MATHEMATICAL MODELING OF A FRAME WITH JOINT ROTATION

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SUMMARY

This paper shows the considerable influence that joint behavior in a frame plays in the response of the frame to a dynamic forcing function. The behavior is demonstrated by means of experimental results obtained when a three-story steel frame is subjected to seismic motions on a shaking table and the results are used to formulate a mathematical model of the frame using system identification.

INTRODUCTION

To date most mathematical models of moment-resisting frames are formulated using the geometric and material properties of the members and assuming that the joints are continuous. This assumption implies that the joints are rigid and that at a joint all members rotate the same amount. As many frames are constructed so that the stiffnesses of the columns and girders are relatively the same, the deformation that occurs when the frames are subjected to earthquake motions will involve joint rotations as well as floor translations; thus, the behavior of a joint during rotation is significant in many frames. When a mathematical model of such a frame is constructed using this traditional method, the model predicts quite poorly the responses recorded when the frame is subjected to an earthquake input on the shaking table at the Earthquake Engineering Research Center, University of California, Berkeley. To our knowledge the first detailed study that associate the shortcomings of the traditional mathematical model with the assumption of continuity of the joints was conducted by Kaya and McNiven[1]. They gained physical insight into the problem by using system identification to construct a number of mathematical models of a frame. They accounted for joint deformation in a somewhat oblique way. They constructed their best mathematical model by introducing a set of parameters; one parameter was associated with the geometric length of each column and girder and an additional parameter accounted for viscous damping. When the cost function was minimized using a Gauss-Newton optimization algorithm and the resulting "best set" of values for the parameters was inserted into the model, it predicted response of the frame extremely accurately. The length of a member times the parameter associated with it was considered to be the "effective" member length. As the effective length differed from the geometric length, Kaya and McNiven concluded that the joints did in fact change shape during the deformation of the frame and that when this change of shape is accounted for, the model is significantly improved.

In this paper we take what we think is the next logical step in improving the formulation of the mathematical model of a moment-resisting frame. The

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step is to isolate each joint to be considered as a separate element in the frame behavior. The formulation described in what follows has a number of advantages over the previous formulation by Kaya and McNiven. First, we have not resorted to static condensation of the stiffness matrix which results in a relationship between translation and joint rotation that depends on the continuity of the joint behavior we know is violated. We are able to reduce from eight to four the number of free parameters to be identified by optimization, thus reducing considerably the computer costs of this operation. We take advantage of symmetry, as did Kaya and McNiven, but further find that the same parameter suffices for all three columns and that no adjustment is needed for the girder lengths. We consider that when a joint deforms it can do so both due to shear and to the moments imposed upon it. An in-depth study, shows that when we associate a parameter with each of these deformations, the parameters, during optimization, do not behave independently, so that one parameter is sufficient to account for joint deformation. We arbitrarily choose to associate the parameter with the shear deformation. The third parameter reveals the viscous damping and the fourth accounts, as in the previous study, for the rocking of the shaking table during excitation.

System identification requires experimental results and we are fortunate to have the results of experiments conducted by Clough and Tang [2]. The experiments were conducted on a three-story steel frame with welded connections between girders and columns. The stiffnesses of the girders and columns were of the same order of magnitude. We are particularly fortunate for this study in that Clough and Tang conducted two series of tests, designated Phase I and Phase II, wherein the only difference in the frames for the two phases was in the joints. For Phase II the joints of Phase I were stiffened significantly. In our mathematical modeling of the frames for the two phases we are able to show that the parameters associated with the columns and the rotation of the table are essentially the same. The damping of the Phase II frame is somewhat higher than that for Phase I, as one would expect. The parameters for the joints, however, differ significantly, the parameter for the stiffened joint being more than twice that for the joint of Phase I.

THE THREE-STORY FRAME

The test structure consisted of two parallel, single-bay, three-story, moment-resistant steel frames. The frames were fabricated from standard rolled shapes of ASTM A-36 grade steel. Two frames, designated A and B, were separated by 6'. They were connected at floor levels by removable cross beams and bracing angles, producing the effect of a floor diaphragm rigid in its own plane. The Total height of the structure was 17'14". The story heights were 6'8", 5'4", and 5'4". The bay width was 12'0". Sections W5-16 and W6-12 were used for columns and girders, respectively.

Fully penetrated welded girder to column connections were used. The panel zone thickness was 1/4" (i.e., the column web thickness) for Phase I of the experiments, and 1" (column web reinforced by 3/8" doubler plates on both sides) for Phase II.

The frames were instrumented with linear potentiometers at each floor to measure floor translation. The frames had strain gauges attached to both flanges at the top and bottom of each column. Assuming a linear variation of bending strain along the length of a column, the relative rotation of the ends will be given by:

$$\theta = \frac{L}{h}\ \frac{\varepsilon_a+\varepsilon_b}{2}$$

where ε_a and ε_b are the bending strains at either end, L is the length of the column, and h is its depth.

The table excitation used was the El Centro earthquake.

CONSTRUCTION OF THE MATHEMATICAL MODEL

The mathematical model associated with the dynamic behavior of an n degree of freedom linear elastic structure subjected to rigid base motion is:

$$\underline{m}\frac{d^2\underline{u}}{dt^2} + \underline{c}\frac{d\underline{u}}{dt} + \underline{ku} = -\underline{mr}\frac{d^2\underline{u}_g}{dt^2} \tag{1}$$

$$\frac{du(0)}{du} = \underline{u}(0) = 0 \tag{2}$$

where $\underline{m}$ is the mass matrix, $\underline{c}$ is the damping matrix, and $\underline{k}$ is the stiffness matrix. $\frac{d^2u}{dt^2}$, $\frac{du}{dt}$, and $\underline{u}$ are vectors for relative acceleration, velocity, and displacement. $\frac{d^2\underline{u}_g}{dt^2}$ is the base acceleration and $\underline{r}$ is a column vector whose elements are static displacements due to a unit displacement of the structure.

It is possible to find a matrix, $\underline{P}$, so that $\underline{M}=\underline{P}^t\underline{mP}$ and $\underline{K}=\underline{P}^t\underline{kP}$ are both diagonal matrices; i.e., $M_{i_j}=0$ if $i \neq j$. If we make the change in variables:

$$\underline{u} = \underline{PY} \tag{3}$$

then the differential equation of motion can be rewritten:

$$\underline{M}\frac{d^2\underline{Y}}{dt^2} + \underline{C}\frac{d\underline{Y}}{dt} + \underline{kY} = \underline{F}(t) \tag{4}$$

where

$$\underline{C} = \underline{P}^t\underline{cP} \quad \text{and} \quad \underline{P}(t) = -\underline{P}^t\underline{mr}\frac{d^2\underline{u}_g}{dt^2} \tag{5}$$

Developed here is a finite element model in which joint panel zones are assumed to be rigid in reacting to flexural and axial forces, but shear distortions are allowed. The column element stiffnesses will be given by:

$$\underline{k} = k' \begin{Bmatrix} 2+\beta & 1-\beta & 0 \\ 1-\beta & 2+\beta & 0 \\ 0 & 0 & \frac{A}{2I}(1+2\beta) \end{Bmatrix} \tag{6}$$

The girder stiffnesses will be given by:

$$k = \frac{3EI}{L(1+2\beta)} \tag{7}$$

The joint stiffnesses will be given by:

$$k = Gbht \tag{8}$$

where:

$$k' = \frac{2EI}{L(1+2\beta)} \qquad \beta = \frac{6EI}{L^2A'G} \tag{9}$$

and E, I, A, and A' denote Young's modulus, moment of inertia, section area, and effective shear area, respectively. The displacement transformation matrices, A_i, for each element are given by:

Girders:

a)	0	0	0	1	0	0	−1/1	0	0	1	0	0	0	0
b)	0	0	0	0	1	0	0	−1/1	0	0	1	0	0	0
c)	0	0	0	0	0	1	0	0	−1/1	0	0	1	0	0

Columns:

a)	−1/1	1/1	0	1	0	0	0	0	0	0	0	0	0	0
	−1/1	1/1	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	−1	0	0	0	0	0	0
b)	0	−1/1	1/1	0	1	0	0	0	0	0	0	0	0	0
	0	−1/1	1/1	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	−1	0	0	0	0	0
c)	0	0	−1/1	0	0	1	0	0	0	0	0	0	0	0
	0	0	−1/1	0	0	0	0	0	0	0	0	0	1	1
	0	0	0	0	0	0	0	0	1	0	0	0	0	−72

Joints:

a)	0	0	0	0	0	0	0	0	0	1	0	0	0	0
b)	0	0	0	0	0	0	0	0	0	0	1	0	0	0
c)	0	0	0	0	0	0	0	0	0	0	0	1	0	0
d)	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Table Spring:

0	0	0	0	0	0	0	0	0	0	0	0	0	72

The global stiffness matrix will be:

$$\underline{K} = \Sigma \underline{A}_i^t \underline{k}_i \underline{A}_i \tag{10}$$

The formulation at this stage accommodates a large family of models. Five models are constructed as groundwork for preparation of the final model, the one in which the joints are considered as separate elements. This preliminary work has two purposes. The first is to confirm the major findings of Kaya and McNiven: (a) if rotation is to be accommodated in the model, the response quantities for the cost function must include joint rotation as well as floor translation time histories and (b) consideration of the pitching of the shaking table has a significant effect on the parameters associated with the girders. The second purpose is to investigate whether the number of parameters introduced by Kaya and McNiven can be reduced without appreciably affecting the ability of the model to predict the experimental frame response.

Identification Using Only Floor Displacements

For the first three models, only the floor translation (displacement) time histories are used and only the first six seconds of response. These time histories are constructed primarily to explore the number of separate parameters needed for association with the columns. In model one, three parameters are used, along with mass proportional damping and optimization resulted in the parameters for the top two floors being almost identical but different from the lowest floor column. Model two uses only three parameters, one for the top two columns, one for the lowest column, and one for damping. The error for the optimized parameters remains unchanged. The third model then uses only one parameter for all three columns but associates three additional parameters with the girders. With damping there are five parameters. The major finding here confirms what was pointed out by Kaya and McNiven: that is, if only the floor translation time histories are used, the parameters associated with the girders are not unique and there could be a family of models that would predict the time histories of the floor translations.

Identification Using Both Displacements and Rotations

It appears that the girder and column length factors do not form an independent set of parameters with respect to displacement response data. However, the rotation data, inferred from strain measurements, are not of the same magnitude as the displacement data. If the rotation data are used directly in identifying the parameters they could be expected to have little or no effect. The rotation data, therefore, are scaled by the modulus of elasticity of the steel, $E=29.6 \times 10^6$ psi. While somewhat arbitrary, this causes the two sets of data to be of the same order of magnitude.

In the fourth model twelve seconds of response data are used with the same set of parameters as model three but using both floor translation and joint rotation responses. The findings of Kaya and McNiven are confirmed. The column parameter and the three girder parameters are of the same order of magnitude, the parameter for the column being greater than one and the three for the girders less than one. However, in this latter case they are unique.

Finally, in model five an additional parameter is introduced to account for the pitching of the table. The initial value of this parameter is the one suggested by Tang [3]. An examination of the parameter resulting from optimization shows that the algorithm tends to "soften" the system by increasing the base stiffness and, to compensate, decreases the effective girder lengths. Thus, it appears that the girder and base parameters do not form an independent set.

Model 6: The Final Model

In the previous models, the parameter adjustment primarily took place in the effective girder lengths. In contrast, model six is an attempt to permit the joints to accommodate the response. Thus, a four-parameter model is formulated, with one parameter associated with the columns, one parameter with the base stiffness, one parameter with the effective joint panel thickness, and one for damping. After identification, parameter values are as follows:

Parameter	Phase I Value	Phase II Value
column	1.09	1.07
base	.96	.97
joint	2.16	5.27
damping	1.35	1.53

Note that the column and base parameters are close to their estimated values and are unaffected by the stiffening of the joints. The Phase II frame has a higher damping factor than the frame for Phase I, which we would anticipate. The most significant observation is that the joint parameter for the Phase II frame is significantly higher than the comparable parameter for the Phase I frame, indicating that joint stiffness plays a major role in the model.

The influence of the joint behavior is exhibited in the figures. The responses predicted by the "traditional" model, neglecting joint deformation, and by model six are each compared to the recorded experimental responses for both floor translations and joint rotations. Examination shows that model six predicts the seismic response of the frame significantly better than the model neglecting joint deformation.

REFERENCES

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2. Clough, R. W. and Tang, D. T., "Earthquake Simulator Study of a Steel Frame Structure, Vol. I: Experimental Results", Report No. EERC-75-6, Earthquake Engineering Research Center, University of California, Berkeley, April 1975.

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DISPLACEMENTS AT THIRD FLOOR
(inches vs. seconds)

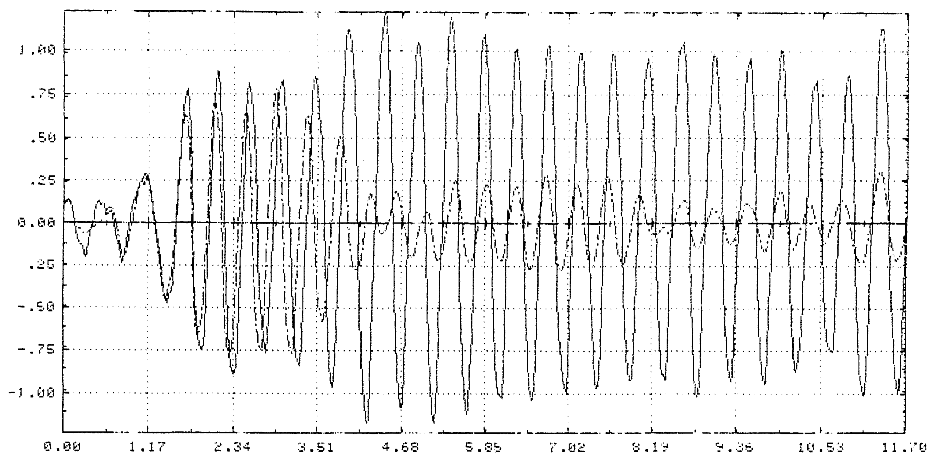

FIGURE 1
Comparison with Response from Traditional Model

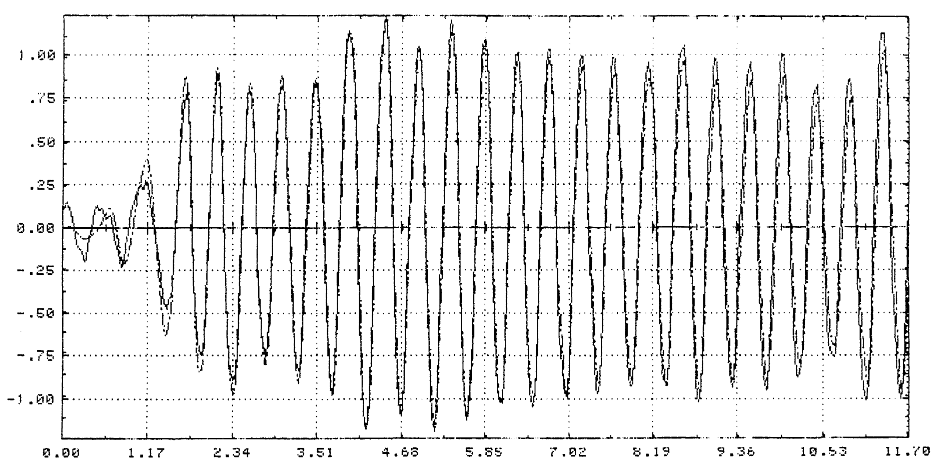

FIGURE 2
Comparison with Response from Model Six

ROTATIONS AT FIRST FLOOR
(milliradians vs. seconds)

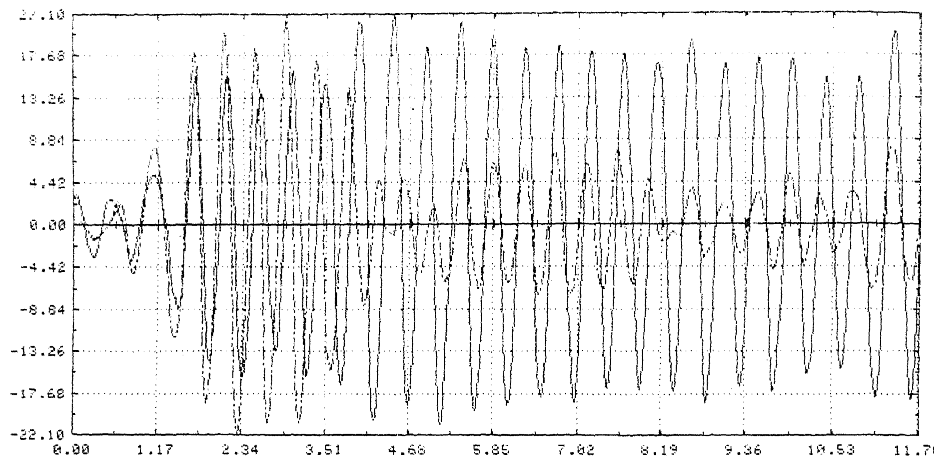

FIGURE 3
Comparison with Response from Traditional Model

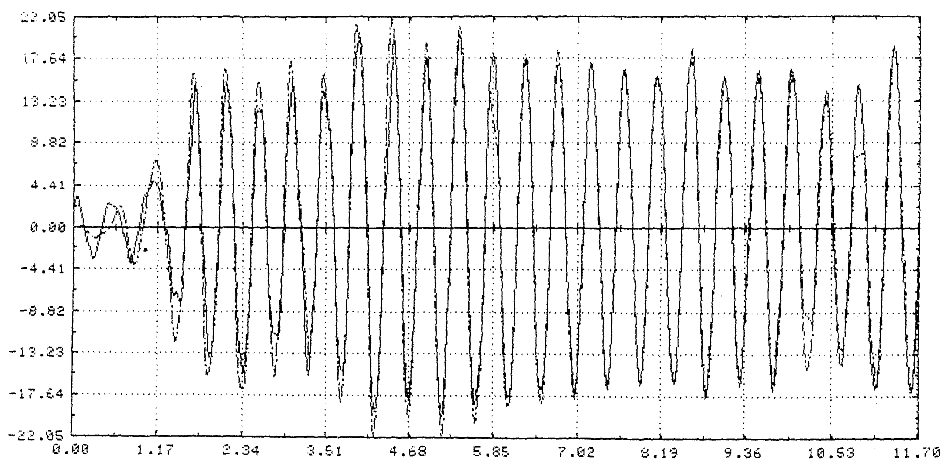

FIGURE 4
Comparison with Response from Model Six