STUDY ON THE RESTORING FORCE CHARACTERISTICS OF RC COLUMN TO BI-DIRECTIONAL DEFLECTION HISTORY

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SUMMARY

The effect of the interaction of bi-directional forces on the aseismic performance of reinforced concrete columns was analytically investigated. The analytical method was already published and its application was extended to the circular column section in this study. Restoring force-accumulated plastic deformation relation, hysteretic energy absorption capacity and ductility of section were discussed. As a result, it was proved that there exists a significant difference on the aseismic performance of the reinforced concrete columns between the case that the section is under bi-axial curvature history and the case that the identical section is under uni-axial curvature history.

INTRODUCTION

It is necessary to study the three dimensional behavior of reinforced concrete structures during an actual earthquake in order to design aseismitically such structures. Authors are making experimental and analytical researches of bi-axial restoring force characteristics of reinforced concrete columns. The test results of rectangular section columns (Ref. 1, 2 and 3) and the analytical method (Ref. 4 and 5) were already published. In this study, the circular section columns were tested and their test results were followed by the analysis. It was found that the three dimensional behavior of long columns could be predicted by the analysis. So the fundamental behavior of reinforced concrete columns subjected to bi-directional horizontal forces will be able to be discussed by the analysis of column section. The main purpose of this study is to estimate the effect of interaction of bi-directional forces on the aseismic performance of reinforced concrete columns.

BI-AXIAL TEST OF CIRCULAR SECTION COLUMN

As shown in Fig. 1, specimens were longitudinally reinforced by twelve deformed bars of 6 mm nominal diameter and laterally reinforced by 30 mm pitch spiral hoop of 4 mm round bar in diameter. Concrete strength was 209 kgf/cm^2. Yield stress of longitudinal reinforcement was 4005 kgf/cm^2 and

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that of spiral hoop was 6195 kgf/cm^2 in 0.2% proof stress. Bending moments about two axes were independently applied by eccentric loads, and average curvatures were calculated from the relative end rotation of specimens. As for detail of loading and measuring, the papers by authors could be referred (Ref. 1).

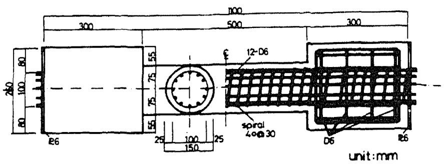

Fig.1 Test specimen of circular section column

ANALYTICAL METHOD OF COLUMN SECTION

Cross sections were partitioned into small elements. It was assumed that each element was uni-axially stressed and that Bernoulli's principle could be applied. In the analysis, uni-axial non-stationary stress-strain hysteresis rules of concrete and reinforcing bar were postulated according to the recent studies on them. As for detail of the analytical method, the papers by authors could be also referred (Ref. 4 and 5).

Experimental and analytical results of the circular section column were shown in Fig.2-(1) through Fig.2-(7). In the experiment, curvatures about two axes were compulsorily applied as shown in Fig.2-(1). The analytical results agree with the experimental results both qualitatively and quantitatively very well. It was found that the analytical method would be applied to the circular column sections. Then authors judged that the three dimensional behavior of reinforced concrete long columns which have sufficient lateral reinforcement could be predicted by this analytical method and that the fundamental behavior of reinforced concrete columns subjected to bi-directional forces will be able to be discussed by the analysis of the column section.

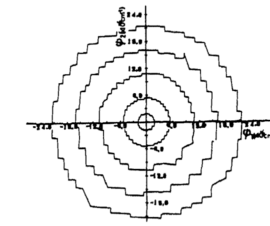

Fig.2-(1) Curvature history

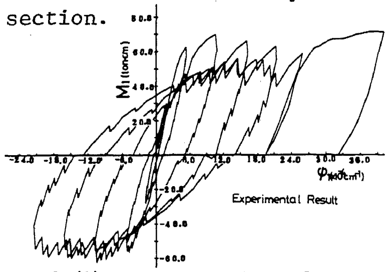

Fig.2-(2) Experimental result of $M_1 - \psi_1$ relation

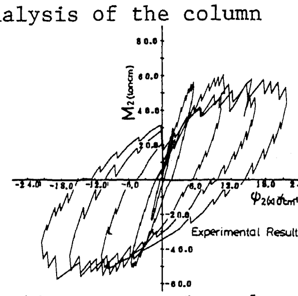

Fig.2-(3) Experimental result of $M_2 - \psi_2$ relation

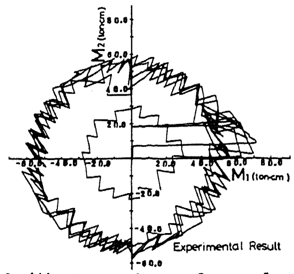

Fig.2-(4) Experimental result of $M_1 - M_2$ relation

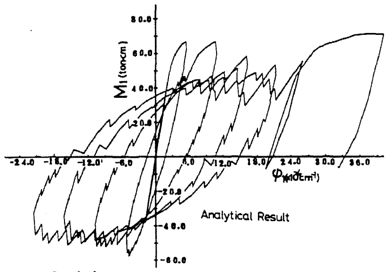

Fig.2-(5) Analytical result of $M_1 - \psi_1$ relation

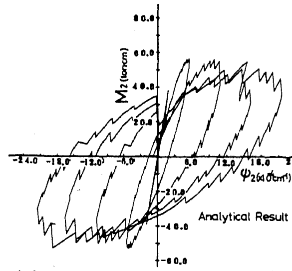

Fig.2-(6) Analytical result of $M_2 - \psi_2$ relation

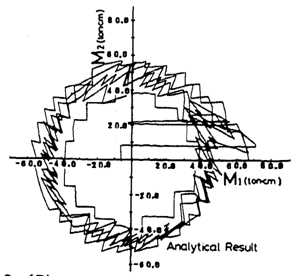

Fig.2-(7) Analytical result of $M_1 - M_2$ relation

PARAMETERS OF COLUMN SECTION

The restoring force characteristics of the columns, which could not be tested because of the difficulties of experimental technics and whose parameters were out of range of experiment, were discussed by the analysis. The analytical model sections were designed by changing the following parameters; concrete strength ($_C\sigma_B$), tensile reinforcement ratio (p_t), yield stress of reinforcement ($_s\sigma_y$) and especially the magnitude of axial force. The combination of the parameters was expressed as follows.

$$q = p_t \cdot {}_s\sigma_y \,/\, {}_C\sigma_B \quad \cdots\cdots\cdots\cdots\cdots \quad (1)$$

Compressive strain of concrete at the maximum strength ($_C\varepsilon_B$) was assumed as follows.

$${}_C\varepsilon_B = 2.2 \times 10^{-4} \times {}_C\sigma_B{}^{0.39} \quad \cdots\cdots\cdots\cdots \quad (2)$$

The analyzed section and its partitioning are shown in Fig.3. The parameters of the analyzed sections are shown in Table 1.

AN IDEAL MODEL FOR NORMALIZATION

Bi-directional and uni-directional deflection histories do not essentially correspond to each other. therefore, it is not so easy that the restoring force characteristics to those deflection histories are directly compared with each other.

A bi-directional restoring force model formulated in this paragraph was used for the normalization of analytical results. An ideal mechanical model was designed as shown in Fig.4, using the analogy to the theory of elasticity and plasticity (Ref. 3, 6, 7 and 8). When the model is subjected to uni-axial curvature history, it has so-called bi-linear restoring force characteristics. Yield locus is assumed to be an ellipse as shown in Fig.4.
In the elastic range, the correlation of bi-directional forces does not exist.

$$\{dM\} = [EI] \cdot \{d\psi\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (3)$$

$\{dM\}$: incremental bending moment vector
$[EI]$: elastic stiffness matrix
$\{d\psi\}$: incremental curvature vector

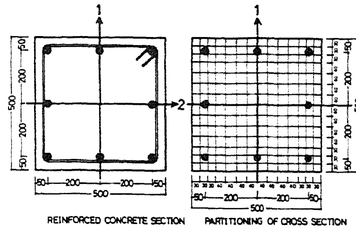

Fig.3 Analyzed Reinforced Concrete Section and its Partitioning

Table 4 Parameters of Analyzed Section

No.	series	$_C\sigma_B$ Kg/cm²	p_t %	$_s\sigma_y$ Kg/cm²	q	N/BD$_C\sigma_B$
1	FC	266	0.96	3600	0.13	0.0
2						0.1
3						0.2
4						0.25
5						0.3
6						0.35
7		230			0.15	0.0
8						0.1
9						0.2
10						0.25
11						0.3
12						0.35
13		203			0.17	0.0
14						0.1
15						0.2
16						0.25
17						0.3
18						0.35
19		180			0.20	0.0
20						0.1
21						0.2
22						0.25
23						0.3
24						0.35
25		138			0.25	0.0
26						0.1
27						0.2
28						0.25
29						0.3
30						0.35
31		115			0.30	0.0
32						0.1
33						0.2
34						0.25
35						0.3
36						0.35
37	PT	180	0.65	3600	0.13	0.0
38						0.1
39						0.2
40						0.25
41						0.3
42						0.35
43			0.75		0.15	0.0
44						0.1
45						0.2
46						0.25
47						0.3
48						0.35
49			0.85		0.17	0.0
50						0.1
51						0.2
52						0.25
53						0.3
54						0.35
55			1.25		0.25	0.0
56						0.1
57						0.2
58						0.25
59						0.3
60						0.35
61			1.50		0.30	0.0
62						0.1
63						0.2
64						0.25
65						0.3
66						0.35
67	SY	180	0.96	2438	0.13	0.0
68						0.1
69						0.2
70						0.25
71						0.3
72						0.35
73				2813	0.15	0.0
74						0.1
75						0.2
76						0.25
77						0.3
78						0.35
79				3188	0.17	0.0
80						0.1
81						0.2
82						0.25
83						0.3
84						0.35
85				4688	0.25	0.0
86						0.1
87						0.2
88						0.25
89						0.3
90						0.35
91				5625	0.30	0.0
92						0.1
93						0.2
94						0.25
95						0.3
96						0.35

From the theory of plastic potential,

$$\{d\psi_p\} = \{\partial F / \partial M\} \cdot d\lambda \quad , \quad d\lambda > 0 \cdots\cdots (4)$$

$\{d\psi_p\}$: plastic incremental curvature vector
F : surface of yield locus
$\{M\}$: bending moment vector

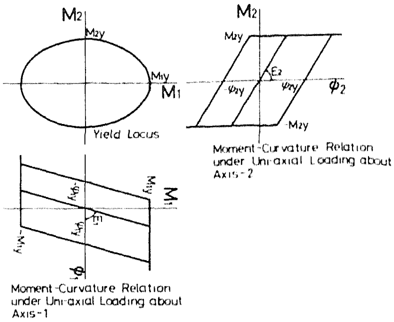

Fig.4 Yield locus and moment-curvature relation under uni-axial loading of normalization model

Assuming that the vector $\{d\psi_p\}$ is orthogonal to the vector $\{dM\}$,

$$\lfloor d\psi_p \rfloor \cdot \{dM\} = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

$$\{d\psi_p\}^T = \lfloor d\psi_p \rfloor$$

The incremental curvature vector $\{d\psi\}$ consists of elastic part $\{d\psi_e\}$ and plastic part $\{d\psi_p\}$.

$$\{d\psi\} = \{d\psi_e\} + \{d\psi_p\} \quad \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

The incremental bending moment was caused by elastic incremental curvature.

$$\{dM\} = [EI] \cdot \{d\psi_e\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (7)$$

Substituting Eq.6 into Eq.7 and that results into Eq.5,

$$\lfloor d\psi_p \rfloor \cdot [EI] \; (\{d\psi\} - \{d\psi_p\} \;) = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

Substituting Eq.4 into Eq.8,

$$d\lambda = \frac{\lfloor \partial F / \partial M \rfloor \cdot [EI] \cdot \{d\psi\}}{\lfloor \partial F / \partial M \rfloor \cdot [EI] \cdot \{\partial F / \partial M\}} \quad \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

When the incremental curvature is given, $d\lambda$ is calculated from Eq.9 and plastic incremental curvature is obtained from Eq.4. The elastic incremental curvature is obtained from Eq.6. As a serult, incremental bending moment is obtained from Eq.7. In this study, the accumulated plastic curvature Φ_p was defined as follows.

$$\begin{aligned} d\Phi_p &= \int |d\psi_p| \\ &= \int \sqrt{(d\psi_{1p})^2 + (d\psi_{2p})^2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (10) \end{aligned}$$

$d\psi_{1p}$, $d\psi_{2p}$: incremental plastic curvatures about axis-1 and axis-2, respectively

CURVATURE HISTORIES

Curvature history is an important factor when uni-axial and bi-axial restoring force characteristics are compared with each other. The restoring force characteristics of columns must be infered from the idealized deflection history (Ref. 2, 9 and 10), because the deflection history in the actual earth-

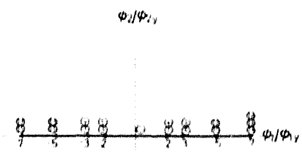

Fig.5-(1) Curvature history CASE-1

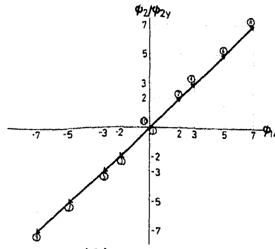

Fig.5-(2) Curvature history CASE-2

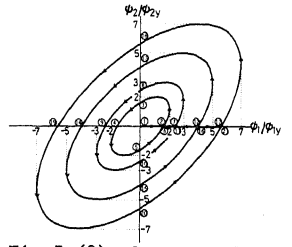

Fig.5-(3) Curvature history CASE-3

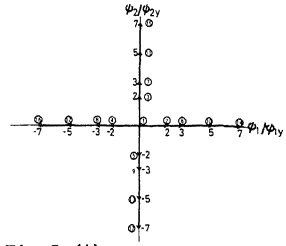

Fig.5-(4) Curvature history CASE-4

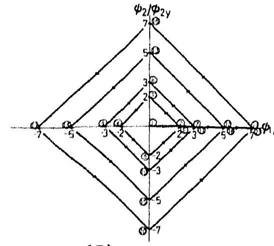

Fig.5-(5) Curvature history CASE-5

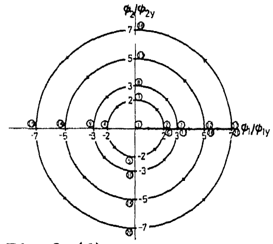

Fig.5-(6) Curvature history CASE-6

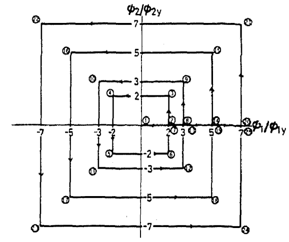

Fig.5-(7) Curvature history CASE-7

quake can not be known in advance. Then, a section shown in Fig.3 was analyzed under various idealized curvature histories as shown in Fig.5-(1) through Fig.5-(7). Its parameters are No.21 in Table 1. The same curvature history was applied to both the section and the mechanical model. Restoring force of the section was normalized by that of the mechanical model. Figure 6-(1) and Fig.6-(2) show the relations between the normalized restoring force f_R and the accumulated plastic curvature of the section $\tilde{\Phi}_p$ when the mechanical model is in plastic range. In Fig.5,

$$f_R = \sqrt{(M_1)^2 + (M_2)^2} \,/\, \sqrt{(M_1{}^*)^2 + (M_2{}^*)^2} \quad \cdots\cdots\cdots\cdots \quad (11)$$

$M_1{}^*$, $M_2{}^*$: moments about axis-1 and axis-2 of the mechanical model, respectively, when the model is inplastic range

$$\tilde{\Phi}_p = \int \sqrt{(d\psi_{1p} \,/\, \psi_{1y})^2 + (d\psi_{2p} \,/\, \psi_{2y})^2} \quad \cdots\cdots\cdots\cdots \quad (12)$$

ψ_{1y} , ψ_{2y}: curvatures at uni-axial bending yield axis-1 and axis-2, respectively

As a result, it was found that the effect of curvature history on the f_R-$\tilde{\Phi}_p$ relation could be ignored when uni-axial and bi-axial curvature history were separated. From the above mentioned fact, two representative uni-axial (case-1) and bi-axial curvature history (case-6) were selected.

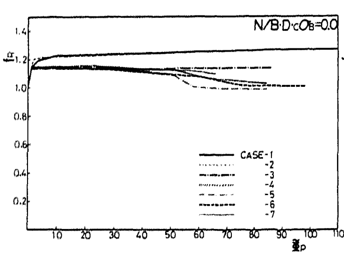

Fig.6-(1) f_R - $\tilde{\Phi}_p$ relation

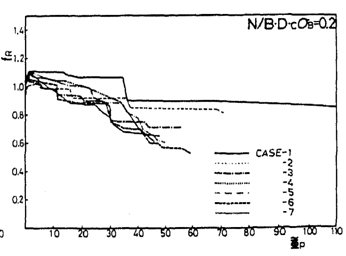

Fig.6-(2) f_R - $\tilde{\Phi}_p$ relation

DIFFERENCE BETWEEN UNI-AXIAL AND BI-AXIAL RESTORING FORCE CHARACTERISTICS

The difference between uni-axial and bi-axial restoring force was examined. Figure 7 shows an example of the relations between the restoring force normalized by the mechanical model (f_R, g_R) and the accumulated plastic curvature ($\tilde{\Phi}_p$), when the identical section is under uni-axial and bi-axial curvature histories.

It was assumed that the difference was expressed by the following equation.

$$\alpha_R = 1 - g_R \,/\, f_R \quad \cdots\cdots\cdots\cdots\cdots \quad (13)$$

f_R , g_R: restoring force ratio calculated from bi-axial and uni-axial restoring force characteristics, respectively

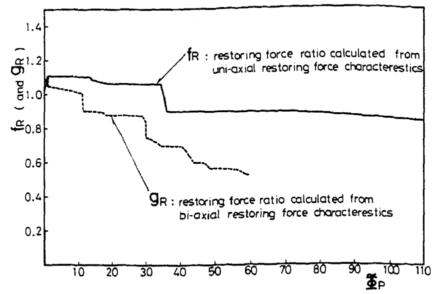

Fig.7 $f_R - \tilde{\Phi}_p$ and $g_R - \tilde{\Phi}_p$ relations

In Fig.13, α_R=0 means that there is no difference. According to the increase of the value of α_R, the difference becomes large. Figure 8 shows the relation between α_R and $\tilde{\Phi}_p$ of the example shown in Fig.7. The straight line in Fig.8 was obtained by regression analysis. The slope of the straight line was considered to be an index of the difference concerning the restoring force when the identical section was subjected to uni-axial and bi-axial bending moments. The relations between the index α and the axial force ratio ($N/B{\cdot}D{\cdot}{}_C\sigma_B$) are shown in Fig.9-(1) through Fig.9-(3).

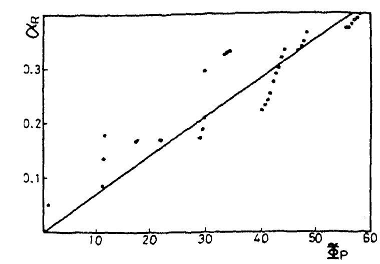

Fig.8 $\alpha_R - \tilde{\Phi}_p$ relation

The difference concerning the restoring force has a tendency to become large according to the increase of axial force ratio and the decrease of parameter q. Especially, the effect of the axial force ratio is great.

The reason of these results would be concerned with the crash of concrete. An example of the relation between the maximum compressive strain of concrete elements (ε_{max}) and the accumulated plastic curvature is shown in Fig.10-(1) and Fig.10-(2). When there is no axial force, the difference of the maximum compressive strain is small between the case that the section is

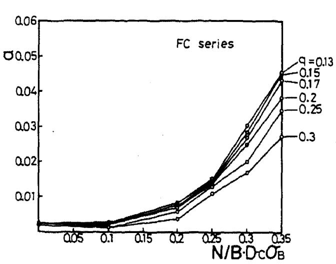

Fig.9-(1) α - $N/B{\cdot}D{\cdot}{}_C\sigma_B$ relations of FC series sections

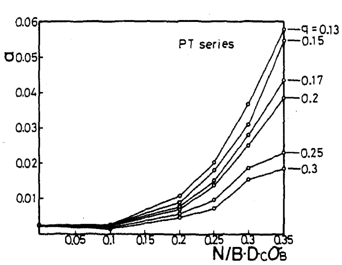

Fig.9-(2) α - $N/B{\cdot}D{\cdot}{}_C\sigma_B$ relation of PT series sections

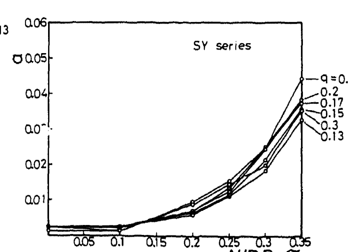

Fig.9-(3) α - $N/B{\cdot}D{\cdot}{}_C\sigma_B$ relations of SY series sections

under uni-axial curvature history and the case that the identical section is under bi-axial curvature history. There is, however, the large difference according to the increase of axial force. The similar investigation was done concerning the hysteretic energy absorption capacity. As a result, it was also found that the magnitude of axial force strongly affected the uni-axial and bi-axial restoring force characteristics.

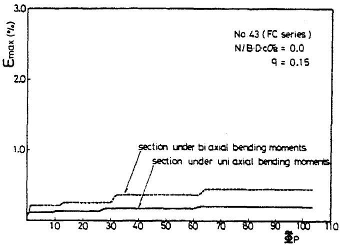

Fig.10-(1) ε_{max} - $\tilde{\Phi}_p$ relations of No.43 section

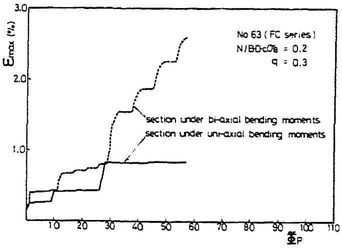

Fig.10-(2) ε_{max} - $\tilde{\Phi}_p$ relations of No.45 section

DUCTILITY OF SECTION AND AXIAL FORCE

Authors judged that the parameters except for axial force could be ignored, because the effect of axial force was especially great. The relation between axial force ratio and the experienced maximum curvature is shown in Fig. 11-(1) through Fig.11-(3) when the restoring force decreased to 90%, 80% and 70% of the restoring force at bending yield, that is, f_R or g_R of Fig.7 is equal to 0.9, 0.8 and 0.7. In Fig.11-(1) through Fig.11-(3), the experienced maximum curvature was expressed as ductility factor μ.

$$\mu = \sqrt{(\psi_1 / \psi_{1y})^2 + (\psi_2 / \psi_{2y})^2} \cdots (14)$$

The size of the circle indicates the number of the plotted points and the zigzag lines connect the average value of the plotted points. The plastic deformation capacity of the section under bi-axial curvature history is inferior to that of the section under uni-axial curvature history, being compared with each other at the same level of axial force. In other words, in order to keep the same plastic deformation capacity the magnitude of axial force should be limited to smaller value in the case that the section would be subjected to bi-axial bending moments than in the case that the identical section would be subjected to uni-axial bending moment.

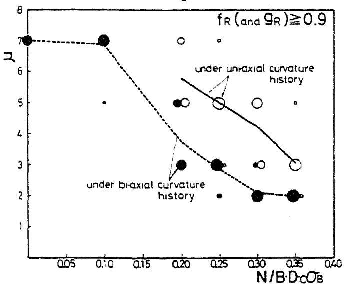

Fig.11-(1) μ - $N/B \cdot D \cdot {}_c\sigma_B$ relations

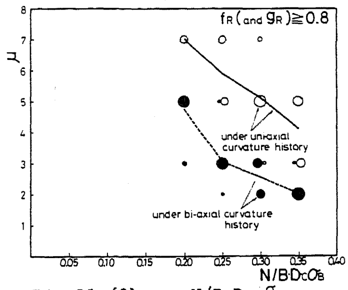

Fig.11-(2) μ - $N/B \cdot D \cdot {}_c\sigma_B$ relations

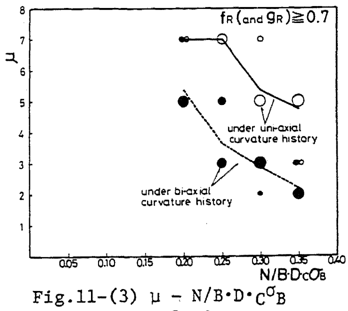

Fig.11-(3) μ - $N/B \cdot D \cdot {}_c\sigma_B$ relations

CONCLUSIONS

The application of the analytical method of column section subjected to bi-axial bending moments was extended to the circular column section. The analytical results agreed with the test results of circular column section very well.

The restoring force characteristics were compared between the case that the section was under uni-axial curvature history and the case that the identical section was under bi-axial curvature history. As the results, it was proved that the interaction of bi-directional forces strongly affects the restoring force and the hysteretic energy absorption when the column section is under high axial load, and that the plastic deformation capacity of the column section under bi-axial curvature history was inferior to that of the column section under uni-axial curvature history. Therefore, the magnitude of axial load should be limited to the smaller value when the column section would be subjected to bi-axial bending moments.

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