CYCLIC STRESS-STRAIN CURVES OF CONCRETE* CONFINED WITH SPIRALS

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SUMMARY

To predict the response of concrete structures subjected to seismic excitation, constitutive models of concrete under cyclic loading are necessary. To develop theoretical models, experiments were conducted on confined concrete subjected to stress as well as incremental strain cycling. The concept of envelope curves was observed to be valid for confined as well as unconfined normal weight, high strength and light weight concrete provided the rate of loading is sufficiently rapid so that cyclic creep is not critical.

A rheological stochastic model is proposed to predict the response of confined concrete subjected to any arbitrary loading history. The three parameters of the model can be calibrated from the envelope curve, and no prior cyclic tests are necessary.

INTRODUCTION

Concrete subjected to cyclic loading represents an important class of problems in structural engineering when designing structures such as buildings subject to earthquake or wind loads, bridges subjected to vehicle loading and offshore structures loaded by sea-waves.

To predict the ultimate behavior of reinforced and prestressed concrete structures subject to this kind of loading, constitutive relationships of concrete under cyclic loading are needed. Plain concrete subjected to cyclic loading exhibits internal microcracking, cyclic creep and path dependency [1]. A constitutive model that would incorporate all the known characteristics of concrete is not available today. Instead, there are a number of models which simulate with acceptable accuracy the response of concrete to particular types of loading. For example, design of columns and beams can often be handled by modeling the concrete behavior in uniaxial loading. This important problem is simplified by the concept of the "envelope" curve which is a curve in the σ-ε space that is never crossed by any uniaxial loading history. It has been postulated that the envelope is unique for a given type of concrete and that it approximately coincides with the stress-strain curve obtained under monotonically applied loading [2]. If this hypothesis is valid, then the complete stress-strain curve provides a basis for a rational and simple analysis for the response of many types of concrete structures subjected to seismic excitation [3, 4, 5].

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The validity of the concept of the envelope curve was primarily based on data from plain, unconfined concrete. The purpose of the research reported in this paper is to examine the validity of the concept for spirally confined concrete and to develop an analytical model for the envelope curves as well as for the response of unconfined and confined concrete subjected to cyclic loading.

EXPERIMENTAL PROGRAM

To examine the validity of the envelope concept for confined normal weight and lightweight concrete, 127 concrete specimens were tested in monotonic and cyclic loading under two different strain rates (32 μstrains/sec and 15000 μstrains/sec). [6, 7].

The specimens were 3x6 in. cylinders without longitudinal reinforcement. The confined specimens had practically no cover and the spiral was fabricated from 1/8 in. diameter wires with a yield strength of 60 ksi (413 MPa). The amount of confinement was changed by varying the spiral spacing; two different spacings 1 and 1½ in. were used. Note that the volumetric ratio of 1 in. spacing is more than the minimum required by the American Concrete Institute for earthquake loading and for concrete strength about 6000 psi (41MPa) whereas the 1½ spacing provides less volumetric ratio than the ACI requirement [ACI-318-77, Sec. A.6.5.2].

The specimens were loaded in a closed-loop testing machine. During strain cycling a fixed amount of strain was imposed during each cycle. A typical stress-strain curve obtained for this type of loading is shown in Fig. 1. During stress cycling the specimens were repeatedly loaded between a fixed value of maximum and minimum stress (Fig. 2).

From the strain controlled cyclic stress-strain curves an envelope curve can be drawn (Fig. 1). A range of envelope curves was obtained from the results of cyclic loading of identical specimens and were then compared with the range of the complete stress-strain curves of the corresponding control specimens obtained from the monotonically increasing loading (Fig. 1). It can be seen that the range of these two sets of curves are essentially the same. Similar results were obtained from other tests of normal and lightweight concrete and also of various strengths (f'_c) and spiral spacing as well as unconfined specimens [6].

A comparison of stress controlled cyclic stress-strain curve with the monotonic curve of a confined normal weight concrete is shown in Fig. 2. Point A in this figure is defined as the largest value of strain when it was still possible to apply the programmed maximum stress value and point B is defined as the value of strain when the maximum stress was 95% of the programmed value. Comparison of these two points with the range of monotonic curves is shown in Fig. 3 where various ranges of loading employed are also reported. The behavior of specimens with different concrete strength (f'_c) and confinement as well as of lightweight concrete was similar and thus, it was concluded that the envelope concept is valid for stress controlled tests and high strain rates. For low strain rates, however, it was found that for some specimens the failure strain exceed the envelope (Fig. 4). Similar phenomenon was also reported by Maher and Darwin [8] and is probably due to the creep deformations and incremental damage. In earthquake, however, the strain rates are much higher than the strain rates of these slow tests [5]. Therefore, for all practical purposes the concept of envelope curve seems to be valid for unconfined as well as confined concrete and also for lightweight concrete.

ANALYTICAL MODEL

Based on the concept of envelope curve many researchers have developed models to predict cyclic stress-strain curves [9, 10]. Most of these models require several sets of equations and a number of constants which may not have any physical meaning.

In this investigation a rheological, stochastic model is proposed for uniaxial cyclic loading. The three constants needed to calibrate the model can be determined from the monotonic stress-strain curve. Use of rheological models to predict hysteretic behavior of metals was proposed by Timoshenko [11]. He used a model consisting of Jenkin's elements (an elastic spring in series with a Coulomb element) connected in parallel. Similar models were used by other investigators to study the dynamic behavior of metals [12] and to predict the response of concrete and rocks subjected to cyclic loading [13, 14].

Four-Element System

The principal characteristics of the proposed model can be easily understood by considering a four element system as shown in Fig. 5. Each element consists of a spring and a slider connected in series and the elements are connected in parallel. The σ-ε curve of a typical element is shown in Fig. 5b and it is characterized by three parameters; the spring constant K, the strain η which is the elastic limit of the element and the strain θ which is the total plastic strain that the element is allowed to undergo (in either direction) and after which the element fractures and loses its capacity to resist deformation. The θ-parameter is associated with the irreversible work consumed by the slider.

At any strain (ε) the stress of the system is equal to the sum of the stresses of those elements that have survived the given loading history. In Fig. 5c are shown the σ-ε curves of the four elements subject to a loading history and in Fig. 5e is shown the response of the system. As the elements enter their plastic stage the system exhibits knees (nonlinearities, e.g. points 1, 2, 7, 8). If the loading is monotonic the elements break progressively and we have a progressive drop of the system stress (strain softening). In cyclic loading when elements break we also have a drop in the system stress (Fig. 5e, point 9) and as a result, the system does not reach the point at which the unloading had commenced and the reloading branch meets the unloading branch at a lower point (common point). Due to progressive failure of elements we also have a decrease of stiffness (stiffness degradation). With increasing the strain, the stress tends to the envelope curve (dotted line in Fig. 5e).

In a system with N elements the spring constant of each element is $K = E_c/N$ where E_c is the modulus of elasticity of concrete. The other two parameters vary from element to element and they are drawn from two independent populations each with an exponential distribution, in a random fashion. The η-parameter of the i-th element is

$$\eta_i = \frac{1}{b} \ln (1 - p) \tag{1}$$

where p is a random number ($0<p<1$) drawn from the p-population (all values of which are equally likely), and b is the constant of the exponential distribution of the η-population. The θ-parameter of the same (i-th) element is

$$\theta_i = - \frac{1}{a} \ln (1 - p) \tag{2}$$

where p is another random number from the same p-population and a is the constant of the distribution of the θ-parameter. The η- and θ-parameters of the four-element system are shown in Fig. 5d. Note that the system is defined by three parameters, a, b, and K (that is E_c).

Continuous System

It is seen that the simple system of four elements captures all the main characteristics of concrete behavior. The discontinuities of the model response can be eliminated by increasing the number of elements. In the limit, when the number tends to infinity, we have the continuous case with spring constant $dk = Kdp$, where dp is an infintesimal interval in the p-population mentioned earlier. The elements can be arranged according to their ascending order of their η-parameters (Fig. 6). For a given strain ε, the elements having $\eta > \varepsilon$ behave elastically; those are the elements with p values varying between $1 - e^{-b\varepsilon}$ and $p = 1$ (Fig. 6) and the stress contribution of these elements is given by:

$$\sigma_e = \int_{1-\exp(-b\varepsilon)}^{1} K\varepsilon dp = K\varepsilon \int_{1-\exp(-b\varepsilon)}^{1} dp = K\varepsilon e^{-b\varepsilon} \tag{3}$$

The elements with $\eta < \varepsilon$ have yielded and are in their plastic stage. However, some of them may have already broken at this strain and, therefore, they do not make any contribution to the systems resistance.

We define the density $\lambda(p)$ as the ratio of the active elements to the total number of elements in a small interval Δp ($\Delta p \to dp$) as shown in Fig. 6a. To determine the number of elements that are active in Δp we rearrange these elements within Δp according to the ascending order of their θ-parameter as it is shown in Fig. 6b. (Note that within Δp the n-parameters have approximately constant value which tends to $\eta(p)$ as Δp tends to infinetisimal dp). Since the two parameters η and θ are independent, the distribution of θ in the interval Δp is the same as the distribution of the population from which θ was drawn (θ-population), that is, exponential. From Fig. 6 it is clear that the elements that are active are those that have $\eta + \theta > \varepsilon$, therefore, the density is

$$\lambda(p) = \frac{\Delta p_1}{\Delta p} = 1 - \frac{\Delta p_2}{\Delta p} \tag{4}$$

but from Fig. 6b it is seen that

$$\frac{\Delta p_2}{\Delta p} = 1 - (1 - p)^{-a/b} e^{-a\varepsilon} \tag{5}$$

and finally

$$\lambda(p) = (1 - p)^{-a/b} e^{-a\varepsilon} \tag{6}$$

The density variation is shown in Fig. 6.

It can be shown that the density $\lambda(p)$ is the probability that an element has to survive a given strain ε. Obviously the density of the elastic elements is $\lambda(p) = 1$ (Fig. 6).

The stress contribution of an element in its plastic stage is $d\sigma = K\eta\ dp$ and the total contribution of the plastic elements that are active is

$$\sigma_p = \int_0^{1-\exp(-b\varepsilon)} \lambda(p)\ K\eta(p)\ dp$$

and substituting $\lambda(p)$ and $\eta(p)$ from Eqs. 6 and 1 respectively, we find

$$\begin{aligned}\sigma_p &= -\frac{Ke^{-ae}}{b} \int_0^{1-\exp(-b\varepsilon)} (1 - p)^{-a/b}\ \ell n(1 - p)\ dp \\ &= \frac{Ke^{-ae}}{(a-b)} \left[\left(b\varepsilon - \frac{b}{a-b}\right) e^{(a-b)\varepsilon} + \frac{b}{a-b} \right]\end{aligned} \tag{7}$$

The total stress of the system in monotonic loading is given by the sum of Eqs. 3 and 7: $\sigma = \sigma_e + \sigma_p$.

The derivation of the system response subject to cyclic loading is, in principle, possible but more complicated due to discontinuity of the element $\sigma - \varepsilon$ curve. One alternative is to construct a system with adequately large number of elements and by a computer porgram to keep track of each element's response. The response of such a system consisting of three hundred elements is shown in Fig. 7 which is a simulation of the experimental data of Fig. 1.

The Proposed Rheological Element

To overcome the difficulty stemming from the discontinuity, an element with continuous stress-strain curve was introduced. The monotonic $\sigma - \varepsilon$ relationship of the element is

$$\sigma = K\eta(1 - e^{-K\varepsilon}) \tag{8}$$

with an asymptote at $\sigma = K\eta$. The response of the element to cyclic loading is shown in Fig. 8. The η-parameter in Eq. 8 is a random parameter with exponential distribution as in the elastic-plastic elements. The second parameter of the new proposed element is the critical work (Wcr) which is the work that the element absorbes before it breaks (Fig. 8) and has an exponential distribution similar to θ-parameter. With these new element it is possible to derive explicit formulas for the cyclic response of a continuous system subject to any arbitrary loading history. The formulas are in closed form and of the total strain type. The response of the system consisting of an infinite number of proposed elements subjected to the same loading history as the experiment of Fig. 1 is shown in Fig. 9. More details of the derivation of the formulas giving the stress in loading and unloading during any cycle are given in Ref. 15.

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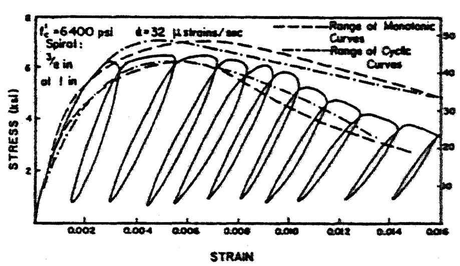

Fig. 1 - Strain Cycling Concrete

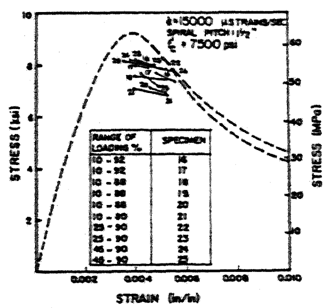

Fig. 3 - Fast Loading

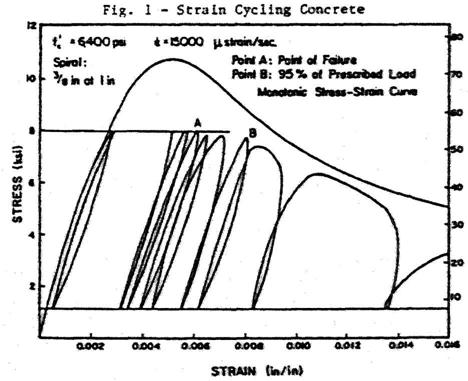

Fig. 2 - Stress Cycling Concrete

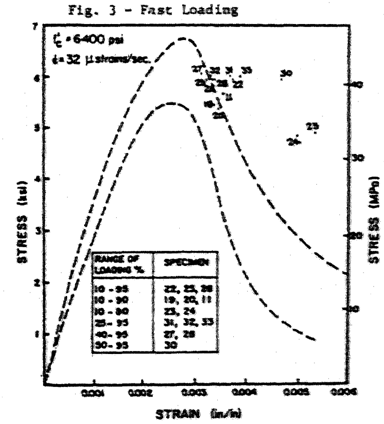

Fig. 4 - Slow Loading

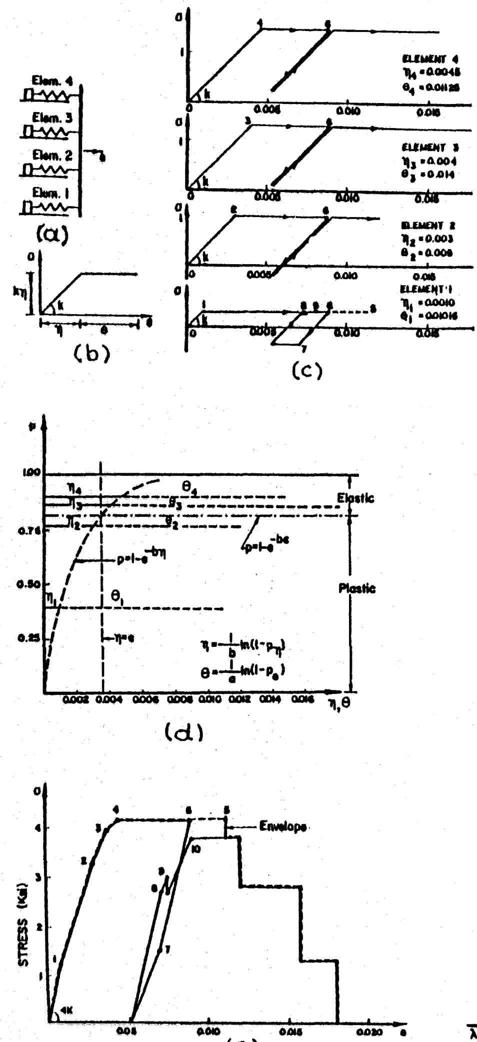

Fig.5 Four-Elem. System

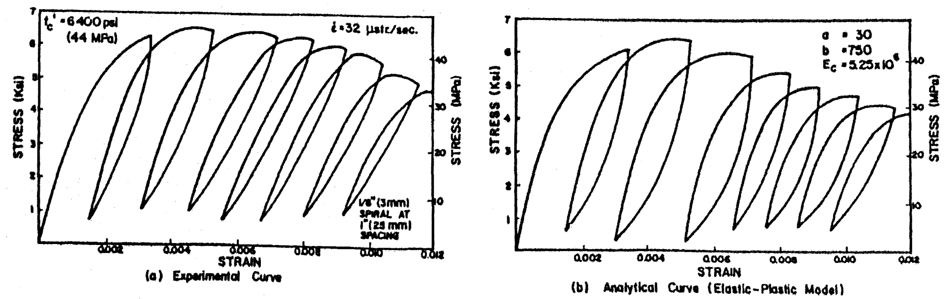

Fig. 7 Experimental and Analytical Curves

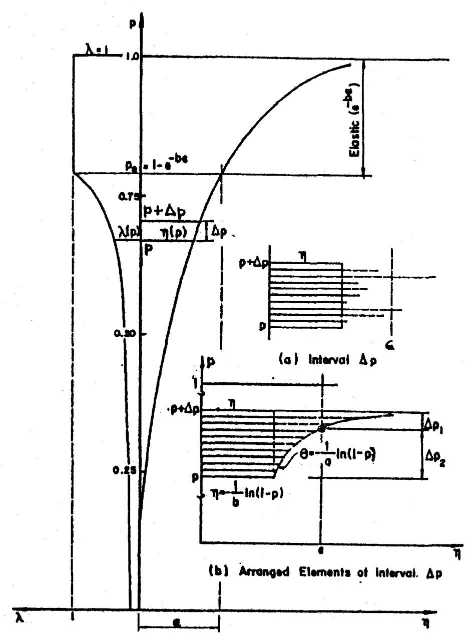

Fig. 6 Density Distribution

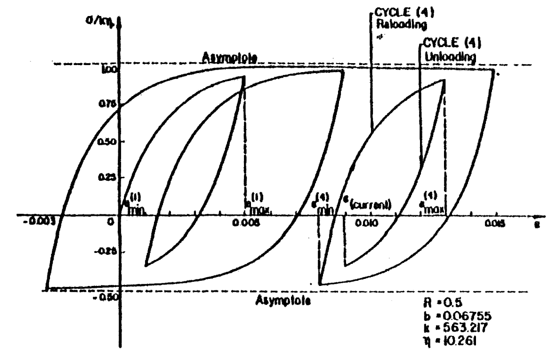

Fig.8 Element Response

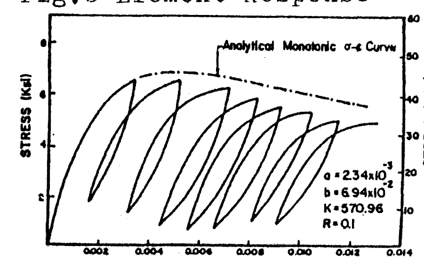

Fig.9 Model Response