EFFECTS OF AXIAL DEFORMATIONS AND FORCES OF THE COLUMNS ON THE DYNAMIC COLLAPSE OF STEEL FRAMES UNDER STRONG GROUND MOTIONS

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SUMMARY

The effects of the axial deformations and the axial forces of the columns on the dynamic collapse of multi-story plane steel frames under strong ground motion is clarified using the time to collapse dynamically derived from the plastic excursion analysis of steel frame models.

The restoring force characteristics of the steel frame models used in the analysis were decided basing on the tests of H-section steel columns under the repeated horizontal load and the constant axial load and the simple plastic analysis introducing the yield function of the column-end sections and the idealization of the column section by the two-flange section.

INTRODUCTION

When a multi-story steel frame is subjected to a strong earthquake motion, the horizontal displacement is accumulated in one direction and finally the frame collapses dynamically by losing the restoring forces by the excessive gravity effect. The dynamic collapse behavior is mainly effected by the degradation of the restoring force.

In the aseismic design of steel frames, the plastic axial deformation and the added axial forces of the columns ,caused by the shear forces of the beams, are usually neglected by the reason of the complexity of the analysis. But the plastic axial deformations of the columns cause to degrade the restoring force of the steel frame and, in some cases, they effect on the dynamic collapse behavior of steel frames under strong ground motion.(Ref.9),(Ref.10)

In this paper, the degradation of the restoring force of multi-story steel frames caused by the plastic axial deformations and the axial forces of the columns and the effects of them on the dynamic collapse behavior of steel frames are investigated.

TESTS OF STEEL H-COLUMNS

Specimens and Loading Apparatus

The specimens of steel H-columns are explained in Fig.1, Table 1 and Table 2. The end of the specimen is fixed and the other end, which allowed to move only in the axial direction by roller supports, is subjected to constant axial load P, as shown in Fig.2. The center of the specimen is supported not to rotate and to move only in the vertical plane and subjected to the alternating repeated horizontal load H. The variables of this test are the slenderness ratio λ, the axial force ratio P/Py and the hysteresis of loading, shown in Table 1. There are three kinds of the hystereses of loading, which are the monotonic loading (ML), the alternating loadings to increase the amplitude in every cycle (IL) and in every five cycles (SL).

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Test Results

The test results are shown in Fig.3 and Fig.4. Fig.3(A) and Fig.3(B) show the examples of the relations between the horizontal displacement δH and the axial deformation δV and the restoring force characteristics respectively. In these figures the dashed lines and the real lines distinct between the monotonic loading and the alternating loading and the chain lines show the full plastic deformation in Fig.3(A) and the mechanism line in Fig.3(B). The axial deformation δV includes the two kinds of deformations which are caused by the deflections and by the axial strain of the column. The axial deformation caused from the axial strain is approximately given by the value of δV when the horizontal displacement takes the value of zero. It is seen in Fig.3(A) that the axial deformation caused from the axial strain is accumulated with the loading cycles. The accumulated plastic axial deformations are summarized in Fig.4. The abscissa is the summation of the products of the axial force ratio P/Py devided by the slenderness ratio λx and the incremental plastic horizontal displacement devided by the column length ΔdHP.

As is seen in this figure the gradients of the curves are approximately constant and almost equal to each other. From these results it is pointed out that the accumulated plastic axial deformations are not effected by the hystereses of loading, which are (ML),(IL) and (SL).

ANALYSIS OF PLASTIC AXIAL DEFORMATION OF STEEL COLUMNS

The accumulated plastic strain of a steel column under the repeated axial load and the horizontal load has been analyzed in a few papers.(Ref.1)-(Ref.3) But in this paper, it is aimed that the simple relation between the plastic axial deformation and the plastic horizontal displacement of a steel column, which is suitable to analyze the bending deformation of steel frames, is derived. The following assumptions are introduced here.

a) A cantilever steel column is subjected to the axial load and the horizontal load at the free end, which deforms only in this plane. The bending moment along the column axis, including P-Δ effect, is distributed linearly.

b) The section of a steel column is idealized by the two-flange section whose sectional area and moment inertia are the same of the original section.

c) The mechanical material properties are given by the rigid-hardening type.

d) The local buckling and the shear deformation of the column are neglected.

The strain distributions εA and εB of the two concentrated flanges, named A-Flange and B-Flange here, are expressed by Eq.(1).

$$\varepsilon_A=-2d\xi_A'' \ , \ [0 \leqq z \leqq z_A]$$

$$\varepsilon_B= 2d\xi_B'' \ , \ [0 \leqq z \leqq z_B] \qquad (1)$$

in which

$\xi A, \xi B$: the horizontal displacement to cause the strain εA in A-Flange and the strain εB in B-Flange respectively,

2d : the distance between the two flanges,

z : the coordinate along the column axis,

zA,zB : the zone length where plastic strain occurs in A-Flange and B-Flange respectively,

' : differentiation with z.

According to the assumptions of the linear distribution of the bending moment and the bi-linear stress-strain relation, ξ_A'', ξ_B'' is a linear function of z and shown as Eq.(2).

$$\xi_A''=\frac{1}{2}\lambda\frac{\sigma_y}{E_p}(n_r+m_r-1)(1-\frac{z}{z_A})\frac{1}{l}$$
$$\xi_B''=\frac{1}{2}\lambda\frac{\sigma_y}{E_p}(-n_r+m_r-1)(1-\frac{z}{z_B})\frac{1}{l} \qquad (2)$$

where

σy : yield stress

Ep : Tangent stiffness in the stress-strain relation

nr : compressive axial force ratio to yield axial force

mr : bending moment ratio at the fixed end of the column to the full plastic moment

λ : slenderness ratio of the column

The ratio of the plastic axial deformation to the column length dVS and the ratio of the plastic horizontal displacement to the column length dHP at the free end of the column are given by Eq.(3).

$$d_{VS}=-\frac{1}{l}\left(\int_o^{z_A}\frac{1}{2}\varepsilon_A dz+\int_o^{z_B}\frac{1}{2}\varepsilon_B dz\right)$$
$$d_{HP}=\frac{1}{l}\{(\xi_A)_{z_A}+(1-z_A)(\xi_A')_{z_A}+(\xi_B)_{z_B}+(1-z_B)(\xi_B')_{z_B}\} \qquad (3)$$

By the use of Eq.(1), Eq.(2) and the boundary conditions at the fixed end of the column, Eq.(4) is derived.

$$d_{VS}=\frac{\sigma_y}{Ep}n_r\frac{m_r-1}{m_r}$$
$$d_{HP}=\frac{\sigma_y}{Ep}\frac{1}{6}\lambda\{2m_r-3+\frac{1}{m_r^2}(1+3n_r^2)\} \qquad (4)$$

From the rate equations of Eqs.(4), we get

$$\Delta d_{VS}=\alpha\frac{1}{\lambda}n_r\Delta d_{HP} \quad \text{where } \alpha=\frac{3m_r}{m_r^3-(1+3n_r^2)} \qquad (5)$$

ΔdVS,ΔdHP : the increments of dVS,dHP respectively.

The value of α in Eq.(5), which is related to the axial force ratio nr and the strain hardenig ratio, is assumed to be constant here. Under this condition it is approximately expressed by Eq.(6) from the test results shown in Fig.4.

$$\alpha = 3.0 \qquad (6)$$

The result of Eq.(5) and Eq.(6) is shown in Fig.4 with the chain line. It is a fairly good approximation with the test results.

LOAD-DEFORMATION RELATION OF STEEL FRAMES

A multi-story plane steel frame model, shown in Fig.5, is analyzed under the following assumptions:

a) The frame is rigid except the two columns corresponding to the first story columns.

b) The both ends of the columns are yielded.

When the equivalent resultant reactions from the base, working at the point 0, are expressed by Mo,No,Qo, the equilibrium equations are given as follows:

$$
\begin{aligned}
&f_x+q_o=0\\
&f_z+n_o=0\\
&f_x m_p(\kappa\cos\theta+\cos\theta_c)-\frac{1}{2}f_z u+\frac{1}{4}\beta m_o=0
\end{aligned}
\tag{7}
$$

where $\beta=l_B/l$, $\kappa=L/l$, $u=\delta/l$, $m_p=M_p/(N_y l)$

$m_o=M_o/(\beta N_y l)$, $n_o=N_o/(2N_y)$, $q_o=Q_o l/(4M_p)$

$f_x=F_x l/(4M_p)$, $f_z=F_z/(2N_y)$

Mp : full plastic moment
Ny : yield axial force

The other notations, used in Eqs.(7), are explained in Fig.5.

The M-N interaction equation of the column-end section is assumed to be expressed by Eq.(8).

$$\frac{|M|}{M_p}+c\left(\frac{N}{N_y}\right)^2=1 \tag{8}$$

in which c : constant.

From the four M-N interaction equations at the four column-end sections and expressing the column-end forces by mo,no,qo, the relation among mo,no,qo is shown by Eq.(9).

$$c\left(m_o-2\frac{m_p}{\beta}q_o\right)^2+(2c\bar{d}_H)m_o-\left(1+4c\frac{m_p}{\beta}\bar{d}_H\right)q_o+\left(-1+cn_o^2+\frac{1}{2}\frac{\beta}{m_p}\bar{d}_H+c\bar{d}_H^2\right)=0 \tag{9}$$

in which $\bar{d}_H=n_o*\theta c/\beta$

The horizontal displacement of the gravity center G, shown in Fig.5, is given by Eq.(10).

$$u=\sin\theta_c+\kappa\sin\theta\doteqdot\theta_c+\kappa\theta \tag{10}$$

Since the bending deformation of the frame θ is given by Eq.(5), u becomes to be a function of $\bar{d}_H$ as follows:

$$u=\left\{1+\frac{1}{2}en_m+e\left(-\bar{d}_H-m_o+2\frac{m_p}{\beta}q_o\right)\right\}\frac{\beta}{n_o}\bar{d}_H \quad \text{where } e=\frac{\kappa}{\beta}\frac{\alpha}{\lambda} \tag{11}$$

nm : the difference between the two axial force ratios of the columns when $\bar{d}_H = 0$. From Eq.(7),Eq.(9) and Eq.(11), fx is expressed by a function of $\bar{d}_H$.

$$
\begin{aligned}
f_x=\frac{1}{8}&\left(\frac{1-2e\bar{d}_H}{1+2\kappa}\frac{\beta}{m_p}\right)\left[\left\{\frac{16}{c}\left(1-cn_o^2+\frac{1}{2}\frac{en_m-2\kappa}{1+2\kappa}\frac{\beta}{m_p}\bar{d}_H\right)+\left(\frac{1}{c}\frac{1-2e\bar{d}_H}{1+2\kappa}\frac{\beta}{m_p}\right)^2\right\}^{\frac{1}{2}}\right.\\
&\left.-\frac{1}{c}\left(\frac{1-2e\bar{d}_H}{1+2\kappa}\frac{\beta}{m_p}\right)\right]-\frac{1}{2}\frac{1+en_m}{1+2\kappa}\frac{\beta}{m_p}\bar{d}_H
\end{aligned}
\tag{12}
$$

The relation between the horizontal load and the horizontal displacement of the gravity center G of the frame model is derived eliminating $\bar{d}_H$ from Eq.(11) and Eq.(12) and adding the elastic deformation to the plastic deformation.

As an example the load-deformation relation is shown in Fig.6, where the effects of the plastic axial deformations and the axial forces on the restoring forces of the frames are compared. In Fig.6, B-Frame shows the calculation where the axial deformations and the added axial forces are both included, S-Frame shows the one in which the axial deformations are neglected. In the calculation of PS-Frame, the axial deformations and the added axial forces are not considered.

TIME TO COLLAPSE DYNAMICALLY

To clarify the effects of the plastic axial deformations and the axial forces of the columns on the response of steel frames under strong ground motion, the dynamic collapse of steel frame models, shown in Fig.5, is calculated.

The equation of motion of the frame, expressed by a function of δ, is given by Eq.(13). In this equation the damping force is neglected.

$$\left\{M_s+\frac{I}{l^2}\,\frac{1}{(1+\kappa)\kappa}\,\frac{\gamma}{1+\gamma}\right\}\ddot{\delta}+4\,\frac{M_p}{l}q_x=-M_s\ddot{\delta}_g \quad \text{in which } q_x=f_x \tag{13}$$

Ms : mass of the frame,
I : rotatory moment of inertia,
$\ddot{\delta}g$: acceleration of ground motion,
γ : ratio of u to θc. The value of γ, which is decided by Eq.(11), is nearly constant and given by the value in case of $\bar{d}\bar{H}=0$.

The dynamic collapse of steel frames under strong ground motions has been studied in many papers.(Ref.4)-(Ref.11) In this paper the simplest analysis method among them is used, in which the behavior that the frame model vibrates under the sine wave ground motions is analyzed. The frequency of the sine wave is equal to the initial natural frequency of the frame model. The numerical analysis was excuted by the plastic excursion analysis method (Ref.11) to simplify the calculation. The restoring force characteristics of the frame model is idealized with the bi-linear model of the Masing-type, as defined with the dashed lines in Fig.6. According to this method the mechanism lines in Fig.3(B) are corresponding to the restoring forces of PS-Frames.

As an example the time histories of the horizontal displacements of B-Frame,S-Frame and PS-Frame are shown in Fig.7. In this figure $(\delta/\delta y)B$,$(\delta/\delta y)S$ and $(\delta/\delta y)PS$ show the ductility ratios when the restoring forces of B-Frame,S-Frame and PS-Frame become zero respectively. As is seen in this figure the horizontal displacements drift with time and the frames collapse when they reach to the values of $(\delta/\delta y)B$,$(\delta/\delta y)S$ and $(\delta/\delta y)PS$ respectively. The times when the horizontal displacements of the frame reach to $(\delta/\delta y)B$,$(\delta/\delta y)S$ and $(\delta/\delta y)PS$ respectively, which are expressed by (tcr)B,(tcr)S and (tcr)PS respectively, are used in this paper to explain the effects of the axial deformations and the axial forces of the columns on the dynamic response behavior of steel frames under strong ground motion.(Ref.4)

The numerical results are shown in Fig.8 and Fig.9. In these results the amplitude of the acceleration of the ground motion in every case is the value of Qpc/Ms (Qpc : the full plastic shear strength of PS-Frame). The values of tcr in Fig.8 and Fig.9 are effected by the amplitude of the accelerations of the ground motions but the ratios of (tcr)B and (tcr)S to (tcr)PS are not almost influenced by it as is seen in Fig.10.

CONCLUSION

1) The plastic axial deformation of steel H-columns subjected to the axial forces and the horizontal forces and bent about the strong axis is approximately given by Eq.(5) and Eq.(6) under the monotonic loading and also under the repeated loading.

2) The added axial forces of the columns do not almost influence the aseismic capacity ratio of steel frames.

3) The plastic axial deformations of the steel columns lower the aseismic capacity ratio of steel frames as the constant axial force ratio of the columns caused by the dead load and the height of the frames increase.

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Table 1 Test Specimens

Specimen	1 (cm)	B/tf	H/tw	λx	λy	Mpx (t.cm)	Mpy (t.cm)	P/Py	Hysteresis
H-205-M	83.5	12.8	17.4	19.7	33.5	286.7	133.5	0.5	(ML)
H-305-M	125.3	13.4	17.0	30.5	51.0	249.5	117.6	0.5	(ML)
H-405-M	167.3	13.5	17.1	40.8	68.1	248.3	117.2	0.5	(ML)
H-505-M	208.9	12.8	17.5	49.3	83.8	285.8	133.1	0.5	(ML)
H-201-I	83.5	12.8	16.8	20.2	33.8	267.4	126.5	0.1	(IL)
H-203-I	83.5	13.1	16.7	20.2	33.8	262.5	123.9	0.3	(IL)
H-205-I	83.5	13.3	16.9	20.2	33.9	259.4	122.1	0.5	(IL)
H-301-I	123.2	13.1	16.5	30.0	50.0	261.5	123.9	0.1	(IL)
H-303-I	123.4	12.9	16.4	30.0	50.0	258.9	122.8	0.3	(IL)
H-305-I	123.4	12.9	16.1	30.0	50.2	260.7	123.1	0.5	(IL)
H-401-I	167.0	11.6	16.4	39.8	67.5	315.7	147.9	0.1	(IL)
H-403-I	167.1	13.8	17.2	40.0	68.4	261.4	119.3	0.3	(IL)
H-405-I	167.1	11.6	16.0	40.0	67.6	316.3	148.2	0.5	(IL)
H-401-S	167.0	13.1	16.6	40.4	67.8	258.5	121.6	0.1	(SL)
H-403-S	167.1	13.0	16.9	40.4	67.6	258.5	122.0	0.3	(SL)
H-405-S	167.1	13.5	17.2	40.7	68.0	252.8	119.0	0.5	(SL)

Mpx (Mpy) : Full plastic moment
λx (λy) : Slenderness ratio
H : Height
B : Width of flange
1 : Length of specimen
P/Py : Axial force ratio
tf : Thickness of flange
tw : Thickness of web

Table 2 Mechanical Properties

Flange			Web		
σ_y (t/cm²)	σ_u (t/cm²)	$\varepsilon_{st}/\varepsilon_y$	σ_y (t/cm²)	σ_u (t/cm²)	$\varepsilon_{st}/\varepsilon_y$
3.18	4.74	11	3.34	4.87	12

σ_y : Yield stress
σ_u : Ultimate tensile stress
ε_y : Yield strain
ε_{st} : Strain-hardening strain

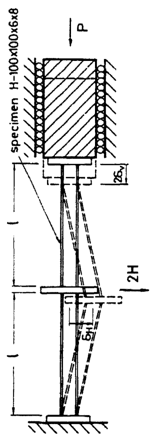

Fig.2 Schematic Diagram of Loading

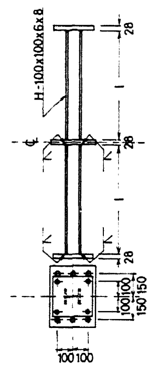

Fig.1 Specimen

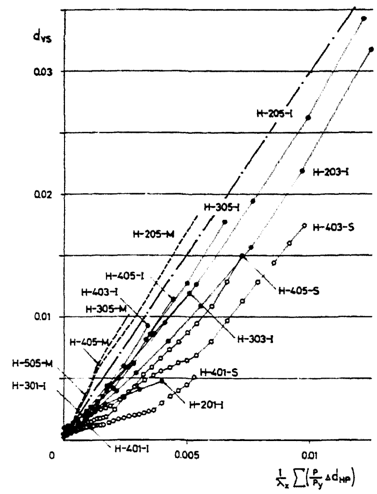

Fig.4 Accumulated Plastic Axial Deformation

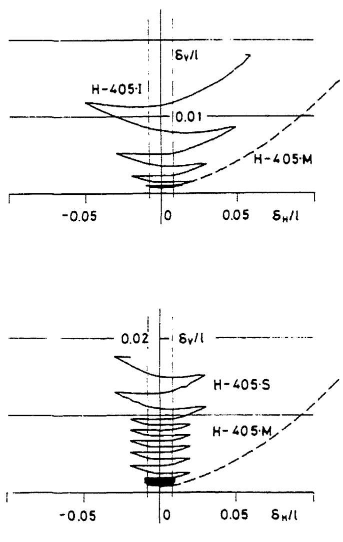

Fig.3(A) Relation between Horizontal Displacement and Axial Deformation

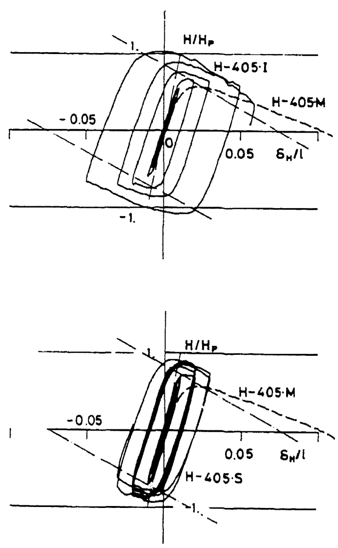

Fig.3(B) Restoring Force Characteristics

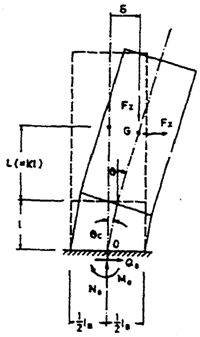

Fig.5 Analyzed Steel Frame Model

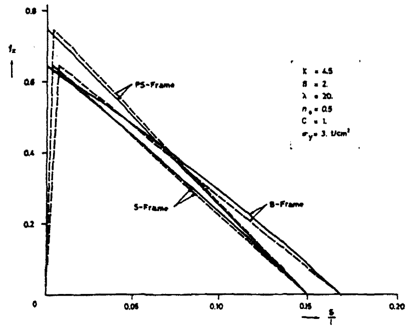

Fig.6 Load-Deformation Relation

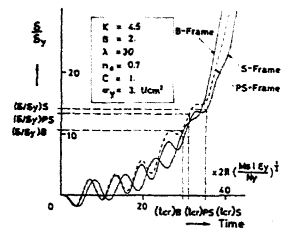

Fig.7 Time History of Horizontal Displacement

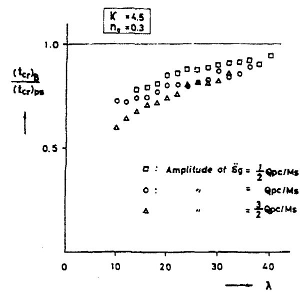

Fig.10 Time Ratio to Collapse Dynamically
(β=2,c=1,σy=3t/cm2)

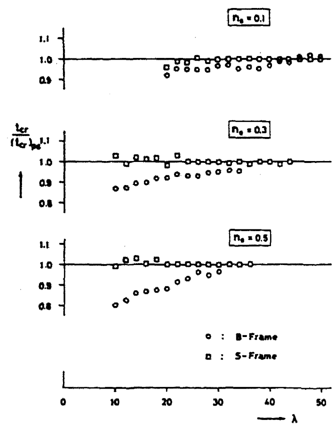

Fig.8 Time Ratio to Collapse Dynamically
(κ=2,β=2,c=1,σy=3t/cm2)

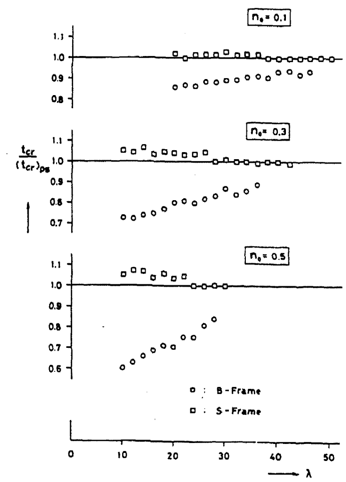

Fig.9 Time Ratio to Collapse Dynamically
(κ=4.5,β=2,c=1,σy=3t/cm2)