

THE EFFECT OF SPECIMEN RESONANCES ON  
ACCURATE CONTROL OF MULTIPLE DEGREE-OF-  
FREEDOM SERVOHYDRAULIC SHAKING TABLES

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SUMMARY

The modern multiple degree-of-freedom shaking table for laboratory earthquake simulation with a resonating and eccentrically offset specimen has the characteristics of a strongly coupled multiple input, multiple output dynamic system. Such a system is challenging to control accurately. It is well known from basic feedback control theory that the requirements of high accuracy and stability are often conflicting.

Using a mathematical model for a typical system, the effects on stability of specimen to table mass ratios, specimen to table frequency ratios, and specimen eccentric offset were studied.

Present practical state-of-the-art analog and digital compensation and equalization techniques for such a resonant specimen-table system are discussed.

INTRODUCTION

Seismic research and qualification testing is generating a requirement for multiple degree-of-freedom shaking tables. Recently, several laboratories have acquired systems which have a full six degree-of-freedom testing capability.

Successful tests require such systems to be at a minimum stable, and at best, have good accuracy with low sensitivity to specimen dynamic characteristics. The problem has been investigated and reported (Ref. 1, 2) for the single axis case. It is much more difficult for the highly coupled multiple loop systems. Each coupled loop lowers the achievable gain and, thus, the accuracy of the total system for duplicating dynamic command signals.

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In addition to the lower achievable loop gain, the problem of obtaining the proper stabilization settings becomes very complex. For a six degree of freedom shaking table that has full coupling between its axes, the number of adjustment knobs are 18 feedback and 12 feed-forward for the MTS shaking table controllers.

The approach of this paper was to study the problem in a different way. First, an example system was optimally tuned with no specimen. The effect of the eccentric specimen resonances was studied with respect to this tuning. The ideal shaking table system would require no adjustment with changing nonlinear complex specimens. A compensation technique approaching this ideal was examined.

#### MATHEMATICAL MODEL

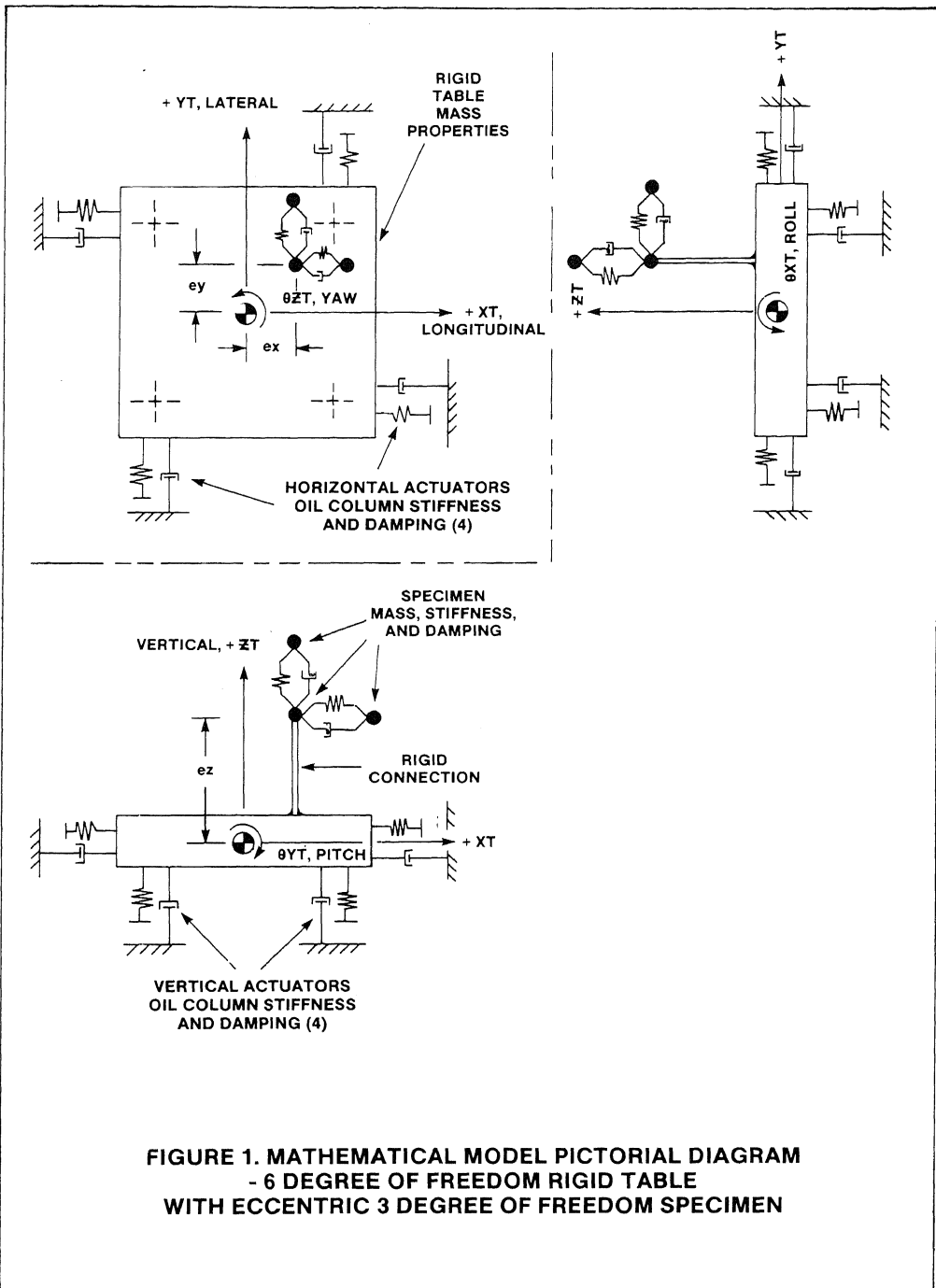
Figure 1 shows the mathematical model used to study specimen parameter variations and compensation techniques. The table is modeled as a six degree of freedom rigid mass which is actuated by compressing the actuator oil column springs. The specimen is modeled with three degrees of freedom and is excited by compressing its table mounting springs and dampers by table base excitation motion.

Figure 2 gives the parameters studied for an example table and specimen system. This example is representative of several recently installed systems in use now.

Figure 3 shows the coupling effect in the table equations of motion of the eccentric specimen. The magnitude of the specimen mass and eccentricity determines the coupling strength and the natural frequency determines the frequency range where it is effective. Note from Figure 3 that eccentricities in x,y, and z strongly couples all 6 equations of motion to each other. For the fully coupled case, the system dynamic order is 36: six second order oil column resonances, six second order servovalve resonances, six first order piston integrations, and three second order specimen resonances. If there are multiple lightly damped resonances of the table and specimen in the same frequency range, it is an extremely difficult feedback control problem.

#### STABILITY SENSITIVITY

The basic control loop for each axis consists of a servovalve which flows oil to the actuator based on the error signal between command and feedback. This flow compresses the oil column spring to actuate the table. Basic stabilization loops are obtained from table transducers which monitor acceleration and velocity. This stabilizes the resonances listed in Figure 2 and allows the gain of the loop to be raised.



SPECIMEN

Mass:  $m_{sx} = m_{sy} = m_{sz} =$  13,400 kg; 26,800 kg; 53,600 kg  
(1/2, 1, and 2 times the mass of the table)

Natural Frequencies: 10, 20, 40, and 80 Hz

Damping Ratio: 0.10

Eccentricities: Case 1 -  $e_x = e_y = e_z = 0$   
Case 2 -  $e_y = 1$  meter;  $e_x = e_z = 0$   
Case 3 -  $e_x = e_y = 0$ ;  $e_z = 2.0$  meter  
Case 4 -  $e_x = e_y = e_z = 1.0$  meter

TABLE

Axis	Mass and Rotational Inertia	Natural Frequency, Hz	Damping Ratio
Longitudinal	26,800 kg	23.4	.10
Lateral	26,800 kg	23.4	.10
Vertical	26,800 kg	38.0	.10
Roll	66,000 kg.m <sup>2</sup>	42.6	.114
Pitch	66,000 kg.m <sup>2</sup>	42.6	.114
Yaw	105,000 kg.m <sup>2</sup>	29.1	.10

Servo valve (all axes): Natural frequency = 100 Hz  
Damping ratio = .90

Figure 2. Representative Table and Specimen Dynamic Characteristics

Table Coordinate

Table Equation of Motion	XT	YT	ZT	$\theta$ XT	$\theta$ YT	$\theta$ ZT
Longitudinal, XT	msx	0	0	0	msx(ez)	msx(ey)
Lateral, YT	0	msy	0	msy(ez)	0	msy(ex)
Vertical, ZT	0	0	msz	msz(ey)	msz(ex)	0
Roll, $\theta$ X	0	msy(ez)	msz(ey)	msy(ez) <sup>2</sup> , msz(ey) <sup>2</sup>	msz(ex.ey)	msy(ex.ez)
Pitch, $\theta$ Y	msx(ez)	0	msz(ex)	msz(ex.ey)	msz(ex) <sup>2</sup> , msx(ez) <sup>2</sup>	msy(ey.ez)
Yaw, $\theta$ Z	msx(ey)	msy(ex)	0	msy(ex.ez)	msx(ey.ez)	msx(ey) <sup>2</sup> , msy(ex) <sup>2</sup>

msx = x direction specimen mass  
 msy = y direction specimen mass  
 msz = z direction specimen mass  
 ex = specimen x eccentricity from the table c.g.  
 ey = specimen y eccentricity from the table c.g.  
 ez = specimen z eccentricity from the table c.g.

Figure 3 Specimen Mass and Eccentricity Coupling of the Table Six Degrees of Freedom

As discussed earlier, the approach taken by the authors was to study specimen parameter sensitivity with respect to an ideal shaking table. The bare table without specimen was tuned to give optimum results. This tuning was maintained and the total system stability as a function of specimen parameters was monitored. In actual practice, depending on the specimen, the control system would be retuned to give the best response.

The results of the analysis are shown in the left half of Figure 4. The performance of the system degrades from the optimum stable and damped response of the bare table as the mass of the specimen increases. Other qualitative results show performance is especially sensitive to specimen resonances close to the oil column resonances of the table (20-40 Hz). Finally, a general trend of an enlarging instability region can be observed as the eccentric distance and the number of axes coupled increases. These results are all plausible physically and qualitatively correlate with field experience.

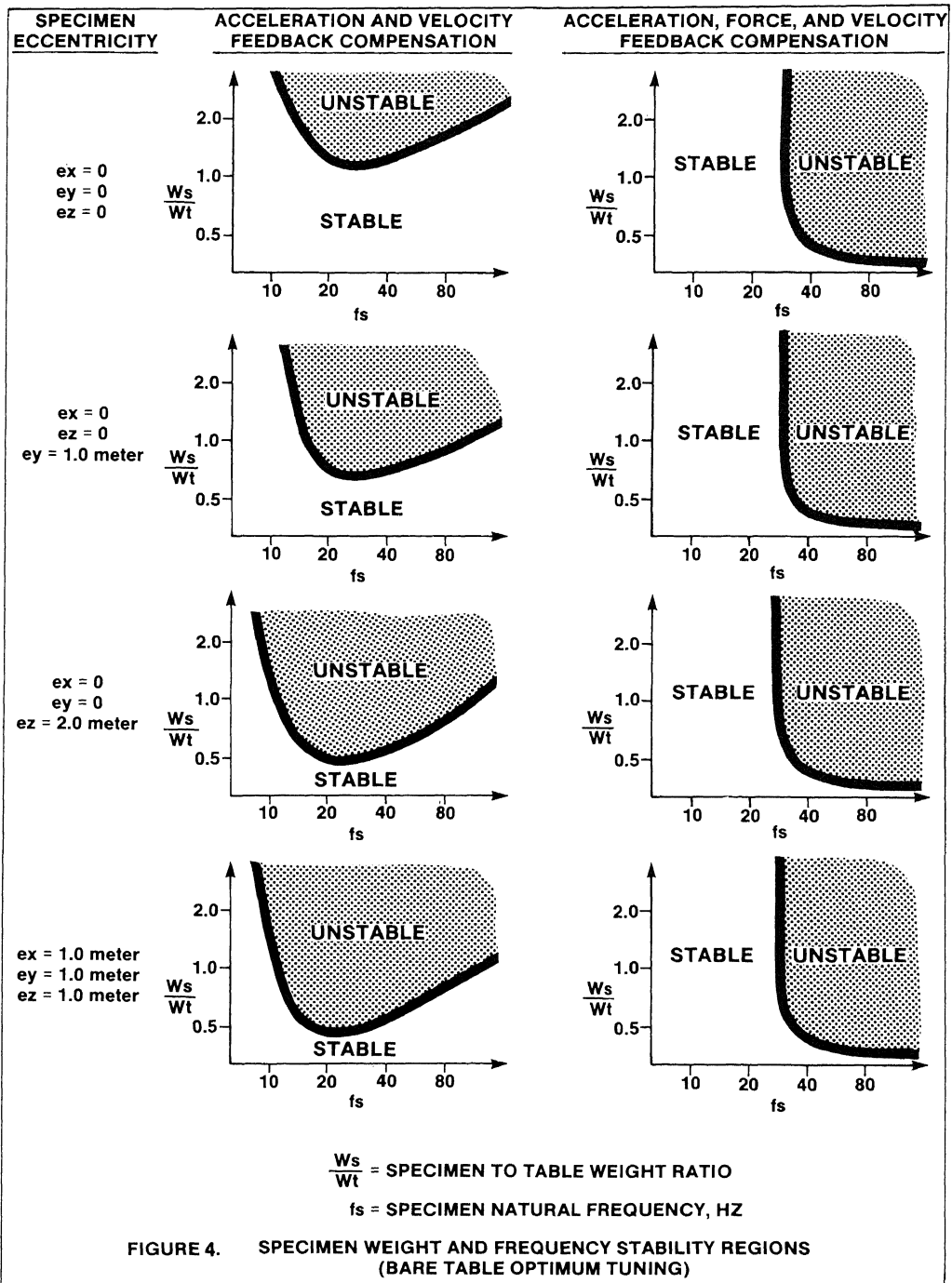
#### ANALOG AND DIGITAL COMPENSATION TECHNIQUES

To perform tests in the unstable regions shown in our numerical studies, the present practice is to reduce the gains from the bare table tuning until stability is reached. To achieve better control accuracy, two possibilities exist (both assume the system is stable). The total system dynamic characteristics can be established and compensated with an open-loop prefilter on the command. This is commonly done for single axis systems by amplitude controllers and tracking filters for sine sweep testing, and by digital vibration control systems for random testing.

An offline iterative algorithm has been developed for multiple degree-of-freedom servohydraulic shaking tables that makes use of the repeatability of the specimen and the table motions. The iterative process increases the accuracy of motion for the case of strong complex dynamic coupling and nonlinearities (Ref. 3).

This technique does not directly apply to earthquake research testing of nonlinear specimens to failure. In this case, it is impossible to characterize the specimen at full level. For this type of specimen, it is necessary to either extrapolate from the characterization at low level or use a dummy specimen for characterization at actual test levels.

A new analog technique that uses a blend of acceleration, force, and velocity feedback is currently under research and development. This technique was evaluated with the same eccentric specimens and found to improve performance. As shown by the right half of Figure 4, the unstable region is shifted up to 30 about Hz.



This technique has the ability to compensate for the specimen disturbances. It can operate as part of the analog control system in real-time and, thus, can be used for higher accuracy tests to failure.

#### CONCLUSIONS

1. The six degree of freedom shaking table with certain types of eccentric resonant specimens is seen to be a highly complex difficult system to control accurately.
2. The control problem becomes more difficult as the specimen mass and eccentricity increase when all the axes are coupled, and when the specimen resonances are close to the table's oil column resonances.
3. For the case where high accuracy analog control is not achievable, open-loop compensation can be used to improve test accuracy. If the specimen cannot withstand full level excitation during characterization measurements, a dummy specimen might need to be employed.
4. For tests to failure, a new feedback compensation technique shows promise for higher accuracy tests if the specimen resonances are below about 30 Hz.
5. More research on this difficult control problem is needed to advance the state-of-the-art in earthquake engineering.

#### ACKNOWLEDGMENT

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