

INELASTIC RESPONSE BEHAVIOR OF H-SHAPED STEEL COLUMN  
TO BI-DIRECTIONAL EARTHQUAKE MOTION

H. Taniguchi (I)

K. Takanashi (II)

Presenting Author: H. Taniguchi

SUMMARY

This paper describes the response behavior of H-shaped steel column to bi-directional earthquake motions simulated by the hybrid system of a digital computer and a loading test system. Two analytical models for the "pure" computer analysis are also described. Comparison are made between the response behavior simulated by the on-line system and those computed by using these analytical models. The results indicate that even the simplest model such as bi-linear is available for rough estimation of the response of H-shaped steel column to bi-directional earthquake motions.

INTRODUCTION

It is important in the earthquake resistant design of structures to know the effect of the coupled lateral response of structures to bi-directional earthquake motions. Many researches have been reported on the response of structures to bi-directional earthquake motions (e.g. Ref.1) and several analytical models have been proposed to compute the inelastic behavior of resisting elements such as column under multi-dimensional forces. However, there are very few experimental researches on the inelastic behavior of the H-shaped steel columns under bi-directional cyclic loadings. Experimental verification of the analytical models is still to be performed. A response analysis system, called IIS Computer-Actuator (On-line) Hybrid System, was developed to obtain the inelastic response behavior of structural models. In the analysis by this method, the responses are computed on the basis of the real restoring force characteristics obtained from the computer-controlled loading test of structure or structural element. So to speak, the assumed analytical model of restoring force characteristics in the "pure" computer analysis is replaced by the real one in the on-line system procedure. This paper presents the response behavior of steel H-shaped columns to two horizontal components of recorded earthquake motions analyzed by the on-line system. Then, two numerical methods are presented. One is denoted Fiber-Model in which a tri-linear type stress-strain relationship was used. The other is denoted Parabolic Model which is an extension of Ziegler's kinematic hardening rule with a bi-linear type shear-displacement relationship (Ref.2). The results computed by these numerical models are compared with the results by the on-line system.

A FRAME MODEL

A one-bay square single-story building model which comprises a rigid

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(I) Ohbayashi-gumi, Ltd., Tokyo, Japan.

(II) Professor of Institute of Industrial Science, Univ. of Tokyo, Japan

floor and four H-shaped steel columns as shown in Fig.1 is intended for analyses. It is assumed that four columns are identical and their slenderness ratios are small. The stiffness center on the floor of this frame coincides with the mass center, and the torsional motion of this frame is considered to be negligible. If the torsional motion of the floor is neglected, the rigid floor connecting the tops of columns always moves horizontally without showing torsion. Then the analysis of a single column is considered enough to predict the response behavior of this frame. Therefore, a single column with the lumped mass on the top shown in Fig.2 is taken as an model. The changes of the axial load of the column and the vertical motion of the mass center are also neglected in this analysis.

#### COMPUTER-ACTUATOR ON-LINE ANALYSIS

##### Procedure of On-line Analysis

The principle and analytical procedure of the system have been reported in the previous papers (e.g. Ref.3). Brief review on the analytical procedures of the system is represented here intending the emphasis on the specific feature of the bi-directional response analysis. Principal directions of a H-shaped column section are initially parallel to the east-west and the north-south directions, respectively. In the case that the mass is lumped on the top of the column and the rotation about the column axis is neglected, the equations of motion predicting the response at the top of the column can be expressed as follows;

$$m\ddot{u}^i + Q_x^i = -m\ddot{u}_o^i \quad (1)$$

$$m\ddot{v}^i + Q_y^i = -m\ddot{v}_o^i \quad (2)$$

where  $m$  is the mass,  $\ddot{u}^i$ ,  $\ddot{v}^i$  denote the response accelerations,  $Q_x^i$ ,  $Q_y^i$  the restoring forces, and  $\ddot{u}_o^i$ ,  $\ddot{v}_o^i$  the ground accelerations in the  $x$ ,  $y$  direction, respectively. The superscript,  $i$ , denotes the variable at the time,  $t=i \Delta t$ , where  $\Delta t$  is the time increment used in the step-by-step numerical integration of equations of motion. In the hybrid system, the open-type finite difference method is used to solve Eqs.(1) and (2) numerically. The simplest central difference method gives the following expressions for the accelerations,  $\ddot{u}^i$  and  $\ddot{v}^i$ .

$$\ddot{u}^i = (u^{i+1} - 2u^i + u^{i-1})/(\Delta t)^2 \quad (3)$$

$$\ddot{v}^i = (v^{i+1} - 2v^i + v^{i-1})/(\Delta t)^2 \quad (4)$$

$\ddot{u}^i$  and  $\ddot{v}^i$  in Eqs.(1) and (2) are replaced by the right side of Eqs.(3) and (4). Then, the response values  $u^{i+1}$  and  $v^{i+1}$  at  $t=(i+1)\Delta t$  can be calculated for the ground accelerations at  $t=i\Delta t$ , namely  $\ddot{u}_o^i$  and  $\ddot{v}_o^i$ , since  $Q_x^i$ ,  $Q_y^i$ ,  $u^i$ ,  $v^i$ ,  $u^{i-1}$  and  $v^{i-1}$  are already known. In the on-line system, the values of  $Q_x^i$  and  $Q_y^i$  are provided by the column load test. The response displacement,  $u^{i+1}$  and  $v^{i+1}$ , are imposed in turn on the column specimen to measure new restoring force,  $Q_x^{i+1}$  and  $Q_y^{i+1}$  at time  $t=(i+1)\Delta t$ . This procedure is continued successively until a run of analysis is completed. The fundamental flow of the procedure on the on-line system is shown in Fig.3.

##### Column Specimens and Test Set-up

To measure the restoring force characteristics of the bi-axial bending column, the load test were carried out on the welded built-up H-shaped steel column specimens, H-70×70×6×6. This column is 89cm in length (Slenderness ratio about strong axis bending,  $\lambda_x=32$ , slenderness ratio about weak axis bending,  $\lambda_y=53$ ). Two hydraulic jacks placed horizontally in the principal axis directions of the column impose the calculated response displacements at a moment and then the restoring forces are measured as reaction forces. A jack placed at the bottom of the column provides the constant thrust to the column during the analysis. The thrust was set 30% of the yield force  $P_Y$  ( $=\sigma_Y A$ ;  $\sigma_Y$ =the measured yield stress,  $A$ =the section area).

#### Assumed Frames and Scaled Ground Acceleration Records

A series of on-line analyses were carried out for the frame models listed in Table 1. The column models with the identical section and length were intended in use. The fundamental elastic period with respect to the strong axis bending motion was fixed to 0.5 second. The variables considered were the ground motion characteristics (EW and NS components of 1968 HACHINOHE, and EW and NS components of 1940 EL CENTRO) and the combinations of the intensities of ground accelerations in each direction. The scaled intensities in each direction were decided on the basis of the yield acceleration  $\alpha_{pc}$  ( $=M_{pc}/(mL/2)$ ;  $M_{pc}$ =the full plastic moment under thrust,  $m$ =the mass,  $L$ =the column length).

#### COMPUTER ANALYSIS BY FIBER MODEL

In the pure computer analysis, it is necessary to calculate the restoring forces,  $Q_x^i$ ,  $Q_y^i$  in Eqs.(1) and (2) by a numerical method. Plastic action is dependent on the loading history and requires the step-by-step calculation procedure, even though the prescribed stress-strain relationship of the material is assumed. It takes much time to calculate by computer even in an approximate solution, so drastic simplifications and idealizations are necessary. The basic assumptions made in this paper are as follow;

- (1) Only the stress and strain component normal to the section are considered.
- (2) The twisting moment and deformation are neglected.
- (3) The axial force is constant along the axis of a beam-column.
- (4) The small deflection theory is applied. The incremental calculation is conducted.
- (5) The curvature of an axis of a column can be expressed in terms of the second derivative of the deflections.
- (6) The cross section does not change its shape.
- (7) The premature local buckling and torsional buckling do not occur.

By symmetry of the analyzed model, the calculation on a canti-lever, a half of a column, can represent the overall behavior. The differential equation of the beam-column was solved approximately by a finite difference method. The finite difference nodes used in the calculations are shown in Fig.4. For carrying out the numerical calculations of the rigidity in the plastic range, the H-section is divided into small elements as shown in Fig.5. The strain and stress in each element are computed as the average values at its centroid. The stress-strain relationship is assumed to be a tri-linear model as shown in Fig.6. This model was determined so that it can predict also well the cyclic behavior of a beam-column due to bi-axial bending moments and thrust.

The computer analyses were conducted for the same column as used in the on-line analyses. For numerical integration of the equations of motion, the linear acceleration method was adopted, where the time increment was set 0.01 second.

#### COMPUTER ANALYSIS BY PARABOLIC MODEL

Non-linear restoring force characteristics such as bi-linear, tri-linear, Ramberg-Osgood type functions etc. have been widely adopted as analytical models for the earthquake response analysis of the planar structures. The analytical models are desired to be as simple as possible from the view point of structural design. We developed a shear force-displacement relationship model for bi-axially loaded beam-columns by extending the bi-linear model into two dimensional one. The model consists of the initial yield condition, a flow rule and a hardening rule.

##### Initial Yield Condition, Flow Rule and Hardening Rule

The initial yield condition is often expressed by an elliptic function, but in this case a set of parabolic function in Fig.7 was taken. The yield function can be expressed as follows:

$$\phi_1 = (q_x - q_{cx}) + (q_y - q_{cy})^2 - 1 \quad (5)$$

$$\phi_2 = -(q_x - q_{cx}) + (q_y - q_{cy})^2 - 1 \quad (6)$$

where  $q_x = Q_x / Q_{pcx}$ ,  $q_y = Q_y / Q_{pcy}$ ,  $q_{cx} = Q_{cx} / Q_{pcx}$ ,  $q_{cy} = Q_{cy} / Q_{pcy}$ .  $Q_{pcx}$ ,  $Q_{pcy}$  denote the full plastic strength in principal directions, and  $Q_{cx}$ ,  $Q_{cy}$  denote the translation components of the yield locus in principal directions. Due to the flow rule of v.Mises, the plastic deformation increment,  $\{\Delta x\}_p$ , lies in the exterior normal to the yield surface Eqs.(5) or (6) at  $\{Q\}$ . Thus it is represented by

$$\{\Delta x\}_p = \{\partial \phi_i / \partial Q\} \lambda, \quad \lambda > 0 \quad (i = 1 \text{ or } 2) \quad (7)$$

$$\text{where } \{\Delta x\}_p^T = \{\Delta u_p, \Delta v_p\}, \quad \{\partial \phi_i / \partial Q\}^T = \{\partial \phi_i / \partial Q_x, \partial \phi_i / \partial Q_y\}$$

The definition of a Ziegler-hardening material is completed by assuming that the yield surface moves in the direction of the vector CP connecting the center of the yield surface with the stress point as shown in Fig.7 (Ref.4). It is represented by

$$\{\Delta Q_c\} = \mu (\{Q\} - \{Q_c\}), \quad \mu > 0 \quad (8)$$

$$\text{where } \{Q\}^T = \{Q_x, Q_y\}, \quad \{Q_c\}^T = \{Q_{cx}, Q_{cy}\}$$

##### Incremental Force-Displacement Relationship

By using the above assumptions, the incremental force-displacement relationship may be obtained as follows:

$$\{\Delta Q\} = ([K_e] - (1-r) \frac{[K_e] \{\partial \phi_i / \partial Q\} \{\partial \phi_i / \partial Q\}^T [K_e]}{\{\partial \phi_i / \partial Q\}^T [K_e] \{\partial \phi_i / \partial Q\}}) \{\Delta x\} \quad (9)$$

where  $[K_e]$  is known as elastic matrix, and  $r$  denotes the work hardening coefficient. The incremental translation of the yield surface is given as follows;

$$\{\Delta Q_c\} = \frac{\{\partial \phi_i / \partial Q\}^T \{\Delta Q\}}{\{\partial \phi_i / \partial Q\}^T (\{Q\} - \{Q_c\})} (\{Q\} - \{Q_c\}) \quad (10)$$

The difference in the work hardening coefficient between the strong axis bending and the weak axis bending is taken into considerations, and the corresponding uni-axial shear-displacement relations in each direction are the bi-linear model shown in Fig.8. It is assumed that the work hardening coefficient under the bi-axial loading,  $r$ , varies according to the loading point on the yield function as follows;

$$r = \sqrt{(r_x \cos \theta)^2 + (r_y \sin \theta)^2} \quad (11)$$

where  $\theta = \tan^{-1}((Q_y - Q_{cy}) / (Q_x - Q_{cx}))$ ,  $r_x$  and  $r_y$ , the work hardening coefficient in the weak and the strong axis bending direction. The numerical analyses using this model were conducted on the same columns as used in the on-line analyses. For numerical integration of the equations of motion, the linear acceleration method was adopted, where the time increment was set 0.001 second.

## RESULTS OF ANALYSES

### Results of On-line Analyses

Some results are selected and shown in Figs.9 to 13. The time history of the response displacements in the weak axis bending plane and those in the strong axis bending plane on DBC-C-3 are shown by solid lines in Figs.9 and 10, respectively. The restoring force-displacement relationship in the weak axis bending plane and that in the strong axis bending plane on DBC-C-3 are shown in Figs.11 and 12. The maximum response displacement in each direction are summarized in Fig.13. We recognize that the restoring force characteristics loop, especially in the weak axis bending plane, results in very complicated curves. The large drift in the weak axis bending direction occurred in the case that the response displacement in the strong axis bending direction became larger than a level shown by a chain line in Fig.13.

### Comparison with Computer Analyses

The results of the "pure" computer analysis by Fiber Model are represented by dashed lines in Figs.9 and 10. The results of the "pure" computer analysis by Parabolic Model are represented by chain lines in Figs.9 and 10. The restoring force-displacement relationships by Parabolic Model in the weak and strong bending planes are shown in Figs.14 and 15. Comparison with the results of on-line analysis reveals many similarities.

## CONCLUDING REMARKS

The IIS Computer-Actuator On-line System was applied to the earthquake response analyses of a square single-story frame subjected to the two components of earthquake acceleration records. Two computer analysis procedures, using Fiber Model and Parabolic Model, are developed. The conclusions obtained are as follows.

(1) The IIS Computer-Actuator On-line System is available to obtain the response behavior of frames subjected to the bi-directional earthquake motions.

(2) The restoring force characteristics in the weak axis bending plane results in very complicated curves.

(3) The large drift in the weak axis bending direction occurred in the case that response displacement in the strong axis bending direction became larger than the level, say,  $4v_{pc}$ .

(4) The numerical analysis based on Fiber Model with the tri-linear type stress-strain relationship is available for estimation of response of H-shaped steel column to bi-directional earthquake motions. However, this method takes much time to calculate by computer.

(5) The results of the response calculation using Parabolic Model coincides satisfactorily with the results by the on-line analysis. Even the simplest model based on bi-linear shear-displacement relationship can predict the earthquake response behavior fairly well.

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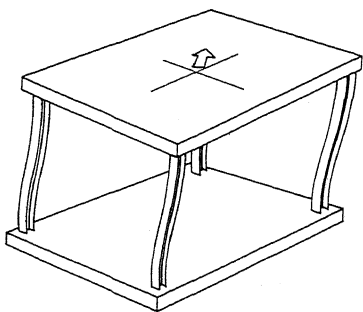


Fig.1 A square single-story frame model

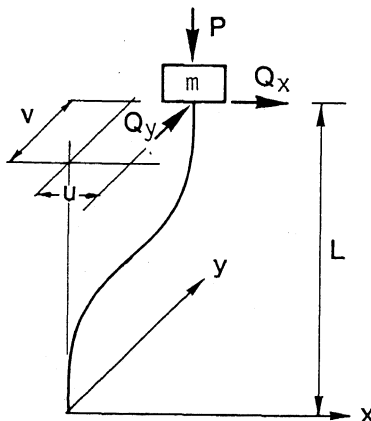


Fig.2 A lumped mass model



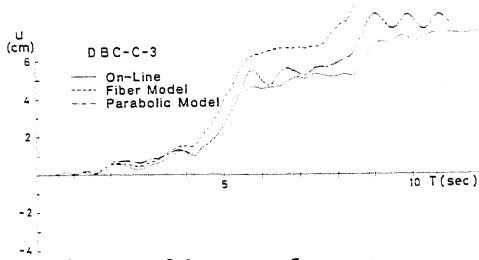


Fig. 9 Time history of response displacements in x-direction

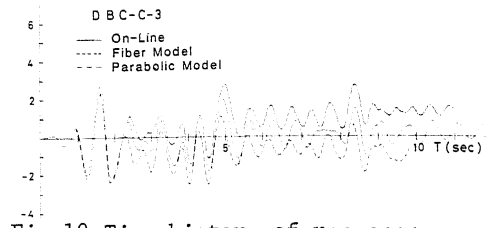


Fig. 10 Time history of response displacements in y-direction

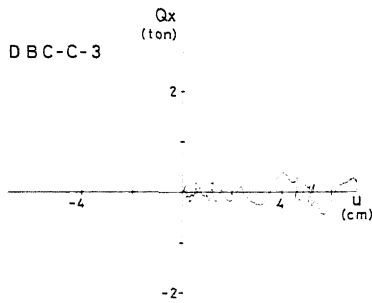


Fig. 11  $Q_x$ - $u$  relation by On-line simulation

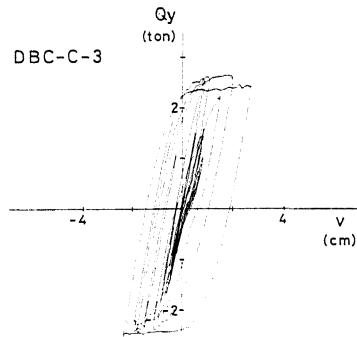


Fig. 12  $Q_y$ - $v$  relation by On-line simulation

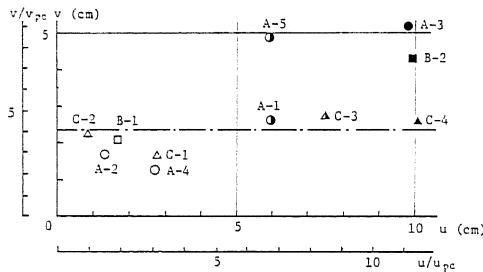


Fig. 13 Maximum displacements by On-line simulation

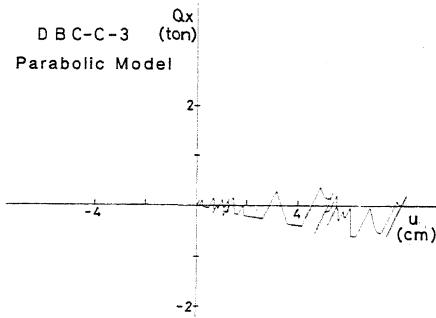


Fig. 14  $Q_x$ - $u$  relation by Parabolic Model

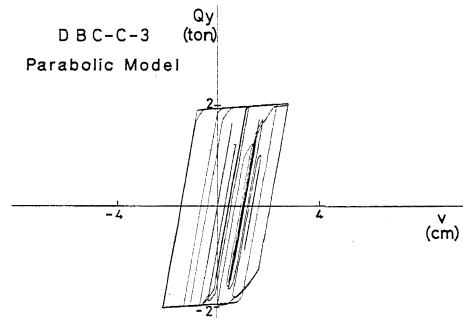


Fig. 15  $Q_y$ - $v$  relation by Parabolic Model