

## ANALYSES OF EARTHQUAKE DAMAGE OF BRICK SMOKESTACKS

Yuxian Ru (I)  
Yong Ru (II)  
Xi-Weng Chen (III)  
Presenting Author: Yuxian Ru

### SUMMARY

Damage pattern of unreinforced brick smokestacks from past earthquakes and numerical analysis are studied with special attention to the relative importance of horizontal and vertical vibrations. Maximum tensile stresses due to bending from horizontal vibration and tension from vertical vibration are computed and, after dividing by the allowable tensile stress, are used as crack index. It is concluded that horizontal vibration of the stack is generally the main cause of damage, while vertical vibration may make equal contribution under special occasions.

### EARTHQUAKE DAMAGE

Within last 20 years, China has been attacked many times by strong earthquakes, such as 1965 Drumqi, 1966 Xingtai, 1970 Tonghai, 1975 Haicheng and 1976 Tangshan, and experience filed from damages of smokestacks. On the basis of these earthquakes and many others in the world, features of damage of unreinforced brick smokestacks may be summarized as follows. (1) There are different possible patterns of failure, such as: long segment or segments falling to one side or debris falling to all sides of the stack; top fallen or remained in twisted or inclined position; cracked horizontally, diagonally or randomly. (2) Stack failure occurred not only near but also far away from epicenters. Top portions usually fell for stacks near epicenters of very strong earthquakes but only cracked, mostly horizontally, and remained on top for distant earthquakes. (3) Several horizontal cracks, sometimes closely spaced, occurred more often in the top half, especially for tall stacks. (4) Based upon observations of a few eye-witnesses, earthquake vibration of stacks was three-dimensional whipping with occasional opening of the cracks and dancing of the top portion.

Although most of the above-mentioned phenomena are widely accepted, their interpretations may be so different as to lead to contrary conclusions. The phenomena of twisting of the top portion, several horizontal cracks, closely spaced near top, and randomly scattered debris have been mentioned (Ref. †) as evidence of domination of vertical vibration failure over horizontal, irrespective of epicentral distance and foundation soils, because, as these investigators believe, no horizontal vibration

- (I) Professor, Institute of Engineering Mechanics, Academia Sinica, Harbin, China
- (II) Former Graduate Student, (III) Research Associate, Institute of Engineering Mechanics, Academia Sinica, Harbin, China

could produce these patterns of failure; but many others (Ref. 2,3,4) do not consider them as such evidence because either horizontal or vertical vibration or their combination is capable of producing these patterns, and the relative importance of horizontal and vertical vibrations on stack response has to be studied otherwise.

#### A POSSIBLE EXPLANATION OF SEVERAL HORIZONTAL CRACKS

It is assumed by some investigators that a crack-isolated top portion of a stack has to fall under horizontal vibration (Ref. 1). This assumption is a wrong extension of static response to dynamic. Studies on rocking and overturning of rigid blocks during earthquakes (Ref. 5) suggest that the rocking and falling of top of stacks is a complicated nonlinear dynamic problem. A dynamic analysis is given below to show the mechanism of the formation of a second horizontal crack, sometimes close to the first, with the top portion standing.

In the analysis, the stack is considered as a beam, subjected to ground motion in one horizontal direction only, and divided by the first crack into two portions. The portion above the crack is considered a rigid body and the portion below the crack another beam, coupled at the crack with the rigid top.

The governing equation of the rigid top are (Ref. 5)

$$\begin{aligned} m\ddot{x} &= T_x \\ m\ddot{y} &= T_y - mg \\ I_c\ddot{\theta} &= -T_x R \cos(\theta_c - \theta) - T_y R \sin(\theta_c - \theta) \end{aligned}$$

where  $m$  is the mass and  $I_c$  is the moment of inertia about center of gravity  $C$  (Fig. 1).

An important assumption introduced then for crack formation is that the crack penetrates instantly thw whole section once the maximum vertical tensile stress  $p$  reaches the allowable  $p_a$  at one edge, and two portions are in contact only at the other edge ( $O$  or  $O'$  in Fig. 1). Under this assumption, following relations may be obtained for accelerations  $a_x$  and  $a_y$  of the point of contact

$$\begin{aligned} a_x &= \ddot{x} - \ddot{\theta} R \cos(\theta_c - \theta) \\ a_y &= 0 = \ddot{y} - \ddot{\theta} R \sin(\theta_c - \theta) \end{aligned}$$

The final equation of motion of the rigid top is then

$$(I_c + mR^2) \ddot{\theta} = -mR[a_x \cos(\theta_c - |\theta|) + g \text{sign}(\theta) \sin(\theta_c - |\theta|)] \quad (1)$$

and reactions at point of contact are

$$\begin{aligned} T_x &= m[a_x + \ddot{\theta} R \cos(\theta_c - |\theta|)] \\ T_y &= mg + m \ddot{\theta} R \text{sign}(\theta) \sin(\theta_c - |\theta|) \end{aligned} \quad (2)$$

where  $\text{sign}(\theta) = 1$  when  $\theta$  is positive,  $0$  when  $\theta$  is  $0$  and  $-1$  when  $\theta$  is negative.

The governing equation of the lower beam may be written in matrix form as follows

$$\underline{M}_1 \ddot{\underline{x}} + \underline{C}_1 \dot{\underline{x}} + \underline{K}_1 \underline{x} = -\underline{M}_1 \underline{I}_a \underline{g} - \underline{T}_x \underline{I}_o + \underline{T}_n \underline{K}_n$$

Where all matrices are referred to the lower beam,  $\underline{M}_1 = \text{diag}(m_1, m_2, \dots, m_n)$ ,

$$T_m = \text{sign}(\theta) T_v D/2 = \text{sign}(\theta) \cdot mgD/2 + \frac{1}{2} DRm\ddot{\theta} \cdot \sin(\theta_c - |\theta|) \quad (3)$$

is the moment due to the vertical force  $T_v$  at point of contact and  $n$  is the number of elements of the lower beam. By noting  $a_x = a_g + \ddot{x}_1$  and eliminating  $T_x$ , the final governing equation of the lower beam is

$$\underline{M} \ddot{\underline{x}} + \underline{C} \dot{\underline{x}} + \underline{K} \underline{x} = - \underline{M} \underline{I} a_g - mR\underline{I}_0 \cdot \ddot{\theta} \cdot \sin(\theta_c - |\theta|) + T_m \underline{K}_n \quad (4)$$

where  $\underline{I} = [1, 1, \dots, 1]^T$  of rank  $n$  is a unit vector,  $\underline{I}_0 = [1, 0, 0, \dots, 0]^T$ ,  $\underline{K}_n$  is a column vector representing the influence of moment  $T_m$  on the lower beam, which transfers  $T_m$  into a set of horizontal forces giving same beam deflection, and  $\underline{M} = \text{diag}(m+m_1, m_2, \dots, m_n)$ .

The motion of the cracked stack is governed by Eqs. (1), (3) and (4). The initial conditions directly after the formation of the first crack are assumed as follows. For the rigid top, the angular rotation  $\theta$  and velocity  $\dot{\theta}$  are taken as

$$\begin{aligned} \theta_0 &= 2 \sum (x_i - x_0) / (n_0^2 \cdot H) \\ \dot{\theta}_0 &= 2 \sum (\dot{x}_i - \dot{x}_0) / (n_0^2 \cdot H) \end{aligned}$$

where  $x_i$  and  $\dot{x}_i$  are respectively the linear displacement and velocity of section  $i$  of the top portion before crack, with  $i=0$  for the cracking section,  $n_0$  is the number of elements and  $H$  the height of the element.

At the instant  $t_c$  of formation of crack, because the bending moment  $M_0$  at the cracking section just before crack changes instantaneously to  $T_m$  after crack according to the assumption of instant crack of the whole section, an incremental moment ( $T_m - M_0$ ) is suddenly applied on the cracked section of both the top and the lower portions and incremental accelerations are obtained and added to the corresponding values at step of cracking.

The simultaneous equations (1) and (4), with Eq. (3) for  $T_m$  are solved by iteration for unknowns  $\theta$  and  $\underline{x}$ . Stacks are divided into 30 elements of equal length and are of 2% modal damping. A coefficient of restitution of the rigid top is taken as 0.925. 1971 Pacoima and other recorded motions are used as input.

Fig. 2 shows the time histories of two possible cases after first crack, while top portion remains standing, one with a second horizontal crack and one without.

Several points can be summarized from our numerical results as follows. (1) First crack occurs usually in the top half for tall stacks. (2) If the ground motion is not very strong, top portion after crack may remain on top, with long-period motion equal to rigid-body period. (3) If the ground motion is strong enough, a second crack may appear, usually very close to the first and directly after, largely due to the static effect of the eccentric moment  $T_m$  and the horizontal force  $T_x$  at the first crack from the rigid top.

Experimental study on smokestack models (Ref. 4) proves that several horizontal cracks may occur during one or a few horizontal impacts applied

at base of the stacks, which supports the foregoing analysis.

#### EARTHQUAKE RESPONSE OF SMOKESTACKS

Of the three important elements of ground motion, maximum acceleration and spectral shape may be different for horizontal and vertical components. Fig. 3 compares maximum horizontal and vertical accelerations  $a_h$  and  $a_v$ . It reveals a clear trend of increasing ratio  $a_v/a_h$  (a maximum average of 1.0) at higher accelerations ( $a_h > 0.5g$ ) and closer distances ( $< 10$  km). For smaller acceleration or greater distance, this ratio is approximately  $1/2$ - $2/3$ .

If predominate period  $T_0$  is used as a simple index, the spectral shapes of horizontal and vertical ground motions are not much different and the following values may be accepted as a reasonable average:  $T_0^h = 0.18$  sec. (I), 0.33(II), 0.75(III) and  $T_0^v = 0.15$ (I), 0.30(II), 0.57(III), with I,II,III for rock,hard and soft sites respectively.

Table 1 gives the relevant data of the stacks analyzed for four input motions named in Fig. 4, but with maximum accelerations changed to  $a_h = 0.2g$  and  $a_v = 0.1g$  in Case A and  $a_h = a_v = 0.2g$  in Case B. By comparing the natural periods  $T_i$  of stacks given in Table 1 and typical values of the predominate periods  $T_0$  of ground motions, it is seen that, for sites I and II,  $T_0^h$  is close\* to the second and third periods  $T_2^h$  and  $T_3^h$  of stacks in horizontal direction, and  $T_0^v$  is close to the first period  $T_1^v$  of stacks in vertical.

Fig. 4(a) gives the distribution of maximum tensile stress ratio  $r = p/p_a$  along the height of stack for inputs of Case A. The maximum ratio in the top half is usually smaller than that at bottom but reaches critical value 1.0 earlier for stacks of height  $H$  equal to or greater than 36 m.

In comparison of the relative importance of horizontal and vertical vibrations, it is clear from Fig. 4(a) that the maximum stress ratio  $r$  for horizontal vibration is about 2-11 times that of the vertical, i.e. the horizontal vibration is more harmful.

In order to cover conditions dangerous for vertical vibration, input motions are modified by changing the time scale to make the predominate period  $T_0^v$  equal to  $T_1^v$  given in Table 1 and, at the same time,  $T_0^h = T_2^h$  or  $T_3^h$ . In Fig. 4, the subscript  $i$  of the ratio  $r_i$  means the case of  $T_0$  modified to  $T_i$  while  $i=0$  means no modification of the time scale. Fig. 4(b) gives a set of results for the modified El Centro input. Since the increase in stress ratio in the vertical vibration is not much, horizontal vibration is still more dangerous.

Table 2 gives the results of Case B inputs of modified spectra of  $T_0^v = T_1^v$  and  $T_0^h = T_2^h$ . It is the case exaggerating the vertical vibration to the most possible extreme; the results show again that the horizontal vibration is the main cause of failure for three inputs, with only El Centro input being of equal importance.

\* Since acceleration spectra usually show a flat maximum portion at high frequency band, it is considered here that  $T_0$  is close to  $T_i$  if  $T_0$  is roughly equal to or somewhat greater than  $T_i$ .

## CONCLUSIONS

In summarizing all cases studied by authors, the following results are obtained; (1) responses of smokestacks are dominated by those modes closer to the predominate period of the input motion, fundamental mode in vertical and 2nd or 3rd mode in horizontal vibration; critical sections are mostly in top half or from 0.5H to 0.9H for tall stacks; (2) under average condition of  $a_v = \frac{1}{2}a_h$ , horizontal vibration is the main cause of smokestack failure, while vertical vibration makes significant contributions only at top portion; (3) even for special case of  $a_v = a_h$  with true time scale, horizontal vibration is still the main cause of failure in general; (4) only under rare cases of  $a_v = a_h$  with modified time scale, vertical vibration may contribute as much as the horizontal, which occur only within epicentral areas of very strong earthquakes where peak acceleration is greater than 0.5g.

All the above mentioned results are obtained under the assumption of ground motion input in one horizontal direction only. If vibrations in two perpendicular horizontal directions are simultaneously considered as happened in real cases, it is then safe to say that the horizontal vibration is the main cause of failure of unreinforced brick smokestacks in general.

## REFERENCES

1. Qian, P.F., On Vertical Seismic Load, Earthquake Engineering and Engineering Vibration, v.3, n.2, 1983 (in Chinese).
2. Hu, Y. and Hu, Y., On the Earthquake Response of Smokestacks, ditto.
3. Liao, Z.P. and Wang, F., Vertical Seismic Motion Input to Structures, ditto.
4. Zhou, S., Ding, S.W. and Wang, Y.J., Experimental Study on the Crack Location in the Chimneys Subjected to Shock Vibration, ditto.
5. Yim, C.S., Chopra, A.K. and Penzien, J., Rocking Response of Rigid Blocks to Earthquakes, Earthqu. Eng. Struct. Dyn., v.8, n.6, 1980.

Table 1 Natural Periods(sec)  
of Smokestacks

H(m)	$T_1^h$	$T_2^h$	$T_3^h$	$T_1^v$
20	0.85	0.17	0.07	0.07
24	1.01	0.21	0.08	0.08
30	1.30	0.28	0.11	0.10
36	1.52	0.33	0.13	0.12
40	1.66	0.38	0.15	0.13
45	1.69	0.42	0.16	0.15

Table 2 Max. Ratio r (Case B)

No. Input	Ratio	H=40	30	20 m
1 El Centro	hor. $r_3$	1.15	1.03	0.72
	vert. $r_1$	1.17	0.94	0.76
2 Taft	hor. $r_3$	1.55	1.38	0.90
	vert. $r_1$	0.34	0.32	0.28
3 Pacoima	hor. $r_3$	1.00	0.86	0.58
	vert. $r_1$	0.39	0.31	0.24
4 Qian-an	hor. $r_3$	1.22	1.24	0.99
	vert. $r_1$	0.48	0.41	0.31

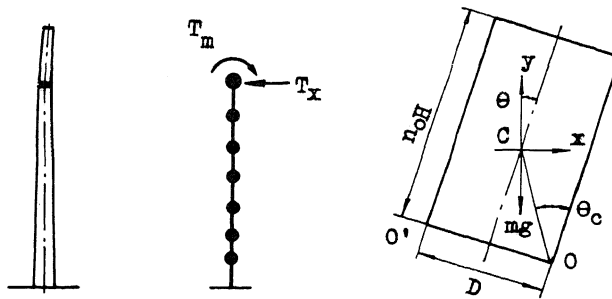


Fig.1 Analytical Sketch of Stack after Crack

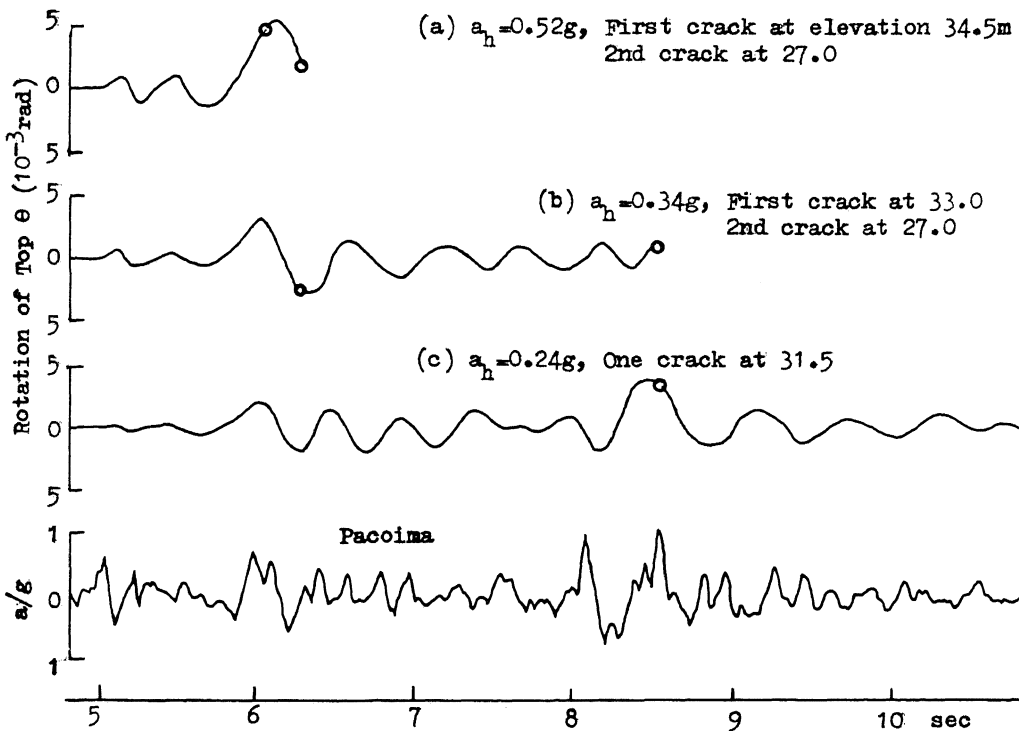


Fig.2 Two Possible Cases of Top Motion without Overturning ( $H=45m$ )  
(a), (b) with Second Crack  
(c) with No Second Crack

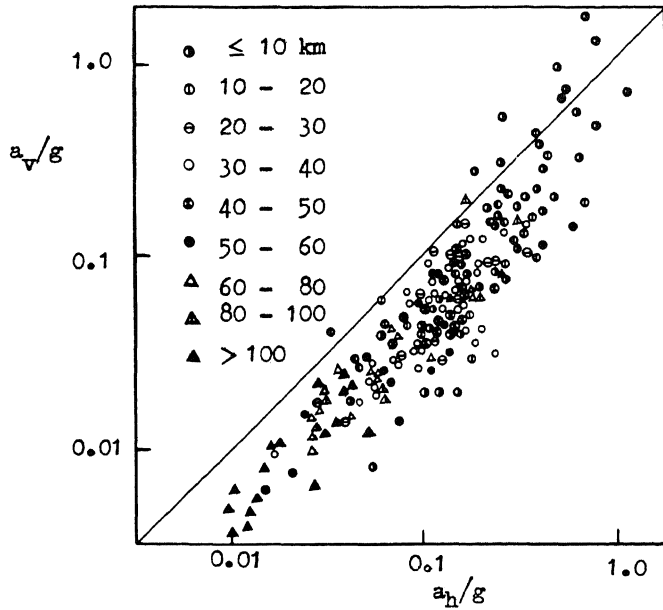


Fig.3 Comparison of Vertical and Horizontal Peak Accelerations

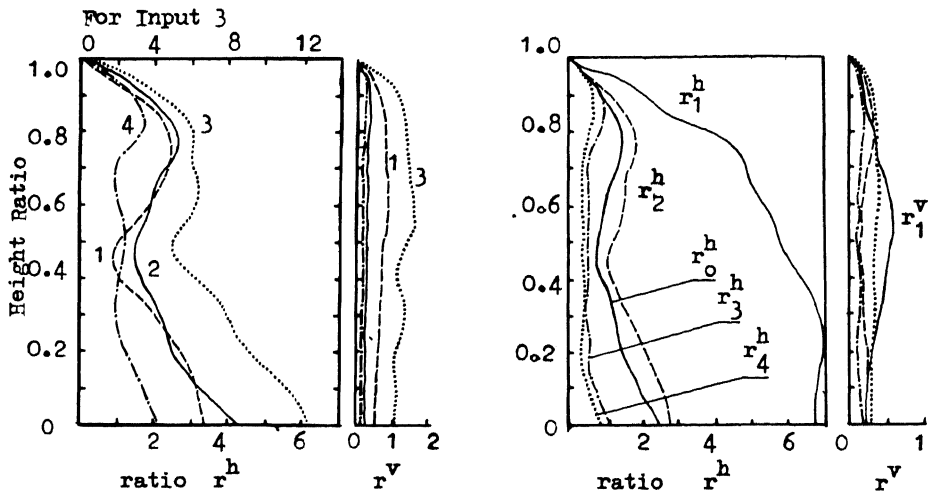


Fig.4 Max. Stress Ratio  $r$  Distributions, Case A,  $H=40$

(a) Four Inputs

- 1 — El Centro
- 2 — Taft
- 3 — Pacoima
- 4 — Qian-an

(b) El Centro

