SEISMIC RELIABILITY ANALYSIS OF STRUCTURES AND PIPING SYSTEMS IN NUCLEAR POWER PLANTS

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SUMMARY

For the purpose of carrying out a system reliability analysis of piping systems contained in nuclear power plants subjected to seismic action, a stochastic analysis procedure is presented which, starting from a design response spectrum, analyses the main structure and then the multi-supported piping system. Maximum local response and floor response spectra are presented as intermediate steps of the procedure.

INTRODUCTION

Special structures whose functionality is essential in some productive processes may require a system reliability analysis. To this purpose, once defined the critical elements and the joint distribution function of the load effects, the reliability can be evaluated as the probability that load effects are contained in the safe domain. In case of seismic action, time dependent load effects must be accounted for, and system reliability analysis involves solution of first up-crossing problems.

In the field of nuclear power plants, seismic action is of permanent concern, and a reliability analysis is frequently required for the equipments contained in the main structures. When the safety verification of an isolated item is required, the floor response spectrum technique is adequate. When the reliability analysis regards a piping system attached to several points along the structure, not only the motions of all supports but their correlation also must be defined. Furthermore non synchronous excitation must also be taken into account.

Procedures for the dynamic analysis of multi-supported structures are well established |3,5|, and system reliability analysis can be dealt with according to |2,7|. Since present design practice in nuclear power plants is not advanced enough in the treatment of these problems (see introduction in | 5 |) it seems appropriate clarify in a unified manner the complete procedure for the seismic system reliability analysis of a multi- supported piping. Importance response quantities, such as the probability of maximum local response, and probabilistic floor response spectra are presented at intermediate stages of the procedure.

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In the present study, the seismic action is modelled by a Gaussian ergodic stochastic process defined by its power density, and the structural behavior is assumed as linear; the response process, completely defined by its covariance matrix, is then used to perform the reliability analysis.

1. GROUND MOTION POWER SPECTRUM

In usual practice for the design of nuclear plant structures, the seismic input is given in the form of a normalized elastic response spectrum; in a probabilistic context, the power density function of the underlying stochastic process (assumed as gaussian ergodic) is required.

The problem of deriving the power spectral density function from the response spectrum has been treated from some time |4||6|, and a satisfactory solution is reported in |6|.

The direct problem is the following: given a gaussian ergodic stochastic process, the response spectrum $R_a(\omega)$ can be derived from the power spectral density function $S_a(\omega)$ by multiplying the response standard deviation by a peak factor r. Following the procedure pointed out in |6| the peak factor will be evaluated for any SDOF system with circular frequency ω, on the basis of the first three moments of the power spectrum:

$$\lambda_h = \int_{-\infty}^{\infty} |H'(\Omega/\omega)|^2 S_a(\Omega)\Omega^h \, d\Omega \qquad h = 0,1,2$$

$$r = \{2\, \ell_n\{2\, m\left[1 - \exp(-\delta_e \sqrt{\pi\, \ell_n\, 2\, m})\right]\}^{\frac{1}{2}} \qquad (1)$$

$$m = \frac{D\sqrt{\lambda_2/\lambda_0}}{2\pi(-\ell_n\, p)} \,; \qquad \delta_e = \sqrt{(1 - \lambda_1^2/\lambda_0\lambda_2)^{1.2}}$$

where: $H'(\Omega/\omega) = (1 - (\Omega/\omega)^2 + i2\nu(\Omega/\omega))^{-1}$

- $S_a(\Omega)$ the power spectrum of the ground acceleration
- D the duration of the motion
- p the exceedance probability associated to the response spectrum ordinates.

The response spectrum is given by the expression:

$$R_a(\omega) = r\left[\int_{-\infty}^{\infty} |H'(\Omega/\omega)|^2 S_a(\Omega)\, d\Omega\right]^{\frac{1}{2}} \qquad (2)$$

In order to treat the inverse problem the power spectrum of the excitation will be expressed by the relation:

$$S_a(\Omega) = \sum_{n=1}^{N} \frac{C_n^2}{2}\, \delta(\Omega - |\Omega_n|) \qquad (3)$$

where $\delta(\cdot)$ is the Dirac function and $\Omega_n = 2\pi n/D$.

The C_n values will be evaluated by solving the integral equation (2). By substituting the expression (3) in (2) the following set of equations is obtained:

$$R_a^2\,(\omega) = r^2 \sum_{n=1}^{N} \frac{C_n^2}{2}\,|H'(\Omega_n/\omega)|^2$$

where the unknown values C_n are multiplied by the factor r which is itself a function of the C_n's through the first of (1). The solution requires an iterative procedure, which can be simplified by putting $H(\Omega_n/\omega) = 0$ for $\Omega_n > \omega$.

With this approximation the C_n values are simply derived from the expression:

$$C_n^2 = 2\,|H(\Omega_n/\omega)|^{-2} \left[\frac{R_{a(\omega)}}{r^2} - \sum_{i=1}^{n-1} \frac{1}{2}\, C_i^2 |H(\Omega_i/\omega)|^2 \right]$$

2. POWER SPECTRA OF LOCAL RESPONSE AND FLOOR RESPONSE SPECTRA

The three ground motion components are assumed to be defined by a single power density function $S_a(\Omega)$ and by the correlation matrix:

$$\underline{C}_a(\Omega) = \begin{vmatrix} 1 & \rho_{xy} & \rho_{xz} \\ \rho_{yx} & 1 & \rho_{yz} \\ \rho_{zx} & \rho_{zy} & \alpha \end{vmatrix} \; S_a(\Omega)$$

The cross-power density matrix (CPDM) of the modal response is given by:

$$\underline{S}_R(\Omega) = \underline{H}(-\Omega)\underline{P}\;\underline{C}_a(\Omega)\underline{P}^T\;\underline{H}(\Omega) \qquad (H(\Omega) = H'(\Omega)/\omega^2) \qquad (4)$$

where $\underline{H}(-\Omega)$ is the frequency response matrix, and $\underline{P}$ is a three column matrix, containing the participation factors related to the three components of the seismic action for all the modes.

The CPDM of the local response can be found as:

$$\underline{S}_r(\Omega) = \underline{V}\;\underline{S}_R(\Omega)\underline{V}^T \qquad (5)$$

where $\underline{V}$ is a modal matrix containing in each column the values of the response in the point of interest relative to each mode. Matrix $\underline{V}$ will contain modal displacement or modal load effects, depending on the intended use. The acceleration of the local response can be obtained by introducing in $\underline{V}$ the modal displacement and then multiplying by Ω^4:

$$\underline{S}_r^a(\Omega) = \Omega^4 \underline{S}_r(\Omega)$$

If the absolute acceleration is required, the dragging motion must be added in the response. Its CPDM is:

$$\underline{S}_r^{ab}(\Omega) = \underline{S}_r^a(\Omega) + \underline{T}\;\underline{C}_a(\Omega)\underline{T}^T + \Omega^2(\underline{T}\;\underline{C}_a(\Omega)\underline{P}^T\;\underline{H}(-\Omega)\underline{V}^T + \qquad (6)$$

$$+ \underline{V}\;\underline{H}(\Omega)\underline{P}\;\underline{C}_a(\Omega)\underline{T}^T) \qquad (\underline{T}\text{: dragging matrix}).$$

A first level of use of $S_r(\Omega)$ is the evaluation of the maximum value of the local response for a given exceedance probability. In this case the direct procedure presented in paragraph 1 will be used. The diagonal value $S_{rkk}(\Omega)$ is the power density function of the response in point k. The peak value r will be evaluated using the process moments:

$$\lambda_h = \int_{-\infty}^{\infty} S_{rkk}(\Omega)\Omega^h \, d\,\Omega \qquad\qquad h = 0,1,2$$

and the maximum response, with the probability used in evaluating r| 6 |,is given by: $X_k = r\ \lambda_0$.

With this procedure all the correlations among the modes and among the components of the ground motion are correctly accounted for.

The second use of $\underline{S}_r(\Omega)$ is the system reliability analysis of the main structure, which can be carried out as shown in par. 4 for the case in which the structure can be modeled as a series system.

A further use of the CPDM of local absolute acceleration is the evaluation of the floor response spectra. The direct procedure given in par. 1 will be adopted using the diagonal terms $S^{ab}_{rkk}(\Omega)$.In this way the "combined spectra" are directly obtained. Finally if a multi-supported piping system (with a mass light compared with that of the main supporting structure) must be studied, the whole matrix $\underline{S}^{ab}_r(\Omega)$ will be used as illustrated in next paragraph.

3. STOCHASTIC ANALYSIS OF MULTISUPPORTED PIPING SYSTEM

As illustrated in | 4 | and | 5 |, the dynamic analysis of a multi-supported structure subjected to a non-synchronous motion can be carried out in the frequency domain in the usual way after partitioning the total degrees of freedom in two parts: the first one associated with the displacements of the free nodal points, and the second one containing the displacements of the support nodal points.

The mass, damping and stiffness matrices have to be partitioned accordingly. The subscripts f and b will be used to refer to the free nodal points and to the support ones, respectively.

Furthermore the global motion of each point can be regarded as composed by a quasi-static part, related to the displacement of the support points, and a dynamic part related to the dynamic response of the free points. The over-script s will be used for the quasi-static solution, and d for the dynamic part. The global displacement vector will be written as:

$$\underline{x} = \begin{Bmatrix} x_f \\ x_{\underline{y}} \end{Bmatrix} = \begin{Bmatrix} x^s_f \\ x^s_b \end{Bmatrix} + \begin{Bmatrix} x^d_f \\ 0 \end{Bmatrix}$$

With these notations the equation of motion can be written in the following form; where the partitioned $\underline{m}$, $\underline{c}$, $\underline{k}$ matrices have been employed.

$$\underline{m}_{ff}(\ddot{\underline{x}}^s_f + \ddot{\underline{x}}^d_f) + \underline{m}_{fb}\,\ddot{\underline{x}}^s_b + \underline{c}_{ff}(\dot{\underline{x}}^s_f + \dot{\underline{x}}^d_f) + \underline{c}_{fb}\,\underline{x}^s_b + \underline{k}_{ff}(\underline{x}^s_f + \underline{x}^d_f) + \underline{k}_{fb}\,\underline{x}^s_b = \underline{0}$$

The quasi-static displacements of the free points can be related to the supports displacements using the equation

$$\underline{k}_{ff}\,\underline{x}^s_f + \underline{k}_{fb}\,\underline{x}^s_b = \underline{0} \rightarrow \underline{x}^s_f = -\,\underline{k}^{-1}_{ff}\,\underline{k}_{fb}\,\underline{x}^s_b = \underline{A}\,\underline{x}^s_b$$

By assuming the damping proportional to stiffness: $\underline{c}_{ff}=\alpha\,\underline{k}_{ff}$; $\underline{c}_{fb}=\alpha\,\underline{k}_{fb}$, disregarding out-diagonal mass terms, the equation of motion can be written as:

$$\underline{m}_{ff}\,\ddot{\underline{x}}_f^d + \underline{c}_{ff}\,\dot{\underline{x}}_f^d + \underline{k}_{ff}\,\underline{x}_f^d = -\,\underline{m}_{ff}\,\underline{A}\,\ddot{\underline{x}}_b^s \qquad (7)$$

where, on the right-hand side, the excitation stochastic process appears in terms of the absolute acceleration of the support points.

The CPDM of this stochastic process is the $\underline{S}_r^{ab}(\Omega)$ matrix derived in paragraph 2. The equations (7) can be solved by the usual modal approach. The modal excitation, for each mode of the multi-supported structure supposed on fixed supports, is the following:

$$\frac{\underline{Y}^T\,\underline{m}_{ff}\,\underline{A}}{\underline{Y}^T\,\underline{m}_{ff}\,\underline{Y}}\,\ddot{\underline{x}}_b^s = \underline{P}\,\ddot{\underline{x}}_b^s \qquad (8)$$

where $\underline{Y}$ collects all the modal displacements vectors, and $\underline{P}$ contains the participation factors of all the modes for each component of supports motion.

The procedure given in (4) provides the CPDM of the modal response $\underline{S}_{pR}(\Omega)$ and by means of a modal matrix $\underline{V}_p$ containing displacements or load effects in the points of interest, the calculation (5) gives the response power density $\underline{S}_{pr}(\Omega)$ related to the dynamic component.

The power density matrix of the quasi-static component is

$$\underline{S}_{ps}\,(\Omega) = \underline{Z}(\underline{S}_a(\Omega)/\Omega^4)\;\underline{Z}^T \qquad (9)$$

where $\underline{Z}$ is a matrix, analogous to $\underline{V}$, containing the displacements (or load effects) of the points of interest relative to each displacement of the supported points.

The total response power density matrix is:

$$\underline{S}_p\,(\Omega) = \underline{S}_{pr}(\Omega) + \underline{S}_{ps}(\Omega) + \underline{Z}\;\underline{S}_{d,a}(\Omega)\underline{P}^T\;\underline{H}(\Omega)\underline{V}_p^T + \\ + \underline{V}_p\;\underline{H}(-\Omega)\underline{P}\;\underline{S}_{a,d}\;Z^T \qquad (10)$$

where $\underline{S}_{d,a}(\Omega) = \underline{S}_{a,d}(\Omega) = -\underline{S}_a(\Omega)/\Omega^2$ is the cross spectral density matrix between displacements and accelerations of the support points on the main structure.

Using the procedure presented in paragraph 2 the maximum values of the local response can be derived from the diagonal terms of the matrix $\underline{S}_p(\Omega)$, taking into account of the simultaneous quasi-static and dynamic effects.

4. PIPING RELIABILITY ANALYSIS

Let $\underline{x}$ be the time dependent vector process which contains the load effects in the critical nodes of the piping, and $\dot{\underline{x}}$ its time derivative. The covariance matrix $\underline{\Sigma}_x$ of $\underline{x}$ is obtained by integrating the CPDM of the total piping response $\underline{S}_p(\Omega)$ given by formula (10), and the covariance matrix of $\dot{\underline{x}}$ is obtained by integrating the CPDM of the derivative process $\Omega^2\underline{S}_p(\Omega)$.

Furthemore let the piping be modeled as a series system with a polyhedral safe domain. The failure probability is measured by the probability of outcrossing such a domain. That probability can be approximately evaluated by means of the generalized Rice's formula, which provides the mean outcrossing rate of the safe region.

By adding the further hypotesis that $\underline{x}$ and $\underline{\dot{x}}$ are mutually indipendent processes, Rice's formula results so simplified:

$$\nu = \sum_{1}^{m} {}_i \frac{\sigma_{\dot{x}ni}}{2\pi} \int_{F_i} f_{\underline{x}}(\underline{x}) dF \tag{11}$$

where $\sigma_{\dot{x}ni}$ is the standard deviation of the component of $\underline{\dot{x}}$ orthogonal to the face F_i of the safe domain, $f_{\underline{x}}(\underline{x})$ is the joint probability density function of $\underline{x}$, and the summation is referred to all F_i.

The integrals contained in the summation (11) can be expressed as the product of the probability density of $\underline{x}$ lying on plane π_i which contains the face F_i, by the conditional probability of $\underline{x}$ lying within F_i given it lies on plane π_i.

Expressing the plane π_i as $a_i + \underline{\alpha}_i^T \underline{x} = 0$ ($|\underline{\alpha}_i| = 1$), the integral (11) becomes:

$$\int_{F_i} f_{\underline{x}}(\underline{x})\ dF = \frac{1}{\sigma_{ii}} \phi\left(\frac{a_i}{\sigma_{ii}}\right) P_r(\underline{x} \in F_i | \underline{x} \in \pi_i) \tag{12}$$

where $\sigma_{ii} = \sqrt{\underline{\alpha}_i^T\ \Sigma_x\ \underline{\alpha}_i}$ is the standard deviation of the component of $\underline{x}$ orthogonal to π_i, and $\phi(\cdot)$ is the standard normal density function.

The conditional probability contained in (12) can be evaluated as the probability of the intersection of domains contained in plane π_i, i.e.:

$$P_r(\underline{x} \in F_i \Big| \underline{x} \in \pi_i) = P_r\left(\bigcap_{\substack{k=1 \\ k \neq i}}^{m} z_k \leq \beta_{k|i}\right) \tag{13}$$

where $\underline{Z}$ is a vector of normal gaussian r.v. with covariance matrix:

$$\left[\rho_{kj|i}\right] = \left[\frac{\rho_{kj} - \rho_{ik}\ \rho_{ij}}{\sqrt{1-\rho_{ik}^2}\sqrt{1-\rho_{ij}^2}}\right] \qquad \left(\rho_{kj} = \frac{\underline{\alpha}_k^T\ \underline{\Sigma}_x\ \underline{\alpha}_j^T}{\sigma_{kk}\ \sigma_{jj}}\right)$$

and $\beta_{k|i}$ are the safety distances between the projection of the origin on plane π_i and each side of the polygon F_i expressed by

$$\beta_{k|i} = \frac{\beta_k - \beta_i\ \rho_{ik}}{\sqrt{1-\rho_{ik}^2}} \qquad \left(\beta_k = \frac{a_k}{\sigma_{kk}}\right)$$

Ditlevesen's bounds | 1 | are useful to evaluate expression (13). For this purpose since such bounds can be applied to unions of events, the intersections contained in (13) must be transformed in the complementary unions:

$$P_r(\underline{x} \in F_i | \underline{x} \in \pi_i) = 1 - P_r\left(\bigcup_{k=1}^{m} Z_k > \beta_{k|i}\right)$$

After evaluation of the mean outcrossing rate, the failure probability can be approximated by one of the following formulas:

$P_f < \nu \cdot D$ or $P_f \simeq 1 - e^{-\nu D}$

NUMERICAL EXEMPLE

The procedure illustrated in the previous paragraphs has been applied to carry out the safety analysis of a piping system contained in a four stories building. In Fig.1 the building and the piping system are sketched. The USNRC response spectrum scaled to the peak ground acceleration a = 0.20 g has been adopted. Fig. 2 shows the response spectrum and the spectral power density derived from it following the procedure presented in par.1. Only the horizontal components of the ground motion have been taken into account.

The procedure involves, at first, the stochastic analysis of the main structure.As it has been shown in par. 2 at this stage of the analysis the local response spectra can be taken out. Fig. 3 contains the floor response spectra relative to the component y of the absolute acceleration referred to two points on second floor and one on fourth floor. Notice that both the ground motion components are taken into account in these spectra, and the correlations among the vibration modes of the structure are accounted for as well. The difference between the spectra n.5 and 7, relative to different points on the same floor should be noticed too; it is due to the torsional modes. The second result of the stochastic analysis of the main structure is the CPDM relative to the motion of the piping supports. The correlation matrix has been derived from it by integration; it is shown in table 1.

The piping analysis, carried out according to par. 3, leads to the CPDM of the piping response. In this application four sections, as indicated in fig. 1b, have been accounted for as critical elements, each one with two components of bending moments.

At the end system reliability analysis has been worked out according to par.4.The critical sections have been assigned of a strength equal to the maximum values previously carried out. The circular strength domain of each section have been approximated by an octagonal domain, tangent in the design points. The mean rate of outcoming of the domain is included between 0.0323 and 0.0329. Since the input process duration has been assumed D=10 sec, the upper bound $P_f \leq 0.329$ of the failure probability can be derived. This value accounts for the presence of more critical sections. As a comparison notice that a single of them offers the failure probability $P_f \leq 0.123$. The values of P_f, of course, depend from the design level adopted and are conditioned to the seismic event taken as input of the analysis.

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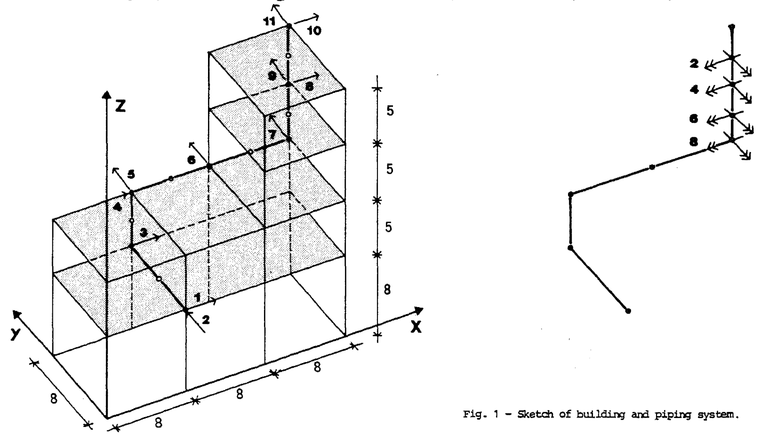

Fig. 1 - Sketch of building and piping system.

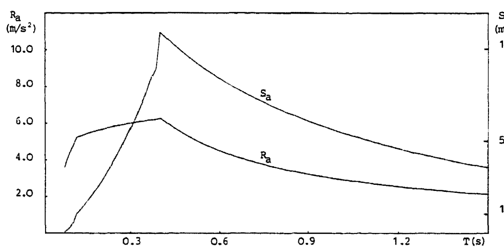

Fig. 2 - USNRC Response Spectrum (for p.g.a. A_{max} = 2g) and derived Power Spectral Density.

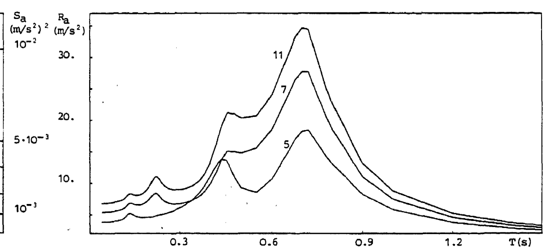

Fig. 3 - Local response spectra for some components of piping excitation (exceed. prob. = 0.85).

1.00	-.011	.937	.916	-.009	.075	.124	.872	.120	.823	.115
	1.00	.011	.007	.953	.835	.636	.007	.607	.007	.589
		1.00	.976	.009	-.075	-.124	.926	-.120	.171	-.115
			1.00	.005	-.076	-.122	.974	-.121	.925	-.118
				1.00	.882	.677	.006	.659	.007	.634
					1.00	.944	-.066	.926	.058	.895
						1.00	-.108	.984	-.096	.953
							1.00	-.106	.986	-.104
								1.00	-.094	.991
									1.00	-.091
										1.00

Table 1: Correlation matrix for excitation of piping supports.

6.35	-.074	.952	-.084	-5.33	.563	-.377	-.447
	125.	-.168	-16.9	.166	16.3	.090	-23.9
		2.10	-.198	-.938	1.09	-1.09	-.819
			8.30	.299	-9.26	.089	9.91
				20.87	-.004	-2.40	-.128
					13.0	-.715	-12.7
						1.09	.504
							26.2
8.87	36.60	4.60	10.1	16.0	12.6	3.40	17.2

Table 2: Covariance matrix ($(Kg\cdot m)^2$) and maximum values ($Kg\cdot m$) for bending moment in critical sections.