

# EXPERIMENTAL INVESTIGATION OF THE STRENGTH, STIFFNESS AND DUCTILITY OF RC STRUCTURAL WALLS

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## SUMMARY

An experimental investigation was made of the inelastic behaviour of reinforced-concrete structural walls under cyclic loading, using three-storey models. Special attention was paid to the characteristic loading stages, such as the occurrence of the first flexural and shear cracks, the load at yielding of the tensile reinforcement and the ultimate load. As a result of the tests, data for strength, stiffness and displacements at all characteristic loading stages were obtained. On the basis of the analysis of the experimental results, analytical procedures have been proposed for determination of the stiffness and displacements in the linear and non-linear range of multi-storey reinforced-concrete structural walls.

## INTRODUCTION

Within the scope of the research project "Seismic Stability of Residential Buildings Constructed in Precast and Monolithic Reinforced Concrete Systems", the Institute of Earthquake Engineering and Engineering Seismology, Skopje, undertook experimental investigations in order to determine the behaviour of reinforced concrete structural walls under cyclic loads. The three-storey models to be used for the experimental testing were designed on the basis of a study of the linear and non-linear characteristics of the prototype building (Ref. 1).

The experimental results concerning the strength and deformability of the tested models have been shown in Ref. 2 and Ref. 3, while this paper presents a description of analytical investigations regarding the stiffness and displacements of structural-wall systems and the correlation with experimental results which were obtained.

## THE STIFFNESS OF STRUCTURAL-WALL MODELS

The stiffness of the tested models was investigated for the purpose of defining an analytical procedure needed to construct the full shear force-displacement diagram.

### The Stiffness of Models Without Cracks

For the case of models without cracks, elastic methods can be applied to determine flexural and shear stiffness. The elastic deformations of each storey can be expressed as a sum of the flexural and shear deformations.

### The Stiffness of Models at Yielding

It is very difficult to define analytically the stiffness of models after the appearance of flexural and shear cracks. Analysis of deformations was performed taking into account the mechanical properties of the materials used, obtained by suitable laboratory tests, the quantity and arrangement of the reinforcement at the ends and in the webs of the models, the axial loading and assuming a linear distribution of strain along the whole cross-section.

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Flexural deformations were determined analytically on the basis of the moment-curvature diagram at yielding, as shown in Fig. 1. Flexural stiffness was defined as the ratio between the analytical shear force at yielding of the vertical reinforcement and the corresponding relative displacement. For definition of the flexural stiffness and storey drifts of a real, multi-storey structure, the elastic moment diagram along the height of each structural wall or column, separately, should be available. In this case, the assumption of a linear  $M-\phi$  relationship up to yielding of the critical section has been made (Ref. 4).

Shear deformations have been analytically determined assuming that only the compressed part of the section is subjected to shear stresses. The position of the neutral axis at yielding has been determined taking into account the section characteristics, similar as in the case of definition of the curvature  $\phi_y$ . The shear stiffness is obtained as a reciprocal of the calculated displacements.

The analytical values obtained for the stiffnesses of the models without cracks and at yielding have been given, together with the measured stiffnesses at yielding, in Table 2. Very good correlation can be observed between the analytical and experimental results for stiffness at yielding, in spite of the simplified model assumed for the definition of shear deformations.

#### The Stiffness of Models Beyond Yielding

After yielding occurred in the vertical reinforcement at the ends of the model, the horizontal force continued to increase in all the models. On the basis of the experimental results an empirical formula was derived for definition of the ratio of stiffness beyond yielding,  $p = k/k_y$ , in the form :

$$p = 10p_{\text{conf.}} + 0.35\rho - 0.07(h_o/d)^2, \text{ where}$$

$p_{\text{conf.}}$  = percentage of vertical reinforcement in the confined part of the section,

$\rho$  = ratio of axial force and the axial compressive load-carrying capacity of the section,

$h_o/d$  = shear span ratio.

The above equation may be applied within the limits :  $p_{\text{conf.}} \leq 0.04$  and  $h_o/d \leq 2.0$ . The experimental and analytical values of  $p$  are given in Table 2.

#### The Stiffness at Unloading

The model stiffness during unloading, for displacements larger than  $\Delta_y$ , gradually decreases in comparison with the stiffness at initial yielding. This stiffness deterioration is due to the occurrence of cracks in the model. Analysing the hysteresis diagrams of all the models, the following stiffness equation for unloading conditions was obtained :

$$k_r = k_y (\Delta_y/\Delta)^q$$

The values of the exponential power  $q$ , obtained according to the test results, are given in Table 2. It is evident that the values are almost constant, except for the model DFM1E3. For analytical evaluation of the  $Q-\Delta$  diagram, it is recommended that a value of  $q = 0.33$  be taken, which is on the conservative side.

The experimentally and analytically obtained hysteresis loops of the model DFM1E3 are shown in Fig. 2, where the dotted line is a diagram in accordance

with the analytical values from Table 2.

The model stiffness during reloading has been defined so that, after the load drops down to zero, the stiffness line passes through the unloading point of the preceding cycle (lines 7, 13 and 19, Fig. 2).

#### DISPLACEMENT AND DUCTILITY

The horizontal displacement of the models has been investigated for different loading stages, so that the total displacements have been shown as a sum of the displacements due to flexure, shear and fixed-end rotation. The displacements due to shear and fixed-end rotation of the model have been measured during the testing of the models, whereas the flexural displacements have been analytically defined and verified through the measured total storey displacements of the models.

Special attention should be given to the definition of yield displacement since a significant change in stiffness takes place in the reinforced concrete elements, when this displacement is reached.

Displacements at ultimate stage define the capacity of the model for working in the non-linear range, while the ratio between the ultimate displacement and the yield displacement has been defined as the available ductility of the model.

#### Shear Displacements

The participation of shear displacements in the total storey displacements is considerable, which points to the necessity for the investigation of these deformations in RC structural walls. The ratio for the transfer of shear force taken by each mechanism changes depending upon the load level and it is difficult to define it quantitatively.

For the analytical evaluation of shear displacements in the structural walls having diagonal cracks, a modified truss analogy has been applied (Ref.5). Studying the photographs of the tested models, it was observed that diagonal cracks appear at an angle of about  $45^\circ$ . Shear displacements of the first storey have been defined for the models, since it is there that the largest shear deformations take place.

Before the yield point was reached, vertical reinforcement at the ends of the models was taken into account for definition of flexural deformations, so that the truss element AC and BD would not sustain any shear deformations (Fig.3). The horizontal web reinforcement and reinforcement in the slab cross-section has been grouped together in the slab level, thus forming the element CD. Space being limited, only the final expression for the displacement of the first storey at yielding will be presented here:

$$\Delta_{1,s}^y = \frac{Q \cdot \ell}{2A_{cd} \cdot E_s} + \frac{Q \cdot d_1^2}{2b \cdot B_e \cdot E_c} \cdot \frac{1}{\cos\theta \cdot \ell} \quad , \text{ where}$$

$$B_e = \frac{2}{3} \ell \cdot \sin\theta$$

If, after the yield state has been reached in the vertical reinforcement, a small increment of shear force is added to the panel ABCD (Fig.3), the following displacement increments have been observed (Ref.5):

1. The chord AC elongates considerably, due to yielding in the longitudinal reinforcement;
2. The chord BD does not shorten very much, since the concrete is effective in compression.
3. The principal elements of the shear mechanism are the horizontal and diagonal bars of the equivalent truss. In fact, the diagonal AD has no tensile axial stiffness after the appearance of diagonal cracks.

The elongation of the diagonal AD will be presented through the deformations of chords AC, BD and the element CD (Fig. 3). The elongation of the chord AC would depend upon the modulus of elasticity after yielding ( $E_{sh}$ ) and on the length of the yielding reinforcement. In the calculations it has been assumed that the yielding length of the reinforcement is  $h/10$ . In this paper only the final expression for the displacement of the first storey at ultimate state will be given.

$$\Delta_{1,s}^u = \Delta_{1,s}^y + \left( \frac{10^{-1}}{2A_s \cdot E_{sh}} + \frac{1}{2A_c \cdot E_c} \right) \cdot \frac{\Delta Q \cdot h^3}{\ell^2} + \frac{\Delta Q \cdot \ell}{2A_{cd} \cdot E_s} + \frac{\Delta Q \cdot d_1^2}{2b \cdot B_e \cdot E_c} \cdot \frac{1}{\cos \theta \cdot \ell}$$

#### Flexural Displacements

The determination of flexural displacements was carried out analytically, based on calculated curvatures at yielding and the ultimate state, according to the diagrams of Fig. 1. The ultimate displacement due to flexure has been calculated assuming that the ultimate moment and the ultimate curvature are reached at the critical section of the model. The length of the plastic hinge  $l_p$ , has been adopted following the proposal of Corley and Mattock:

$$l_p = 0.5d + 0.05 h_o$$

#### Displacements Due to Fixed-End Rotation

The fixed-end rotation of the models was measured directly during the tests. Based on these measurements, it was found that displacements due to fixed-end rotation at yielding amount to 15% of the total first-storey displacements, whereas displacements due to fixed-end rotation at ultimate state amount to 25% of the total first storey displacements.

#### Total Displacements and Ductility

In the evaluation of the total displacements due to flexure, shear and rotation of the first storey of the models, it should be observed that considerable non-linear deformations appear under cyclic loads, and that it is very difficult to define analytically the exact displacements of structural walls.

In multi-storey buildings designed using structural-wall systems, non-linear deformations are observed in the lower storeys, whereas in structural walls with openings non-linear deformations appear also in the coupling beams. Therefore, it is of particular importance to determine the deformational and strength capacity for yielding and ultimate state of these elements.

The proposed analytical methods for the definition of lateral displacements are based on the test results of three-storey models constructed using the

structural wall system, and rather good correlation between the experimental and analytical results has been obtained.

Table 2 shows the displacements of the third storey of the models at yielding and ultimate state, as well as the ductility calculated on the basis of these displacements.

Table 3 shows the experimental results for the first-storey displacements of the model DFM1E2, which had a rectangular section. The components due to rotation and shear are the measured values whereas the flexural displacements are analytical ones. The participation of separate displacement components in the total displacements has been given in Table 3 and Fig. 4.

#### CONCLUSIONS

The behaviour of reinforced-concrete structural walls in multi-storey buildings during strong earthquakes is either in the linear or the non-linear range, depending upon the plan dimensions of the walls and the position of floor in the building. Most probably, the lower storeys experience non-linear deformations, and a plastic zone will form.

The stiffness of the tested models was defined successfully, in spite of the simplified model used for definition of the shear stiffness. The stiffness after yielding, in the case of unloading and reloading has been defined depending upon magnitudes of the non-linear deformations, while the hysteresis diagrams  $Q - \Delta$  have been completely defined by applying an analytical procedure.

For analytical determination of the horizontal displacements of RC walls, deformations due to flexure, shear and fixed-end rotation should be included. Analytical methods for the determination of flexural and shear displacements in the plastic region could be applied for evaluation of the results obtained by non-linear dynamic analysis, that is by a study of ductility supply compared with demand.

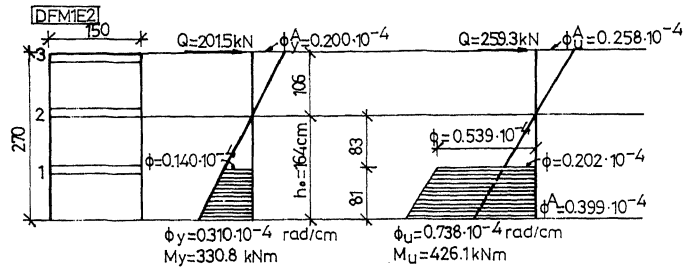


Fig.1 Analytical curvature diagrams

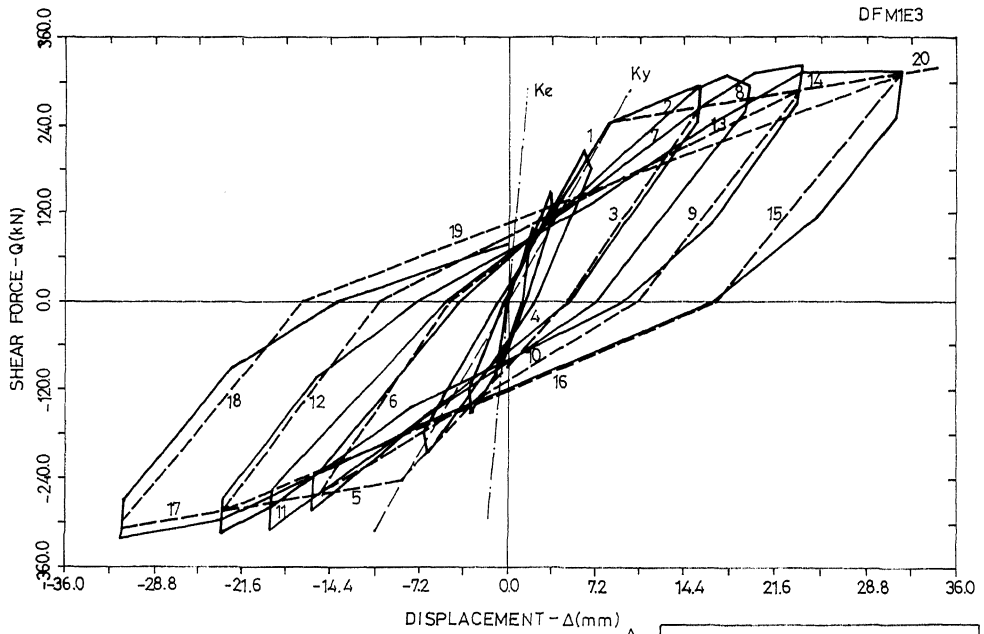


Fig.2 Analytical and experimental hysteretic diagrams  $Q - \Delta_3$

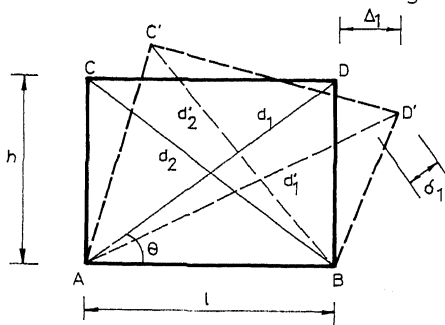


Fig.3 First storey deformation due to shear

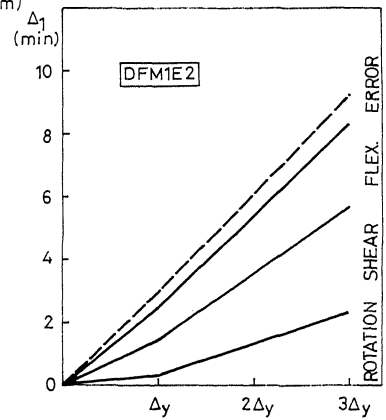


Fig.4 Components of horizontal displacement

Table 1

Model	$A_c$ cm <sup>2</sup>	$\sigma_o$ MPa	$A_s$ cm <sup>2</sup>	$P_v$ %	$L_{conf}$ cm	$P_{conf}$ %	$A_{wv}$ cm <sup>2</sup> /m	$A_{wh}$ cm <sup>2</sup> /m	$h_o$ cm	$h_o/d$
DFM1E2	1071	2.30	1.70	0.16	13	1.87	2.77	2.77	164	1.07
DFM1E3	1224	2.02	3.01	0.25	27	1.39	3.93	3.93	164	1.07
DFM1E4	1071	4.60	3.01	0.28	27	1.59	3.93	6.70	164	1.07
DFM2E1	2632	0.15	2.83	0.12	24	1.68	2.77	3.35	160	0.73
DFM2E2	2632	2.45	2.83	0.12	24	1.68	3.93	7.85	245	1.11

Table 2

Model	$K_{e,3}^A$ KN/mm	$K_{y,3}^A$ KN/mm	$K_{y,3}^E$ KN/mm	$\frac{K_{y,3}^E}{K_{y,3}^A}$	$p^E$	$p^A$	q	$\Sigma\Delta_y$ mm	$\Sigma\Delta_u$ mm	$D_\Delta$
DFM1E2	175.5	27.85	26.90	0.97	0.153	0.150	0.33	7.03	22.08	3.14
DFM1E3	201.4	30.10	29.80	0.99	0.102	0.096	0.23	7.93	31.85	4.02
DFM1E4	175.5	36.53	32.20	0.88	0.177	0.171	0.35	7.62	26.83	3.52
DFM2E1	671.0	96.00	104.40	1.09	0.144	0.134	0.35	2.88	15.32	5.32
DFM2E2	488.0	68.90	72.60	1.05	0.123	0.131	0.33	6.48	22.56	3.48

Table 3

Load Point	$\Delta_r^E$ mm	$\frac{\Delta_r^E}{\Delta E}$ %	$\Delta_s^E$ mm	$\frac{\Delta_s^E}{\Delta E}$ %	$\Delta_f^A$ mm	$\frac{\Delta_f^A}{\Delta E}$ %	$\Sigma\Delta$ mm	$\Delta E$ mm	$D_\Delta$	$Q^E$ kN
40			0.477	32				1.47		139.8
89(Y)	0.349	12	1.094	38	1.03	35	2.47	2.91	1.00	188.8
105			1.652	32				5.12	1.76	214.6
133			2.400	32				7.45	2.56	236.9
174(u)	2.284	25	3.386	37	2.68	29	8.35	9.27	3.19	250.8
208			4.359	39				11.29	3.88	245.2

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