IDENTIFICATION OF A NONLINEAR BUILDING MODEL FROM RESPONSE MEASUREMENTS UNDER EARTHQUAKE EXCITATION

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SUMMARY

A simple and efficient nonparametric identification technique is applied to a multistory building under seismic excitation. Using the limited amount of available excitation and response measurements, a reduced-order nonlinear nonparametric model is constructed. This model is subsequently used to generate estimates of unavailable response measures which allows the analysis of the building as a chain-like system that is amenable to treatment by an even simpler identification technique. It is shown that the building underwent significant nonlinear nonconservative deformations, particularly in the zones involving the interstory motions between the ground and first story, and between the second and third story.

INTRODUCTION

North Hall is a three-story office building constructed in 1960 and located on the campus of the University of California, Santa Barbara (UCSB). Following its rehabilitation in 1975, the building was fully instrumented by strong motion seismic recorders. Thus, the occurrence of the 13 August 1978 Santa Barbara earthquake generated a large data base of strong motion measurements from the installed instruments (see Fig. 1).

This paper is concerned with the development of an appropriate not-necessarily linear building model by the use of the available response measurements of the building under earthquake ground motion. Particular attention is devoted to determining the nature, extent and location of the nonlinear building characteristics.

FORMULATION

Introduction

Consider a discrete nonlinear dynamic system whose motion is governed by

$$M\ddot{\underset{\sim}{x}} + \underset{\sim}{f}(\underset{\sim}{x},\dot{\underset{\sim}{x}}) = \underset{\sim}{p}(t) \quad , \tag{1}$$

where M = diagonal mass matrix of order n, $\underset{\sim}{x}(t)$ = displacement vector = $\{x_1,x_2,\ldots,x_n\}^T$, $\underset{\sim}{f}$ = function that represents nonconservative nonlinear

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forces, and $p(t)$ = excitation vector. Assume that an "equivalent" stiffness matrix K corresponding to the range of motion of interest can be determined.

Restoring Force Estimation

Now solving the eigenvalue problem associated with the linearized version of Eq. (1) results in the transformation

$$\tilde{x} = \phi\, \tilde{u} \tag{2}$$

where ϕ is the eigenvector matrix and $\tilde{u}$ is the vector of generalized coordinates.

Making use of Eq. (2), the system equation of motion Eq. (1) can be converted to the form

$$\tilde{h}(\tilde{u},\dot{\tilde{u}}) = \phi^T \tilde{f}(\tilde{x},\dot{\tilde{x}}) = \phi^T(\tilde{p}(t) - M\,\ddot{\tilde{x}}) \;, \tag{3}$$

where $\tilde{h}$ is a vector corresponding to the transformed nonlinear forces acting on the system. Note from Eq. (3) that if the terms appearing on the right-hand-side (RHS) are known, the time history of each component of vector $\tilde{h}$ can be determined.

Note also that in the case of a linear system, due to the orthogonality condition associated with ϕ, the set of equations represented by Eq. (3) are decoupled; i.e., each component h_i of h depends only on the i^{th} generalized coordinate u_i rather than on all components of $\tilde{u}$.

Guided by the preceding observation, the central idea of the present method is that in the case of nonlinear dynamic systems commonly encountered in the structural engineering field, a judicious assumption is that each component of $\tilde{h}$ can be expressed in terms of a series of the form:

$$h_i(\tilde{u},\dot{\tilde{u}}) \approx \hat{h}_i(\tilde{u},\dot{\tilde{u}}) = \sum_{j=1}^{J_{max_i}} \hat{h}_i^{(j)}\,(v_{1_i}^{(j)},\, v_{2_i}^{(j)}) \;. \tag{4}$$

The approximation indicated in Eq. (4) is that each component h_i of the nonlinear generalized restoring force h can be adequately estimated by a collection of terms $\hat{h}_i^{(j)}$ each one of which involves a pair of generalized coordinates (displacements and/or velocities). The particular choice of combinations and permutations of u_k and $\dot{u}_\ell$ and the number of terms J_{max_i} needed for a given h_i depend on the nature and extent of the nonlinearity of the system and its effects on the specific "mode" i. Note that the formulation in Eq. (4) allows for "modal" interaction between all modal displacements and velocities, taken two at a time.

Series Expansion

The individual terms appearing in the series expansion of Eq. (4) may be evaluated by using the least-squares approach to determine the optimum fit for the time history of each h_i. Thus, $\hat{h}_i^{(1)}$ may be expressed as a double series involving a suitable choice of basis functions,

$$h_i(\underset{\sim}{u},\underset{\sim}{\dot{u}}) \approx \hat{h}_i^{(1)}(v_{1_i}^{(1)}, v_{2_i}^{(1)}) = \sum_k \sum_\ell {}^{(1)}C_{k\ell}^{(i)} T_k(v_{1_i}^{(1)}) T_\ell(v_{2_i}^{(1)}). \qquad (5)$$

Eq. (4) is obtained by fitting the residual error with a similar double series involving other pairs of generalized coordinates that have significant interaction with "mode" i.

Least-Squares Fit

Using two-dimensional orthogonal polynomials to estimate each $h_i(\underset{\sim}{u},\underset{\sim}{\dot{u}})$ by a series of approximating functions $\hat{h}_i^{(j)}$ of the form indicated in Eq. (5), then the numerical value of the $C_{k\ell}$ coefficients can be determined by invoking the applicable orthogonality conditions for the chosen polynomials. While there is a wide choice of suitable basis functions for least-squares application, the orthogonal nature of the Chebyshev polynomials and their "equal ripple" characteristics make them convenient to use in the present work.

Note that in the special case when no cross-product terms are involved in any of the series terms, functions h can be expressed as the sum of two one-dimensional orthogonal polynomial series instead of a single two-dimensional series of the type under discussion.

Special Case of Chain-Like Systems

A major feature of the identification method under discussion is that it is not restricted to any particular structural configuration or class of discrete nonlinear systems. However, in the special case of structures that can be adequately modeled as chain-like systems (e.g., simplified MDOF stick models of buildings) whose adjoining mass points are interconnected by a single nonlinear element, considerable simplification is obtained in the identification procedure since these structures have the property that the nonlinearities in the various links of the chain are independent of each other (i.e., are dependent only on relative motions between masses). Thus, with a suitable transformation of variables, the task of identifying an n DOF system reduces to that of identifying n separate SDOF systems.

Consider a MDOF chain-like structure consisting of n lumped masses each of magnitude m_i. The structure may be subjected to base excitation S(t) and/or directly applied forces $F_i(t)$. The absolute displacement of m_i is measured by $x_i(t)$; the corresponding relative displacement with respect to the moving support is given by $y_i(t) = x_i(t) - S(t)$; and the interstory relative motion is specified by $z_i(t) = x_i(t) - x_{i-1}(t)$ for $i > 1$, and $z_1(t) = x_1(t) - S(t)$. The arbitrary nonlinear elements interposed between the masses are represented by functions G_i which are dependent on the relative displacement and velocity across the terminals of each element.

It follows directly from the equations of motion for such systems that the nonlinear restoring functions can be expressed as

$$G_n(z_n,\dot{z}_n) = F_n(t) - m_n\,\ddot{x}_n \;,$$

$$G_i(z_i,\dot{z}_i) = F_i(t) - m_i\,\ddot{x}_i + G_{i+1}(z_{i+1},\dot{z}_{i+1});\; i = 1,2\ldots,\; n-1 \;. \tag{6}$$

Then, using the procedure outlined above for the general nonlinear MDOF system, each of the "real" restoring forces G_i is estimated by an approximating function $\hat{G}_i$ expressed in terms of two-dimensional orthogonal polynomials involving the associated state variables z_i and $\dot{z}_i$:

$$G_i(z_i,\dot{z}_i) \approx \hat{G}_i(z_i,\dot{z}_i) = \sum_k \sum_\ell C_{k\ell}^{(i)}\; T_k(z_i')T_\ell(\dot{z}_i') \;. \tag{7}$$

APPLICATION

Processing of Measured Data

Analog measurements of the earthquake ground motion and the building response were processed using modern time series analysis techniques. The digitized measurements consisted of the time histories of the acceleration, velocity, and displacement at different locations and directions (Ref. 2).

For the purposes of this study, the building motion in the North-South direction was used. Figure 2 indicates the simplified 3-degrees-of-freedom building model used as well as the corresponding recorded motions. Notice that the motion of m_1 (the second floor) was not measured during the earthquake.

Linearized Structure Properties

A linearized analysis of the building was performed using the ETABS finite element code (Ref. 3). The structure was modeled as having 24 frames and three stories. A summary of the mass distribution and the dominant mode shapes corresponding to the DOF's under discussion are given in Fig. 3.

Nonparametric Identification of North Hall

Using the data in Fig. 3 and the general procedure outlined above (i.e., without invoking the chain-like characteristics of the simplified model), results in the time history of the dominant modal parameters. Plots of $h_1(t)$ and $\hat{h}_1(t)$ are shown in Fig. 4. A three-dimensional representation of the surfaces associated with the measured h_1 and approximating $\hat{h}_1$ are shown in Figs. 5(a) and 5(b) respectively.

Following the procedure in Eq. (5), the identification results shown in Fig. 4(b) are obtained. The relatively good agreement between the measured and estimated time history of the modal parameters shown in Fig. 4 is typical.

By incrementally solving the reduced-order system equations and using the transformation in Eq. (2), the approximate response time histories of all of the building's 3 degrees of freedom were computed (estimated) and are compared to the available measurements in Fig. 6. The computed results shown in Fig. 6 are based on using only the contribution of the first "mode" h_1.

Note from Fig. 1 that no measurements are available for the motion of the first story m_1; thus the computed response $\hat{y}_1(t)$, $\hat{\dot{y}}_1(t)$ and $\hat{\ddot{x}}_1(t)$ are predicted measurements that can be used to estimate the interstory motions, some of which are shown in Fig. 7.

Chain-Like Model of North-Hall

Following the procedure in Eq. (7), North-Hall is treated as a chain-like system and its interstory restoring forces G_i are determined and plotted versus their corresponding state variables in Figs. 8 and 9 from which it is clear that significant nonlinear behavior is present.

The identified coefficients of Eq. (7) corresponding to G_1, G_2 and G_3 are shown in Fig. 10. The adequacy of the identification results is illustrated in Fig. 10 where the time histories of the measured (estimated) $G_i(t)$ are compared to the computed (predicted) $\hat{G}_i(t)$ over the time span corresponding to the strong-motion part of the Santa Barbara earthquake.

The use of additional translational and rotational DOF are expected to yield even better correlation between measured and predicted response based on simplified reduced-order nonliner models.

AKNOWLEDGEMENT

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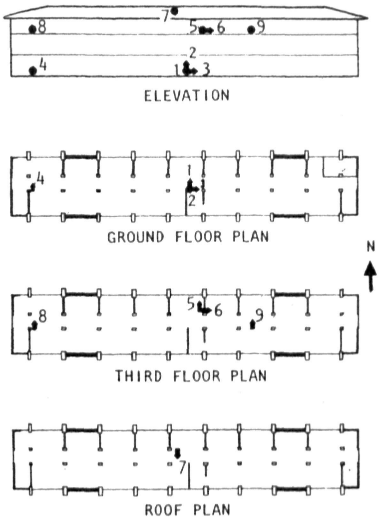


FIGURE 1. LOCATION OF STRONG MOTION INSTRUMENTATION (Ref. 1)

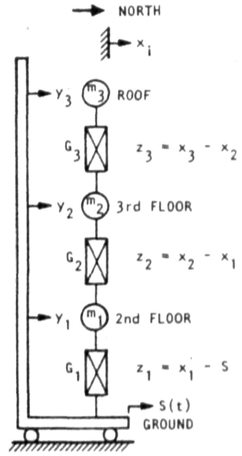


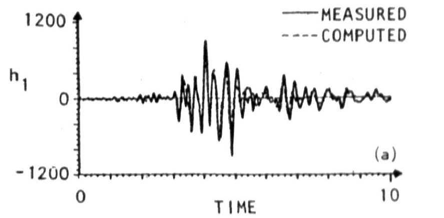
FIGURE 2. SIMPLIFIED MODEL OF NORTH HALL

Masses:	Frequencies:
$m_1 = 3.9086$	$\omega_1 = 29.32$
$m_2 = 3.763$	$\omega_2 = 137.15$
$m_3 = 2.887$	$\omega_3 = 311.68$

Mode Shapes:

	$\phi^{(1)}$	$\phi^{(2)}$	$\phi^{(3)}$
m_1	1.0	1.0	1.0
m_2	3.08	1.0	-0.79
m_3	5.58	-0.96	0.33

FIGURE 3. DYNAMIC CHARACTERISTICS OF LINEARIZED MODEL OF NORTH HALL



Identification Parameters: $\hat{h}_1(u_1, \dot{u}_1)$

Motion Range: $u_{min} = -1.66$ $\quad u_{max} = 1.67$

$\dot{u}_{min} = -31.57$ $\quad \dot{u}_{max} = 31.71$

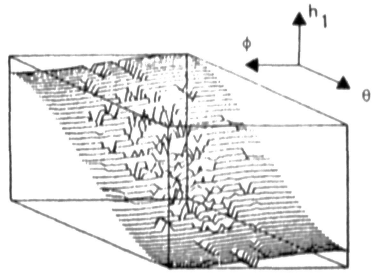
COEFFICIENTS OF 2D CHEBYSHEV SERIES:

$$\hat{h}_i(u_i, \dot{u}_i) = \sum_k \sum_\ell C1^{(i)}_{k\ell} T_k(u_i') T_\ell(\dot{u}_j')$$

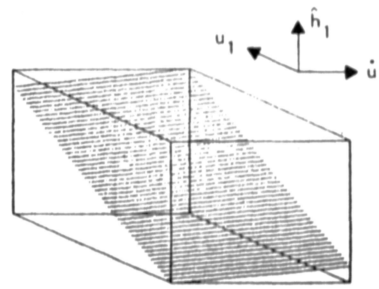
$k \backslash \ell$	$T_0(\dot{u})$	$T_1(\dot{u})$	$T_2(\dot{u})$	$T_3(\dot{u})$
$T_0(u)$	73.47	109.98	41.45	-2.73
$T_1(u)$	860.99	-1.56	9.21	5.49
$T_2(u)$	20.57	7.98	-11.15	10.80
$T_3(u)$	16.74	-3.09	-9.92	6.76

(b)

FIGURE 4. IDENTIFICATION RESULTS FOR h_1



(a) Measured response



(b) Chebyshev approximation

FIGURE 5. IDENTIFICATION RESULTS FOR h_1

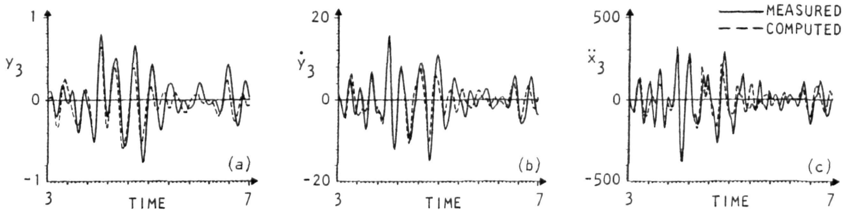


FIGURE 6. RESPONSE TIME HISTORY OF NORTH HALL

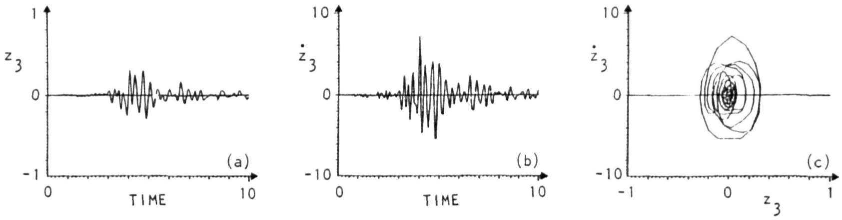
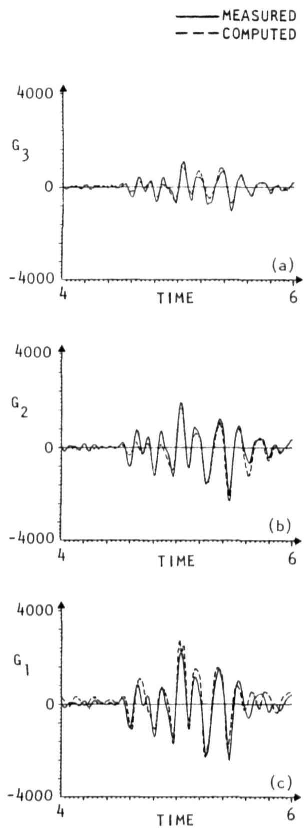


FIGURE 7. INTERSTORY MOTION OF NORTH HALL



Coefficients of 2D Chebyshev Series

$$G(z,\dot{z}) = \sum_{i=0}^{m2} \sum_{j=0}^{n2} C_{ij}\, T_i(z')T_j(\dot{z}')$$

UCSB North Hall Building (G_3)

Motion Range: $z_{min} = -\ 0.287$

$\dot{z}_{min} = -\ 5.307$

$z_{max} = .304$

$\dot{z}_{max} = 7.137$

i \ j	$T_o(\dot{z}')$	$T_1(\dot{z}')$	$T_2(\dot{z}')$	$T_3(\dot{z}')$
$T_o(z')$	162.10	299.50	57.73	-147.60
$T_1(z')$	820.1	-3.38	4.16	13.75
$T_2(z')$	-39.24	-26.78	10.14	-11.04
$T_3(z')$	-111.70	-15.61	-34.05	16.72

(d)

UCSB North Hall Building (G_2)

Motion Range: $z_{min} = -\ 0.419$

$\dot{z}_{min} = -\ 7.886$

$z_{max} = 0.377$

$\dot{z}_{max} = 6.744$

i \ j	$T_o(\dot{z}')$	$T_1(\dot{z}')$	$T_2(\dot{z}')$	$T_3(\dot{z}')$
$T_o(z')$	-282.90	399.60	81.98	175.4
$T_1(z')$	190.30	1.35	-12.02	1.00
$T_2(z')$	12.08	31.08	44.13	39.81
$T_3(z')$	165.7	-9.71	40.43	15.19

(e)

UCSB North Hall Building (G_1)

Motion Range: $z_{min} = -\ 0.106$

$\dot{z}_{min} = -\ 2.079$

$z_{max} = 0.112$

$\dot{z}_{max} = 2.493$

i \ j	$T_o(\dot{z}')$	$T_1(\dot{z}')$	$T_2(\dot{z}')$	$T_3(\dot{z}')$
$T_o(z')$	818.90	257.00	686.80	-34.55
$T_1(z')$	2226.00	17.91	0.67	-20.99
$T_2(z')$	-141.30	-52.38	30.59	12.98
$T_3(z')$	18.97	-10.23	42.16	-11.87

(f)

FIGURE 10. IDENTIFICATION RESULTS FOR CHAIN-LIKE MODEL OF NORTH HALL

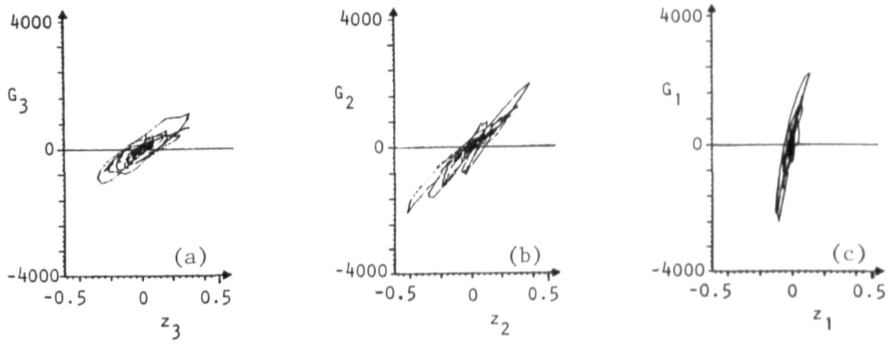


FIGURE 8. MEASURED VARIATION OF INTERSTORY FORCES WITH INTERSTORY DEFLECTIONS

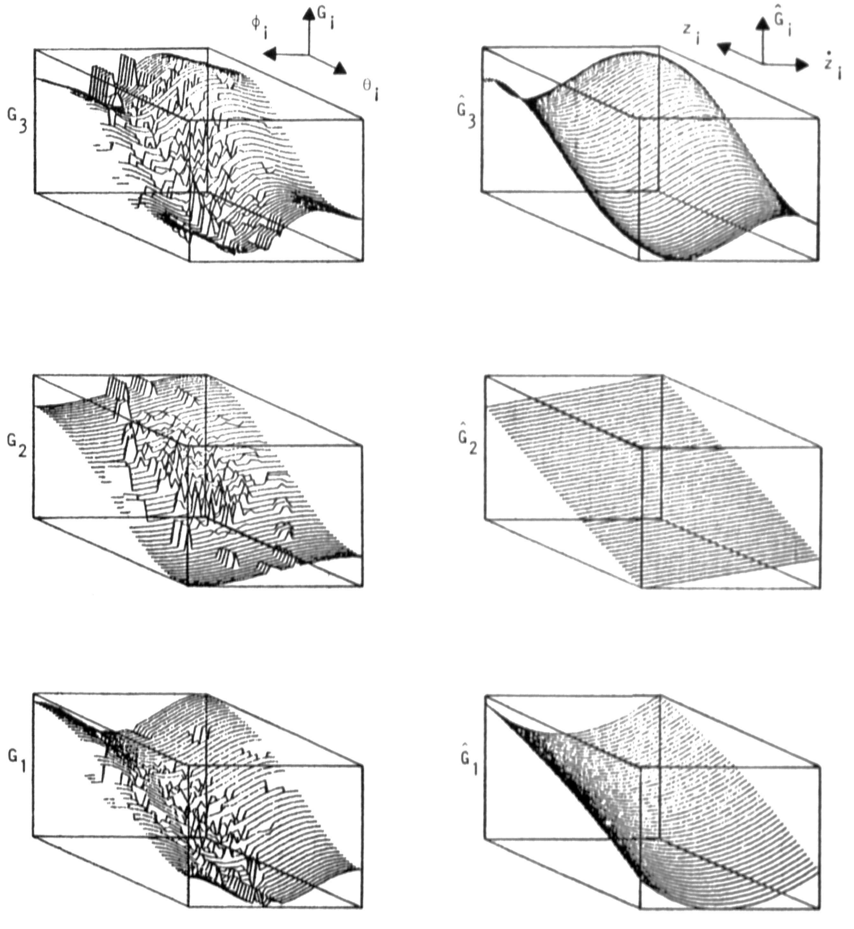


FIGURE 9. MEASURED AND ESTIMATED INTERSTORY FORCES IN CHAIN-LIKE MODEL OF NORTH HALL