HYSTERETIC BEHAVIOR OF K-TYPE BRACED FRAME

M. SHIBATA (I)
M. WAKABAYASHI (II)
Presenting Author: M. SHIBATA

SUMMARY

Experimental and theoretical studies on the hysteretic behavior of K-type braced frame are presented. Five specimens are tested under repeated loading, for two values of the brace slenderness ratio and three values of the strength ratio of the beam to braces. Test results show that the braces in K-type braced frame do not function well as the tension members. A detailed elastic-plastic analysis is applied to this problem. The computed results show a good agreement with the experimental results. The mechanism of the beam plastification is also discussed, and the ultimate strength formula is proposed.

INTRODUCTION

The hysteretic behavior of a K-type braced frame is much different from that of an X-type braced frame. The beam in the K-type braced frame is subjected to the lateral force which is the vertical component of the sum of the axial forces of braces. The hysteretic property of the system much depends on the elastic-plastic behavior of the beam, because the braces do not function well when the beam yields in bending by the lateral force caused by braces.

In this paper, some experimental and theoretical studies are presented to investigate the hysteretic behavior of a K-type braced frame. The study focuses on the interaction effect between braces and frame elements.

EXPERIMENTAL STUDY

Experiment The shape and dimensions of specimens are listed in Table 1 and illustrated in Fig. 1. P_{fo} and P_{bfo} in the table are defined as follows.

$$P_{fo} = \begin{cases} 2(M_{co} + M_{go})/h & (M_{go} \leqq M_{co}) \\ 4M_{co}/h & (M_{go} > M_{o}\ ) \end{cases} \tag{1}$$

$$P_{bfo} = P_{fo} + 2M_{go}/h \tag{2}$$

Where M_{co} and M_{go} are respectively the full plastic moments of the column and the beam, and h is the story height. P_{fo} is the horizontal strength carried by the frame with strong beam, whose mechanism is illustrated in Fig. 3(a). P_{bfo} means the contribution of the frame with rather weak beam, corresponding to the mechanism, shown in Fig. 3(b).

Total five specimens were tested. The slenderness ratio of the brace was about 80 for Series A, and 30 for Series B. The ratio of the bending strength of the beam to the axial strength of the brace

$$C = 2M_{go}/(l_g\ T_o \sin \Theta) = 4M_{go}/(P_{bo}\ h_c) \tag{3}$$

(I) Lecturer, Osaka Institute of Technology, Osaka, Japan
(II) Professor, Kyoto University, Kyoto, Japan

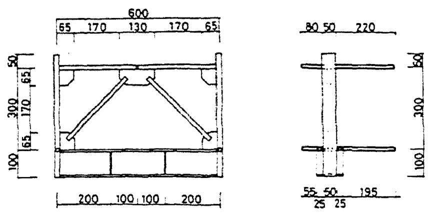


Fig. 1 Test specimen

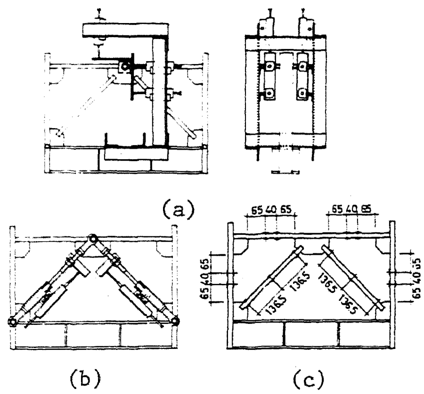


Fig. 2 Measuring equipments

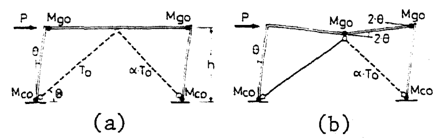


Fig. 3 Collapse mechanism

Table 1 Summary of specimens

		A-1	A-2	A-3	B-2	B-3
Brace	D (cm)	0.555	0.565	0.575	1.57	1.57
	B (cm)	3.19	2.06	4.98	1.88	1.88
	ℓ_b (cm)	27.1	27.1	27.2	27.1	27.1
	σ_y (t/cm²)	2.58	2.58	2.58	2.92	2.92
	$\lambda = \ell_b/(2i)$	84.6	83.1	81.9	29.9	29.9
Beam	D (cm)	2.46	1.60	1.58	2.46	1.58
	B (cm)	4.99	5.03	5.03	4.98	5.03
	σ_y (t/cm²)	2.86	2.92	2.92	2.86	2.92
	M_{go} (tcm)	21.59	9.40	9.17	21.55	9.17
Column	D (cm)	2.78	1.87	1.86	2.77	1.86
	B (cm)	5.03	5.00	5.01	5.00	4.99
	h_c (cm)	16.8	16.9	16.8	16.7	16.7
	σ_y (t/cm²)	2.62	3.07	3.07	2.62	3.07
	M_{co} (tcm)	25.46	13.42	13.30	25.13	13.25
$P_{bo} = 2T_o \cos\Theta$		6.46	4.24	10.44	12.18	12.18
$P_{fo} = 2(M_{co} + M_{go})/h_c$		5.60	2.70	2.68	5.59	2.69
$P_{bfo} = 2(M_{co} + 2M_{go})/h_c$		8.17	3.81	3.77	8.17	3.78
P_{bo}/P_{bfo}		0.791	1.113	2.769	1.491	3.222
$C = 4M_{go}/(P_{bo}h_c)$		0.796	0.525	0.209	0.424	0.180

was also varied from 0.2 to 0.8 (Table 1). In Eq.(3), T_o and Θ are the tensile strength and the cord angle of the brace, respectively. l_g and h_c are the length of the beam and the column, respectively. Beams, columns and braces were made of steel plates, and the foundation was made of a wideflange. Specimens were annealed after welding. Figs. 2(a) and (b), show the arrangement of displacement meters, and Fig. 2(c) shows the positions of strain gauges. The test set-up is illustrated in Fig. 4, and a sample of loading history for A-1 is shown in Fig. 5.

Results The test results for 4 specimens are illustrated in Figs. 6-10. Fig. 6 shows the relationship between the applied horizontal force P and the horizontal displacement u of the midspan of the beam. The ordinate is normalized by P_{bfo} defined in Eq.(2). In the case of Series A, the horizontal force abruptly decreased after the initial buckling of the brace. The horizontal force of A-1, which had a strong beam, slowly increased subsequently, but the increase for A-3 with rather weak beam was small. Under the repeated loading with the constant displacement amplitude, the hysteresis loop converged early, and the load at each reversal point was about P_{bfo} for A-1, and a little greater than that for A-3.

In Series B, the decrease of the load, after the initial buckling of the brace, was not so abrupt as in Series A, but the subsequent increase of the load was small for B-2, and was not observed for B-3. The hysteresis loop under repeated loading with the constant displacement amplitude, deteriorates slowly for B-2, and significantly for B-3. There was not observed the recovery of the restoring force with the increase of the amplitude.

In Fig. 7, the relationship between the vertical and horizontal displacements, v and u of the midspan of the beam is shown. Those characteristics seem to correpond to the elastic-plastic behavior of the beam. In Series A, the beam behaved almost elastically under the reversed loading with the constant displacement amplitude and the plastification developed at the increase of the amplitude. But in A-3, a little development of the beam plastification was observed in the 3rd and 4th displacement amplitude, under the load reversal of the constant amplitude. In Series B, the beam plastification developed not only at the increase of the amplitude, but also under the load reversal of the constant amplitude.

Fig. 8 shows the relationships between the relative axial displacement of braces and the horizontal displacement u. The hysteresis loops with left side down show the behavior of the left brace, and those with right side down show that of the right brace. It was made clear that the braces in K-type braced frame are rarely stretched and that they do not function well as the tension members. The cumulation of the compressive deformation of braces was much dependent on the plastification of the beam, and was significant in Series B.

Fig. 9 shows the relationships between the horizontal force P_f carried by the frame elements and the horizontal displacement u. They were obtained by analyzing the output of the strain gauges attached to columns. Because the strain exceeded its yield limit in some points, the discrete distribution of strain was assumed by dividing the cross section into 10 fiber elements, as shown in Fig. 11(a), and it was also assumed that the each fiber element had the piecewise linear stress-strain relationship, shown in Fig. 11(b). In Fig. 9, P_f was calculated as the sum of the slope of the column bending moment, and is normalized by P_{fo}, defined in Eq.(1). The hysteretic behavior of P_f/P_{fo} - u relationships showed similar characteristics for each specimen, and the maximum value of P_f was about P_{fo}.

In Fig. 10, the horizontal force P_b shared by braces was estimated by the difference between the applied load P and P_f, and normalized by $P_{bo} = 2T_o \cos \Theta$. Since the braces were rarely stretched, as shown in Fig. 8, the restoring force shared by the tension brace did not recover its full strength by stretching, even at the increase of the amplitude. In Series A, the deterioration of the hysteresis loop caused by the repeated loading was not so serious as in Series B, under the reversed loading with the constant amplitude, but the recovery of the restoring force at the increase of the amplitude was small, especially for A-3. The deterioration of the hysteresis loop was significant in Series B, even under the reversed loading with the constant amplitude.

The output of the strain gauges attached to the beam was also analyzed, and the bending moment distributions of the frame at the load reversal point of the first 4 cycles for B-3, is illustrated in Fig. 12. The collapse mechanism shown in Fig. 3(b) was also confirmed experimentally.

THEORETICAL STUDY

An elastic-plastic analysis was made to simulate the hysteretic behavior of a K-type braced frame. Each member of the braced frame was assumed to be composed of three elastic-plastic springs and two straight segments, as shown in Fig. 13. The detail of the analysis was presented in Ref. 1. Some examples of computed results are illustrated by dashed lines in Figs. 6, 7 and 10, and well predict the experimental results. Fig. 14 shows the analytical relationships between the vertical component Q_g of the sum of the brace axial forces and the vertical displacement v of the midspan of the beam. The abscissa is normalized by M_{go}/l_g. The figure shows that the lateral strength of the beam is estimated as $Q_{go} = 2M_{go}/l_g$.

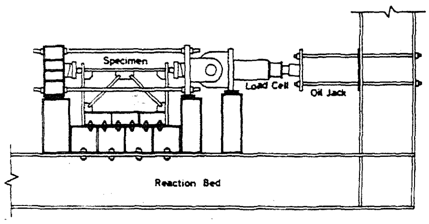


Fig. 4 Test set-up

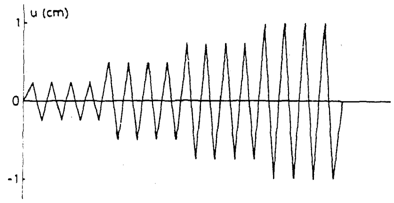


Fig. 5 Sample of loading history

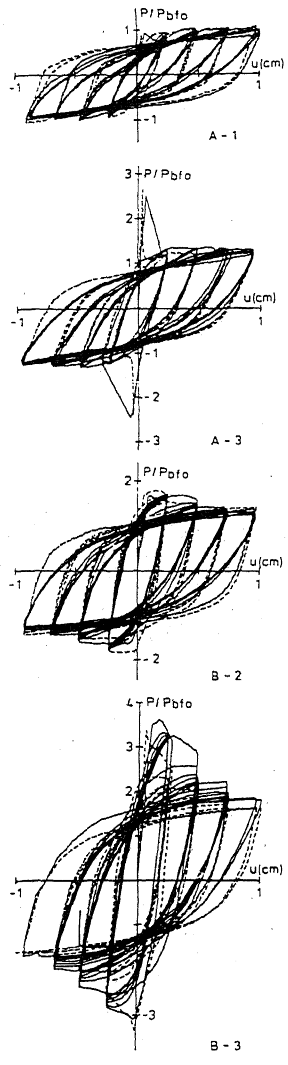


Fig. 6 P/P_{bfo} – u relationship

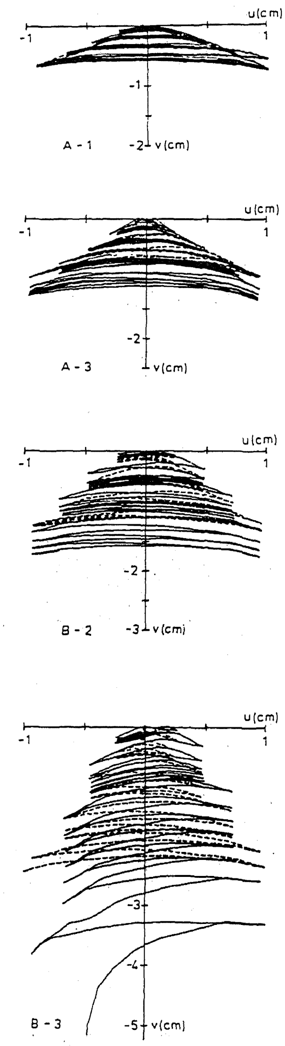


Fig. 7 v – u relationship

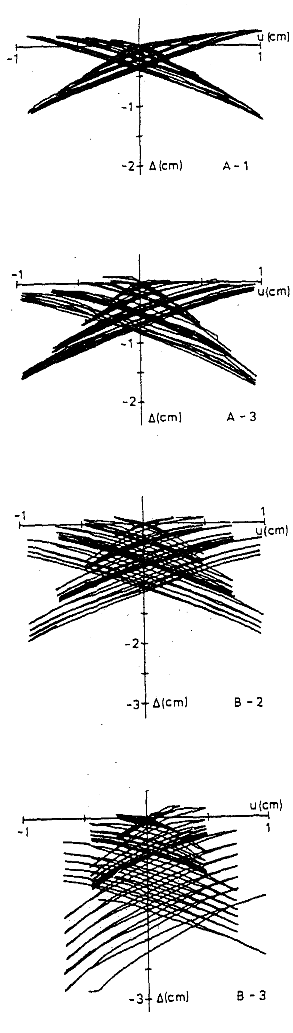


Fig. 8 Δ – u relationship

PREDICTION OF ULTIMATE STRENGTH

Simple plastic theory   Applying the simple plastic theory to the K-type braced frame, assuming the restoring force characteristics of the brace to be as shown in Fig. 15, the plastic collapse load P is obtained by following equations, which correspond to collapse mechanisms, shown in Fig. 3.

$$P_o = P_{fo} + (1 + \alpha)/2 \cdot P_{bo} \qquad (1 - \alpha)\ P_{bo} \leqq 4M_{go}/h \qquad (4)$$

$$P_o = P_{bfo} + \alpha\ P_{bo} \qquad (1 - \alpha)\ P_{bo} > 4M_{go}/h \qquad (5)$$

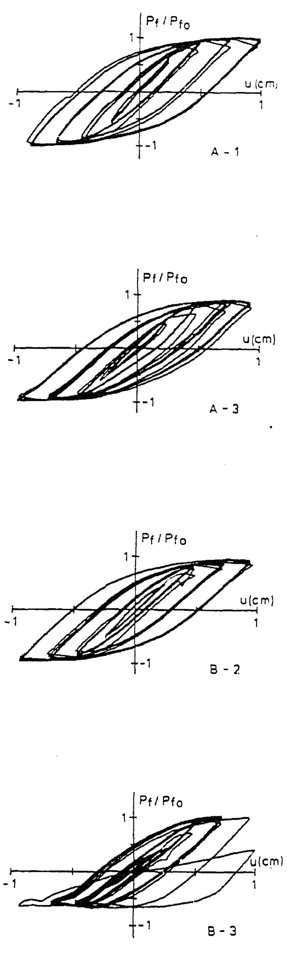


Fig. 9 P_f/P_{fo} - u relationship

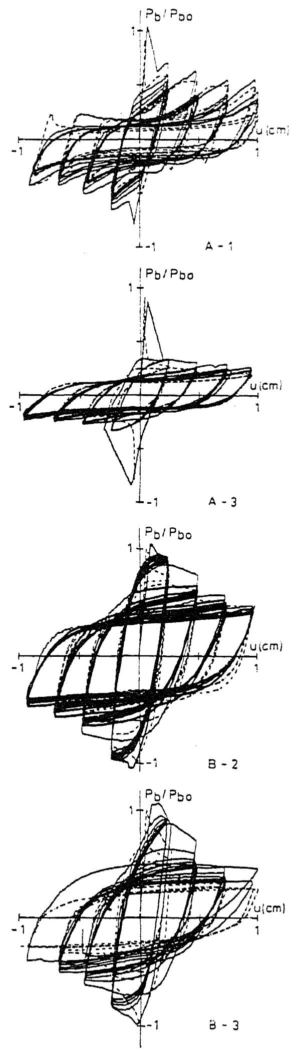


Fig. 10 P_b/P_{bo} - u relationship

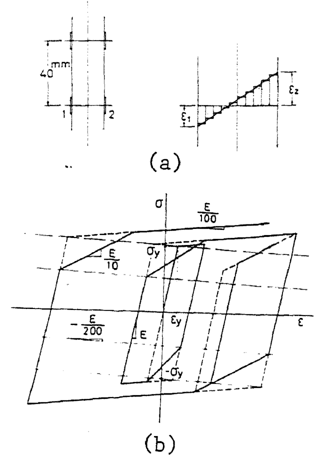


Fig. 11 Analysis of stress distribution

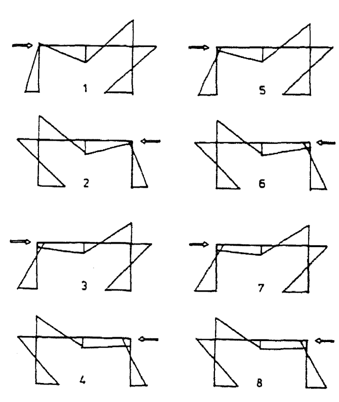


Fig. 12 Bending moment distribution

Equation (4) is applied to the K-type braced frame with strong beam and Eq.(5) to that with rather weak beam. In Eq.(5), the contribution of braces to the collapse load is very small, and only the first term is effective if α is small. The relationship between P_o and P_{bo} in Eqs.(4) and (5) is illustrated in Fig. 16. Dashed lines in the figure show the results obtained by applying the similar method to the X-type braced frame. The brace in K-type braced frame is not so effective, if $(1 - \alpha)\ P_{bo} > 4M_{go}/h$, as that in X-type braced frame where P_o increases linearly with P_{bo}.

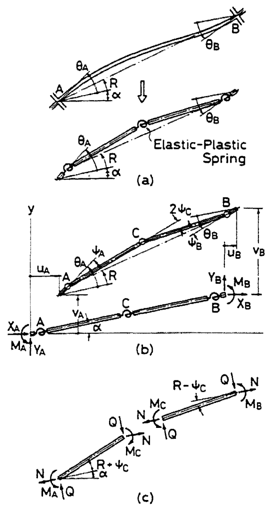


Fig. 13 Model for analysis

Beam plastification   Let us idealize the effect of the beam in K-type braced frame by a rigid-plastic spring, shown in Fig. 17. If the beam is stiff and strong enough, the geometrical relationship between the relative axial displacements Δ_1, Δ_2 of braces and the horizontal displacement u is illustrated in Fig. 17(b). On the contrary, if the lateral strength Q_{go} of the beam is not so large, the beam yields when the vertical component of the difference $\bar{T}$ of the brace axial forces attain the beam strength Q_{go}. In this case, the unloading occurs in the tension brace, where the variation of the relative axial displacement is considered to be little and only the compression brace is subjected to large deformation, as shown in Fig. 17(c).

The relationship between the axial forces T_1, T_2 and the horizontal displacement u keeps stationary state, as shown in Fig. 18(a), provided that the beam does not yield by the lateral force under the constant displacement amplitude. Since the lateral force acting on the beam is estimated by the vertical component $\bar{T} \sin \Theta$ of the difference of the brace axial forces, the beam is subjected to the upward force at the region denoted by (-) mark, and to the downward force at the (+) region. $\bar{T} \sin \Theta$ should be less than the lateral strength Q_{go} of the beam at the load reversal point, in order to prevent the beam plastification.

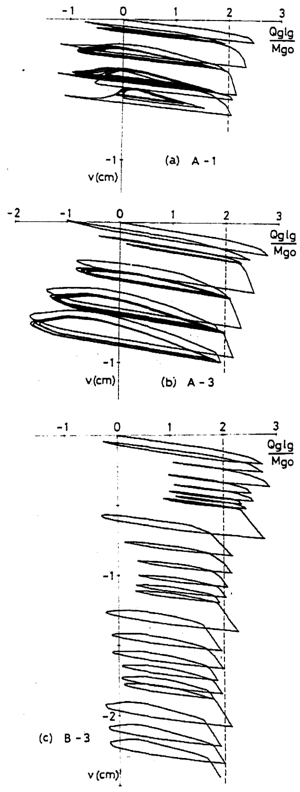


Fig. 14 $Q_g l_g$ - v relationship

On the other hand, if the vertical component $\bar{T} \sin \Theta$ of the difference of the brace axial forces attain the lateral strength Q_{go} of the beam, the beam starts yielding and the unloading occurs in the tension brace at point A in Fig. 18(b), while the large plastic deformation of the compression brace is induced over point A'. Since the variation of the axial displacement of the unloaded brace is considered to be little, the axial displacement of the compression brace increases by $2u_c \cos \Theta$ and the lateral deflection of the beam increases by $u_c \cot \Theta$, where u_c is the necessary increment of the horizontal displacement to attain the prescribed reversal point from the start of beam yielding (Fig. 18(b)). Large plastic deformation of the beam may be induced in the case that u_c is large.

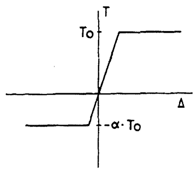


Fig. 15 Assumed restoring force of braces

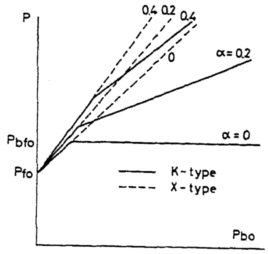


Fig. 16 $P_o - P_{bo}$ relationship

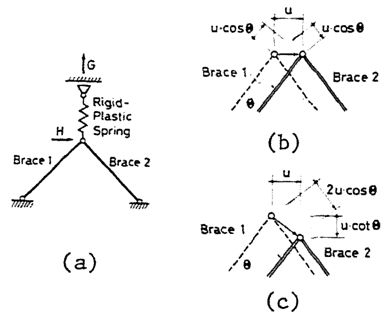


Fig. 17 Rigid-plastic spring model

In Fig. 19, a pair of hysteresis loops of a single brace obtained from the experiments by Wakabayashi et al. (Ref. 2) are illustrated. u_c is large for small value of the brace-slenderness ratio, provided that the other conditions are the same; beam strength Q_{go}, displacement amplitude δ_a, etc.

Ultimate strength under repeated loading If the beam is strong enough, braces behave stationary under the repeated loading with the constant displacement amplitude, as shown in Fig. 18(a). The strength of a K-type braced frame at the reversal point is given by

$$P_o = P_{fo} + (\alpha_t + \alpha_c)/2 \cdot P_{bo} \tag{6}$$

where α_t and α_c are the ratios of the tensile and compressive values of brace axial forces at the reversal point to the limit load T_o in tension.

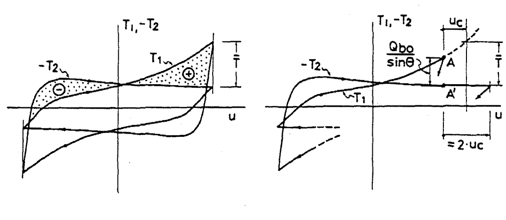
On the contrary, if the vertical component of the predicted difference $\bar{T} = (\alpha_t - \alpha_c)\, T_o$ of the brace axial forces at the reversal point is greater than the lateral strength Q_{go} of the beam, the axial force T_t of the tension brace at the reversal point is evaluated as $T_t = \alpha_c\, T_o + Q_{go}/\sin\Theta$, and the strength of the system at the reversal point is given by

$$P_o = P_{fo} + (T_t + \alpha_c\, T_o)\cos\Theta = P_{fo} + Q_{go}\cot\Theta + \alpha_c\, P_{bo} \tag{7}$$

Assuming $Q_{go} = 2M_{go} / l_g$ which is based on the conclusions of the previous chapter, and taking into account of $\cot\Theta \cong l_g/h$ and Eq.(2), Eq.(7) is rewritten, as follows.

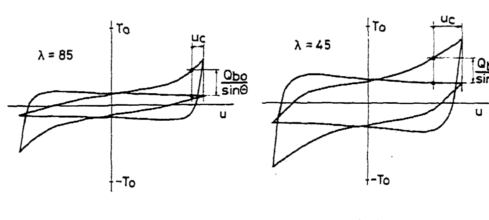
$$P_o = P_{bfo} + \alpha_c\, P_{bo} \tag{8}$$

Equations (6) and (8) are the ultimate strength formulae of a K-type



(a) (b)

Fig. 18 $T_1,(-T_2)$ - u relationship



(a) (b)

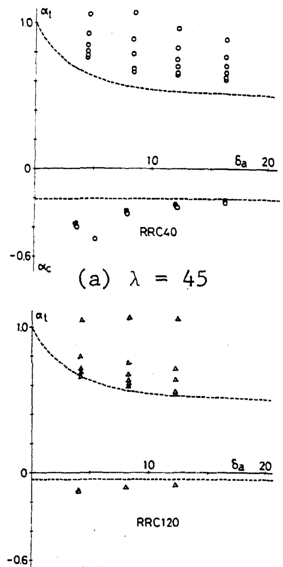
Fig. 19 Hysteresis loop of a single brace braced frame subjected to the repeated loading with the constant displacement amplitude, and are the same form as Eqs.(4) and (5), if $\alpha_t = 1$ and $\alpha_c = \alpha$.

α_t and α_c should be estimated according to the hysteretic behavior of a single brace. Fig. 20 shows some examples of the relationships between the axial force of a single brace at the reversal point and the displacement amplitude δ_a, obtained from the experiment of Ref. 2. α_t and α_c can be expressed, as follows, and illustrated by dashed lines in Fig. 20.

$$\alpha_t = \{ 1 + 1/(\delta_a/20 + 1)^6 \}/2 , \qquad \alpha_c = \exp(-2.526/\sqrt[3]{n_E}\ ) \tag{9}$$

In Eq.(9), n_E denotes the ratio of the Euler load of the brace to T_o.

Comparison with experimental results The experimental values of the strength of a K-type braced frame at the load reversal point are plotted against the normalized value $\Sigma\ \varepsilon_b$ of the cumulated displacement amplitude, in Fig. 21. The ordinate is normalized by P_o obtained by Eqs.(6) or (8). Solid marks jointed by straight lines show the experimental results presented here, and open marks are the results by other Japanese investigators. Proposed formula well predicts the strength of a K-type braced frame at the ultimate state subjected to severe reversed loading, with a few exceptions.



(a) λ = 45

(b) λ = 125

Fig. 20 α_t, α_c - δ_a relationship

CONCLUSIONS

1) Braces in a K-type braced frame is rarely stretched, and do not function well as the tension members. These characteristics are related to the property of the beam plastification, and the hysteretic behavior of a K-type braced frame is much dependent on the elastic-plastic behavior of the beam.

2) The mechanism of the beam plastification is also investigated, and it is suggested that the beam plastification may develop if the brace is stubby.

3) Ultimate strength formula of a K-type braced frame subjected to severe repeated loading is proposed on the basis of the knowledge on the hysteretic behavior of a single brace and the investigation on the beam plastification.

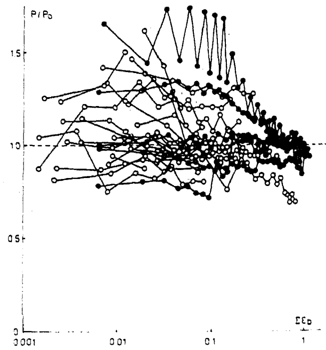


Fig. 21 P/P_o - $\Sigma\ \varepsilon_b$ relationship

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