoop stress in the pipeline for different angles of incidence.

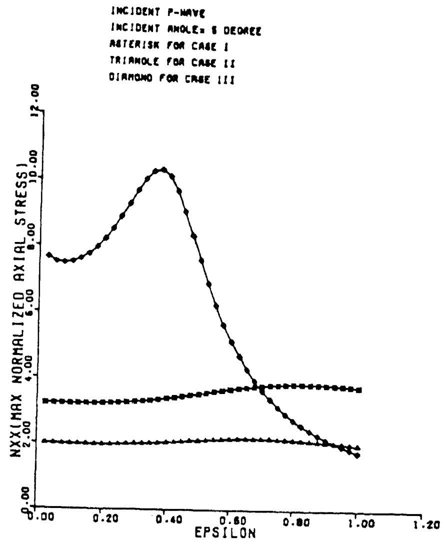

Fig. 4. Variation of maximum axial stress with frequency for different ground and pipeline properties.

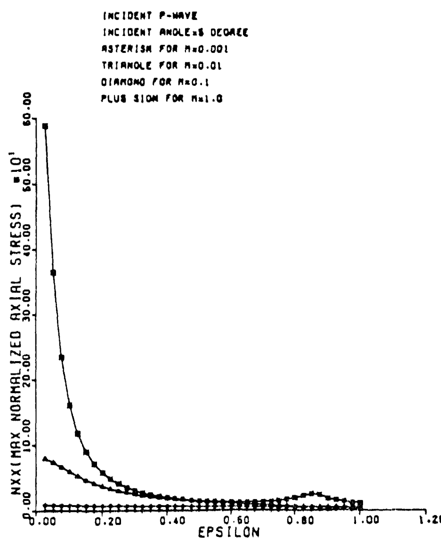

Fig. 5. Variation of maximum axial stress with frequency for different rigidity ratios.

CONCLUSIONS

Maximum dynamic stresses induced in a continuous pipeline due to plane longitudinal waves have been studied in this paper. It is shown that maximum axial stresses are caused by longitudinal waves at small angles of incidence to the pipeline whereas, maximum hoop stresses are caused when the wave is incident nearly perpendicular to the pipe. Maximum dynamic amplification is found at low frequencies when the ratio of rigidities of the soil and the pipeline is small.

ACKNOWLEDGMENT

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