CONFINED CONCRETE SUBJECTED TO AXIAL AND BENDING LOADS

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SUMMARY

This paper presents a procedure to analyze reinforced concrete sections confined by rectilinear ties. An analytical model originally developed for the determination of the stress-strain curve of confined concrete under concentric compression has been extended to include the effects of flexural strain gradient. In order to simplify the application of the model a rectangular stress block representing the actual stress-strain curve is proposed. The provisions of the Appendix A of the ACI Code (318-77, 83) for the confinement steel in ductile frame columns are critically examined.

CONFINEMENT MODEL

In the model proposed (Ref. 1) for concrete confined by rectilinear ties, the increase in concrete strength is calculated on the basis of an effectively confined concrete area, which is less than the core concrete area enclosed by the center line of the perimeter tie. The area of this effectively confined concrete is a function of the distribution of longitudinal and lateral reinforcement. Closer spacing of both the steels results in a higher proportion of the effectively confined area. A detailed development of the stress-strain curve for confined concrete *sections subjected to concentric compression is given elsewhere* (Ref. 1). The suggested curve OABCDE is shown in Fig. 1, where f_{cc} = strength of confined concrete and f_{cp} = strength of plain concrete. The governing equations for a square section are given below:

$$K_s = 1.0 + \frac{2.73\ B^2}{P_{occ}} \{ (1 - \frac{nC^2}{5.5\ B^2})(1 - \frac{s}{2B})^2 \} \sqrt{\rho_s f'_s} \tag{1}$$

$$\varepsilon_{s1} = 0.55\ K_s f'_c \times 10^{-6} \tag{2}$$

$$\frac{\varepsilon_{s2}}{\varepsilon_{oo}} = 1 + \frac{0.81}{C} (1 - 5.0(\frac{s}{B})^2) \frac{\rho_s f'_s}{\sqrt{f'_c}} \tag{3}$$

$$\varepsilon_{s85} = 0.225\ \rho_s \sqrt{\frac{B}{s}} + \varepsilon_{s2} \tag{4}$$

where B = core size measured to the center line of the perimeter tie; C = the distance between the laterally supported longitudinal bars; s =

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spacing of the sets of ties; ρ_s = volumetric ratio of tie steel to the core volume; f_s' = stress in tie steel; P_{occ} = f_{cp} (A_{cc}); A_{cc} = area of concrete in the core; and ε_{oo} = strain corresponding to the maximum stress in plain concrete. All the linear dimensions and areas are in inch units. In Eq. 1 the stresses are in kips per square inch. In Eqs. 2 and 3 the stresses are in pounds per square inch.

Effect of Strain Gradient

For columns subjected to axial load only the model described above predicts the experimental results in a satisfactory manner (Ref. 2). The analytical results for sections under combined axial and flexural loads were, however, found to be consistently conservative at higher curvature values although the predictions from this model were better than those from other available models. At lower curvatures the predictions were quite accurate. In addition to confining concrete, the transverse reinforcement in columns also needs to be provided to insure that shear strength is at least equal to the applied shear at the formation of plastic hinges in the frame. If the moment capacities of the plastic hinges are underestimated, the shear reinforcement will be designed for the shear force which may be much less than the maximum shear force that can develop during the earthquake. This may result in a brittle failure in the structure.

Studies on the effects of strain gradient on the behavior of unconfined and confined concretes have shown varied and often contradictory results (Ref. 3,4,5). In some tests eccentrically loaded specimens showed higher strength and ductility over concentrically loaded specimens (Ref. 5), while in others only larger strains were observed in eccentrically loaded specimens (Ref. 3,4). Based on the analysis of the limited test data available in the literature, the effect of strain gradient has been incorporated in the proposed model following the pattern of Sargin's equation (Ref. 3). Only the equation for ε_{s2} is affected by this change. The modified equation is given below:

$$\frac{\varepsilon_{s2}}{\varepsilon_{oo}} = 1 + \{\frac{0.81}{C} (1-5.0(\frac{s}{B})^2) + 0.25 \sqrt{\frac{B}{c}}\} \frac{\rho_s f_s'}{\sqrt{f_c'}} \tag{5}$$

in which c = depth to the neutral axis from the center line of perimeter tie.

The effect of this modification on the stress-strain curve of confined concrete is shown in Fig. 1. The curve OABCDE is for confined concrete under concentric compression, while for eccentrically compressed concrete the stress-strain curve is OAB'C'D'E. The additional strain BB' represents the effect of strain gradient. The ε_{s85} for eccentric compression is also increased by the same amount over ε_{s85} for concentric compression.

Application

The proposed model was used to predict the results of the tests reported by Scott et al (Ref. 6). The analytical results are compared with the experimental results in Figs. 2 and 3 for one of the specimens. The 1.2 m. long specimen was tested under monotonically increasing eccentric load with an end eccentricity of 49 mm. (1.93 in.). The results were presented in the form of load-strain and moment-strain curves. An analytical procedure using the proposed model will give numerous combinations of strain profile along the section depth, axial load (P) and moment (M). The P,M combinations are very sensitive to small variations in the strain profiles. To ascertain the accuracy of the analytical procedure two approaches have been attempted. Firstly, capacity P-M interaction curves were developed and compared with the column capacities observed during the test (Fig. 2). Secondly, experimental and analytical P vs. ε_c, M vs. ε_c and M vs. ϕ relations were compared (Fig. 3) where ε_c is the extreme fibre compressive strain and ϕ is the curvature of the section. The ε_c values in the analytical strain profiles were equal to the measured ε_c values and the neutral axis depth was slightly varied from the measured values to compute the load combinations comparable to the experimental results. The ε_c values were chosen at or beyond the peak stress on the confined concrete stress-strain curve. The concrete strength in unreinforced specimen, cast and tested in a manner similar to the reinforced column was reported to be equal to 0.86 f'_c. The analytical results are, therefore, evaluated using $f_{cp} = 0.86\ f'_c$. The predictions from the proposed model agree well with the experimental results. The analytical results given by the modified Kent and Park model (Ref. 6) and the ACI method are also shown in Figs. 2. Similar comparisons of experimental and analytical results were observed for other specimens.

Since the proposed stress-strain curve of confined concrete is a function of strain gradient, determination of capacity interaction curves becomes a lengthy and cumbersome procedure involving a number of trials. It is suggested that two interaction curves should be drawn for $\varepsilon_c = \varepsilon_{s85}$. In one curve ε_{s85} should be determined assuming concentric compression and in the other ε_{s85} should be calculated by assuming the neutral axis at the center of the concrete core. The envelope curve drawn over the two interaction curves would then be the required capacity interaction curve for the section.

ACI CONFINEMENT REQUIREMENTS

The proposed model was also used to study the confinement requirements of the Appendix A of the ACI Code (318-77, 83) (Ref. 7) Four sections designed according to the Code requirements are shown in Fig. 4. For the purpose of direct comparison of the behavior of the sections, commercial availability of different sizes of steel bars was not a consideration in their selections. Strain hardening of steel was not considered. The cover concrete was assumed effective up to a strain of ε_{50u} (strain corresponding to 50% of the maximum stress in cylinder on the descending part of the curve).

The moment-curvature behaviors of the sections were studied for various levels of axial loads. Fig. 4 shows the results for only two levels of axial loads. The moments are non-dimensionalized with respect to M_u calculated for unconfined sections at an extreme fibre compressive strain of .003 according to the ACI assumptions. Considering unconfined gross sections the balanced loads in all the cases are approximately equal to 0.37 f_{cp} A_g.

Although the code does not differentiate between various steel configurations, the efficiency of confinement in section OA is considerably less than that in other sections. Sections B and D show the most ductile behavior. Depending on what is expected of a column when subjected to a severe earthquake and the perception about how much detailing is needed in reinforced concrete members, the ACI requirements may be either too severe for columns with well distributed steel or unsafe for columns with only one tie at each level or both.

EQUIVALENT STRESS BLOCK

In the evaluation of flexural behavior of a reinforced concrete or prestressed concrete section, an iterative procedure is commonly employed to find the depth of the neutral axis required for the internal forces to balance the external applied load. When the stress-strain curve of concrete is complicated and dependent on several factors including the depth of the neutral axis, this procedure becomes extremely tedious and cumbersome. A simplified procedure is suggested below for the application of the proposed model. The simplification is made on the pattern of the accepted practice of replacing the actual stress-strain curve up to a certain point with an equivalent rectangular stress-strain curve with dimensions βf_{cc} and $\alpha \varepsilon_{cc}$. The equivalence of the two curves requires equal areas and equal distances from the stress axis to the centroids of the respective areas.

The stress-strain curve is divided into four zones as shown in Fig. 1. The equivalent stress blocks are also shown. The following procedure is worked out for the first three zones only since most of the applications are limited to these zones. To generalize the procedure, three new terms are used which are defined as follows:

$$\Omega = \varepsilon_{cc}/\varepsilon_{s1}; \; D = \varepsilon_{cc}/\varepsilon_{s2}; \; G = \varepsilon_{cc}/\varepsilon_{s85}$$

The equations for the determination of α and β values in different zones are given below:

Zone 1

$$\alpha = \frac{4 - \Omega}{2\,(3 - \Omega)} \tag{6}$$

$$\beta = \frac{2\,\Omega\,(3 - \Omega)^2}{3\,(4 - \Omega)} \tag{7}$$

Zone 2

$$\alpha = \frac{6\,\Omega^2 - 4\,\Omega + 1}{2\,\Omega\,(3\,\Omega - 1)} \tag{8}$$

$$\beta = \frac{2\,(3\,\Omega - 1)^2}{3(6\,\Omega^2 - 4\,\Omega + 1)} \tag{9}$$

Zone 3

$$\alpha\,\beta = 1 - \frac{1}{3\,\Omega} - 0.075\,\left(\frac{G\,D}{D - G}\right)\left(1 - \frac{1}{D}\right)^2 \tag{10}$$

$$\alpha = 2 - \frac{1}{\alpha\beta}\left\{ 1 - \frac{1}{6\,\Omega^2} - \frac{G\,(2D^3 - 3D^2 + 1)}{20D^2\,(D - G)} \right\} \tag{11}$$

Graphical representations of these equations further simplify the application of the proposed model. To generalize the graphical representation two new terms need to be defined: a) $F2 = \varepsilon_{s2}/\varepsilon_{s1} = \Omega/D$ and b) $F85 = \varepsilon_{s85}/\varepsilon_{s1} = \Omega/G$. A family of curves showing α and β as functions of Ω and F85 can be drawn for a constant value of F2. The limited space available here does not permit the inclusion of these curves in this paper.

CONCLUSIONS

A confinement model originally developed for concentric compression only has been extended to include the effects of flexural strain gradient. The model predicts the behavior of columns under combined axial and flexural loads in a satisfactory manner.

There is no conclusive evidence in the limited available data that the strain gradient in a section enhances the strength of confined concrete over that measured under concentric compression. The ductility, however, is significantly improved.

In lieu of the actual stress-strain curve of the confined concrete, a rectangular stress-strain curve can be used. The parameters for the stress block can be obtained by using the equations provided in this paper.

The ACI confinement requirements produce columns which show varied results under axial and flexural loads. Depending upon the criterion against which the columns are tested, the ACI requirements may be either too conservative for columns with well distributed steel or unsafe for columns with only four corner bars fully supported by a tie or both.

ACKNOWLEDGEMENTS

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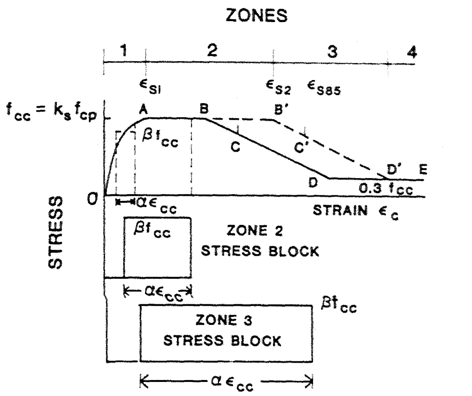


FIG. 1 STRESS-STRAIN CURVES FOR CONFINED CONCRETE

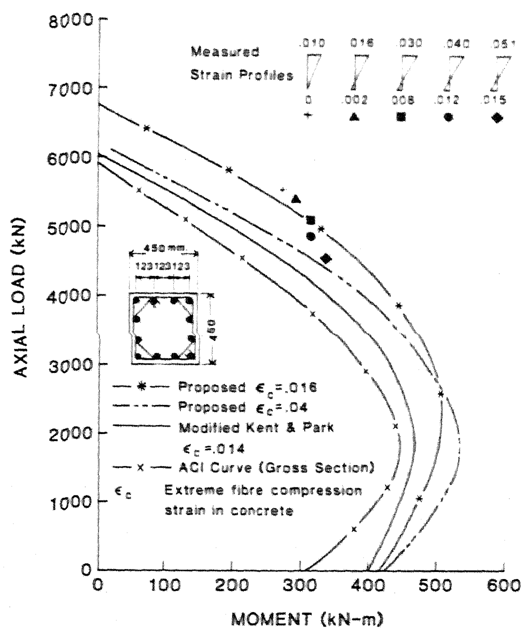
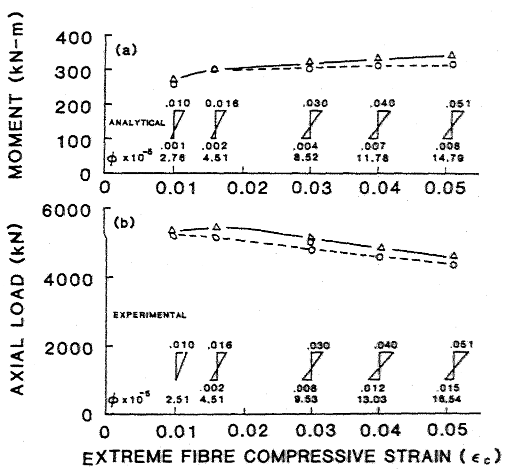


FIG.2 ANALYTICAL CAPACITY INTERACTION CURVES FOR UNIT 4 (REF. 6)



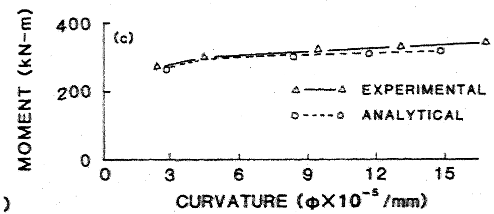


FIG.3 COMPARISION OF EXPERIMENTAL AND ANALYTICAL RESULTS FOR UNIT 4 (REF. 6)

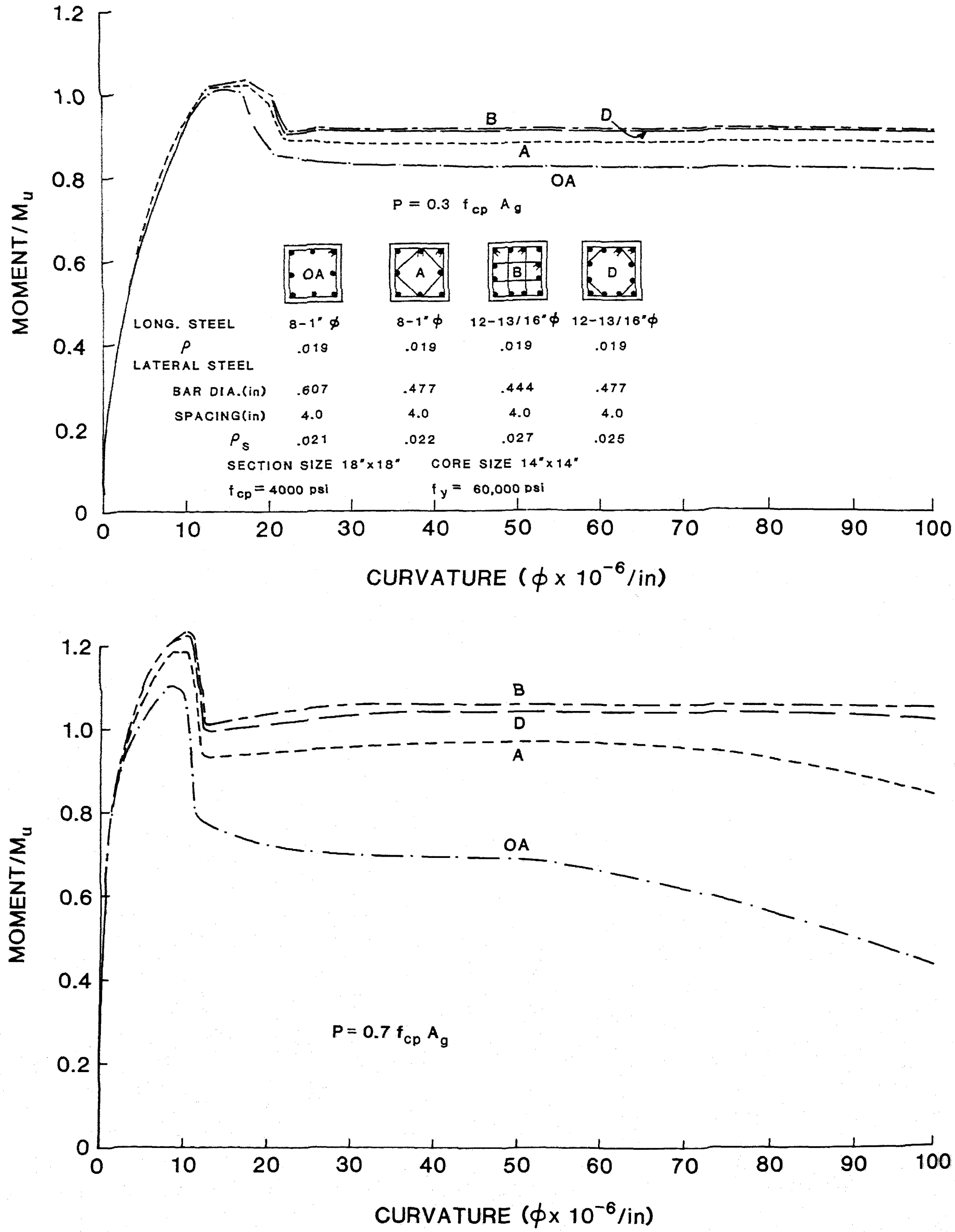


FIG.4 EFFECTS OF AXIAL LOAD LEVEL AND STEEL CONFIGURATION ON MOMENT - CURVATURE BEHAVIOR