EXPERIMENTAL INVESTIGATION OF COMPLETE STRESS-STRAIN CURVES OF CONCRETE CONFINED BY STIRRUPS UNDER CYCLIC LOADING

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SUMMARY

This paper discusses the experimental results of the complete stress-strain curves of concrete prisms with two different stirrup ratios and two different strength of concrete under monotonic and cyclic loading.

This paper points out that concrete confined by stirrups is able to restrain the lateral strain and to resist the sliding along the slant cracking surface of the prism. So, the strain under the peak stress is much larger than that of the plain concrete and the descending branch of the stress-strain curve is also much higher. Thus the ductility of the concrete is improved. The equations of the complete stress-strain curves of concrete confined with stirrups are similar to those of plain concrete, but the parameters are different.

EXPERIMENT

There are total 58 specimens in 26 groups with two different strength of concrete, 500 kg/cm^2 (H) and 300 kg/cm^2 (L) (actual strength: 543 kg/cm^2 (H) and 287 kg/cm^2 (L) respectively); four different stirrup ratios: μ_t = 0%, 0.5%, 1%, 2% and two groups 2a and 2b with same μ_t = 1% but with different spaces of stirrup. They were tested under three different kind of loadings: monotonic loading (A), cyclic loading with equal strain increment (B) and repeated cycles of loading under each strain increment (E) as shown in Fig. 6. The size of prism is 10 X 10 X 30 cm.

The diameter of the stirrup is 4 mm cold drawn steel wire. The yield strength is about 4484 kg/cm^2. The skeleton construction of the reinforcement is shown in Fig. 2.

All the experiments were tested by an ordinary hydraulic test machine with two hydraulic jacks placed at sides of the specimen as shown in Fig. 1.

MECHANICAL BEHAVIOURS AND RUPTURE DEVELOPMENT OF THE SPECIMEN

The rupture development of concrete prism confined with stirrups is similar to the plain concrete under monotonic and cyclic loading. The similarities of the complete stress-strain curves (σ-ε) and the lateral

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strain (ε') are shown in Fig. 3,4. Fig. 3,4 are also shown that the strain (ε_{1K}) under the peak stress (σ_{1K}); the slopes of the descending branchs and the residue strength are obviously increased than those of the plain concrete.

Fig. 5 is a typical diagram showing the cracking development and the different stages of strain increment. As the figure shown, at the initial stage when the specimen is under elastic stage ($\sigma < 0.4\ \sigma_{1K}$) the lateral strain (ε') is very small and the increase of the strain of stirrup is very slowly.

The first crack usually appears at the side surface in the direction of casting and along the out side of the longitudinal steel. With increasing the characteristic values of stirrup $\lambda_K = \mu_t \cdot R_g/R$ (Where Rg is the yield strength of stirrup and R is the cubic strength of concrete.) the lateral strains at cracking are increased.

At the descending branch part, the bearing strength of the specimen is decreased with increasing strain. When the stirrup steel yields, the complete stress-strain curve of the specimen is shown the turning point on the descending branch ($d^2Y/dx^2=0$). When the strain increases to (4000-6000) X10^{-6} or (2-3)ε_{1K}, critical slant crack is formed. On the complete stress-strain curve, it is shown the point of Max. curvature on the descending branch ($d^3Y/dx^3= 0$).

When the longitudinal strain reaches about (10000-12000) X 10^{-6} or (4-5) ε_{1K}, the concrete covering of the stirrup begin to spall. The stirrup of steel is exposed, but the specimen still can keep certain amount of the residue strength.

EFFECT OF DIFFERENT KIND OF LOADINGS TO BEHAVIOUR OF CONFINED CONCRETE

The envelopes under cyclic loading B,E have no obvious difference to the complete stress-strain curve under monotonic loading and the envelopes of the lateral strain(ε') are also similar to those of monotonic loading as shown in Fig.6.

The test results and the parameters of the complete stress-strain curve of confined concrete with stirrup are shown in Table 1. The parameters a_K , α_K under different loading are nearly same.

EFFECT OF DIFFERENT STIRRUP RATIOS TO STRENGTH AND DEFORMATION OF CONFINED CONCRETE

For convenience in comparison, $Y= \sigma/\sigma_{1K}$ and $X = \varepsilon/\varepsilon_{1K}$ are used in coordination to draw the tested curves with different λ_K as shown in Fig.7. Due to the similarity of the complete stress-strain curve of concrete confined with stirrup and plain concrete, the equation of curves for plain concrete under different loadings can also be used for concrete confined with stirrups.

$$x=\varepsilon/\varepsilon_{1K} \leq 1 \qquad Y=\sigma/\sigma_{1K}=a_K x+(3-2a_K)x^2+(a_K-2)x^3 \qquad \ldots\ldots \quad (1)$$

$$x=\varepsilon/\varepsilon_{1K} \geq 1 \qquad Y=\sigma/\sigma_{1K}= x/\left(\alpha_K (x-1)^2+x\right) \qquad \ldots\ldots \quad (2)$$

where a_K and α_K are the parameters of the ascending and descending branch respectively.

With increasing the value of λ_K, the values of a_K are increasin; in different strength of concrete specimens and α_k are decreased (see Table 1). These can be shown that concrete confined with stirrups can improve the behaviour of concrete and increase the residue strength.

When λ_K is less than 0.3 and the space between the stirrup is over 10 times the diameter of longitudinal steel, with increasing λ_K , ε_{lK} increases largely, but the increase of peak stress is not great.

The least space between stirrup (2b , s = 3 cm) has the best behaviour and if the space of stirrup is too large (2a, s = 12 cm) the stirrup can't confine the concrete between the adjecent stirrup effectively.

EQUATION OF COMPLETE STRESS-STRAIN CURVE OF CONCRETE CONFINED WITH STIRRUPS

As in the previous description, the equation of the complete stress-strain curve of concrete confined with stirrups is same as the equation of plain concrete under monotonic and cyclic loadings but with different parameters (see Eq. 1,2).

Under the condition when the space of stirrup is larger than 10 times the diameter of longitudinal steel, the prism strength of the plain concrete is used for the peak stress of the confined concrete. The ratio of ε_{lK} of confined concrete to ε_l of the plain concrete varies with different values of λ_K as shown in Fig. 8a. Same equation can be used in different strength of concrete as shown in Table 2. The parameters a_K and α_k vary with the different value of λ_K as shown in Fig. 8b,c, but values of α_k have larger difference in different strength of concrete. It seems suitable to use different equation in expression, but same expression can be used for values of a_K .

2. UNLOADING CURVE

During each unloading, from any point (ε_u, σ_u) on the envelope, unloaded to zero stress, the variation of the residue strain ε_p can be described in the form of power function:

$$X_p = G\ (X_u)^H \qquad \ldots\ldots\ (3)$$

where $X_p = \varepsilon_p / \varepsilon_{lK}$, $X_u = \varepsilon_u / \varepsilon_{lK}$

Values of G and H can be determined as shown in Table 3.

If $\xi = \frac{\varepsilon - \varepsilon_p}{\varepsilon_u - \varepsilon_p}$ and $\eta = \frac{\sigma}{\sigma_u}$ are the abscissa and ordinate of unloading curve of the specimen, a group of curves can be obtained and equation for plain concrete can be used as:

$$\eta = \xi^n \qquad \ldots\ldots\ (4)$$

where n can be obtained from $n = 1 + \sqrt{X_u}$ .... (5)

3. RELOADING CURVE

Each reloading starts from zero stress (ε_p , o) to the point tangent to the envelope. The tangent point is (ε_r, σ_r). The relationship between ε_r and ε_p from testings, can be obtained as:

$$X_r = L\ (X_p)^M \qquad \ldots\ldots\ (6)$$

where $X_r = \varepsilon_r / \varepsilon_{1K}$, $X_p = \varepsilon_p / \varepsilon_{1K}$

Parameters L, M can be determined and the results are shown in Table 4.

If the abscissia and ordinate of the reloading curves use $\xi = \frac{\varepsilon - \varepsilon_p}{\varepsilon_r - \varepsilon_p}$ and $\eta = \sigma / \sigma_r$, the curves are shown in Fig. 10.

The equations of ascending and descending part of the reloading curves can be taken as:

$$\varepsilon_r / \varepsilon_{1K} < 1 \qquad \eta = \xi^{1.2} \cdot (1 + Q \cdot 1.2 \cdot \sin \pi \xi) \qquad \ldots\ldots (7)$$

$$\varepsilon_r / \varepsilon_{1K} \geqslant 1 \qquad \eta = \xi^{T} \cdot (1 + 0.43\, T \sin \pi \xi)$$

$T = 1.4$, $Q = 0.167$ are determined by the method of least square.

CONCLUSION

1. With increase of stirrup ratio, the strain under peak stress of concrete confined with stirrup increases greatly, and the descending part of the complete stress-strain curve is lifted very much. These behaviours improve the ductility of the concrete structure.

2. The envelopes of the stress-strain curve of concrete confined with stirrup under cyclic loading (B,E) are similar to the curve under monotonic loading, and also to the curve of plain concrete.

3. The equations to calculate the complete stress-strain curve and unloading and reloading curve for plain concrete can also be used for concrete confined with stirrup, but the parameters should be modified with different characteristic value of stirrup λ_K.

4. The effects of concrete confined with stirrup are shown in two respects: restraing lateral strain (ε') of the core concrete and resisting the slip along slant cracking surface.

REFERENCE

I. Guo Zhen-hai et al, Experimental Investigation of Complete Stress-Strain Curve of Concrete. Journal of Building Structure, No. 1, 1982

2. Guo Zhen-hai, Zhang Xiu-qin, Experimental Investigation of Complete Stress-Strain Curve of Concrete under Cyclic Loading Res. Lab. of Earthquake and Blast Engr., Qinghua University Technical Reports TR-3, Qinghua University Press, 1981

Results of Testings (Average Group Value)

Table 1

Series No:	λ_K	σ_1' (kg/cm²)	σ_{1K} (kg/cm²)	ε_{1K} (10^{-6})	$E_{0.4}$ (10^5 kg/cm²)	Parameters of Complete Stress-Strain curve a_K	α_K	Number of Specimens
LA0	0	197	197	1525	2.644	2.225	0.85	4
LB0	0	224	224	1430	2.993	2.15	0.51	2
LA1	0.0756	238	228	2120	2.380	2.5	0.33	1
LB1	0.0765	245	234	1980	2.443	2.4	0.415	2
LE1	0.0774	212	202	2100	2.557	2.95	0.365	2
LA2	0.1531	220	209	2445	2.232	2.9	0.155	2
LB2	0.1531	211	201	2460	2.145	2.9	0.19	2
LE2	0.1512	242	232	2160	2.361	2.8	0.21	1
LA3	0.3061	225	215	2860	2.500	3.0	0.015	2
LB3	0.3061	245	235	2760	2.410	3.0	0.04	2
LE3	0.3061	246	236	2850	2.422	3.0	0.05	2
HA0	0	445	445	1871	3.300	1.383	2.465	7
HB0	0	460	460	1900	3.521	1.3	3.26	2
HA1	0.042	400	390	2360	2.772	1.8	1.38	1
HB1	0.042	451	441	2320	2.787	1.45	1.5	2
HA2a	0.079	452	442	1940	3.433	1.533	1.033	3
HB2a	0.079	478	468	2170	3.382	1.65	1.23	2
HE2a	0.080	459	449	1860	3.382	1.3	0.87	1
HA2	0.0809	473	463	2427	3.128	1.8	0.863	3
HB2	0.0817	461	451	2310	3.162	1.675	0.713	4
HE2	0.0793	465	455	2250	3.153	1.65	0.615	2
HA2b	0.0725	471	461	2370	3.244	1.9	0.66	2
HB2b	0.0725	471	462	2490	3.101	1.85	1.02	2
HE2b	0.0725	484	474	2400	3.150	1.7	0.53	2
HA3	0.1679	468	458	2520	3.516	2.2	0.3	1
HB3	0.1679	477	467	2900	2.959	2.1	0.365	2

Note: (1) The first alphabet of series No representing the strength of concrete, the second representing form of loading and the third representing the amount of stirrup steel.

(2) σ_1', the average compressive stress.

Parameters of Complete Stress-Strain Curve of Concrete Confined with Stirrup

Table 2

Strength of Concrete	Strength of Cement	Plain Concrete a	α	ε_1 (10^{-3})	Concrete Confined with Stirrup a_K/a	α_K/α	$\varepsilon_{1K}/\varepsilon_1$
200~300	400	2.2	0.4	1.40	$1+1.8\lambda_K$	$1-0.8\lambda_K^{0.55}$	$1+3\lambda_K$
	500	2.2	0.8	1.60			
400	500	1.7	2.0	1.80	$1+3\lambda_K$	$1-1.5\lambda_K^{0.35}$	$1+2.5\lambda_K$
Unified formula		2.0			$1+2.5\lambda_K$		$1+2.5\lambda_K$

Parameters of Calculated Equation of Unloading and Residue Strain

Table 3

Strength of Concrete	Plain Concrete	Concrete Confined with Stirrup	Standard deviation
200~300	$X_p = 0.32 X_u^{1.58}$	$X_p = 0.378 X_u^{1.439}$	0.089
≥ 400	$X_p = 0.22 X_u^{1.83}$	$X_p = 0.35 \cdot X_u^{1.511}$	0.143
Unified	$X_p = 0.27 X_u^{1.7}$	$X_p = 0.359 X_u^{1.489}$	0.128
Formula	$X_p \quad 0.31 \cdot X_u^{1.6}$		

Parameters of Calculated Equation of Reloading and initial Strain

Table 4

Strength of Concrete	Plain Concrete	Concrete Confined with Stirrup	Standard deviation
200~300	$X_r = 2.28 X_p^{0.63}$	$X_r = 2.348 X_p^{0.62}$	0.127
≥ 400	$X_r = 2.58 X_p^{0.54}$	$X_r = 2.39 X_p^{0.563}$	0.192
Unified	$X_r = 2.4 X_p^{0.6}$	$X_r = 2.375 X_p^{0.583}$	0.191
Formula	$X_r = 2.38 \cdot X_p^{0.59}$		

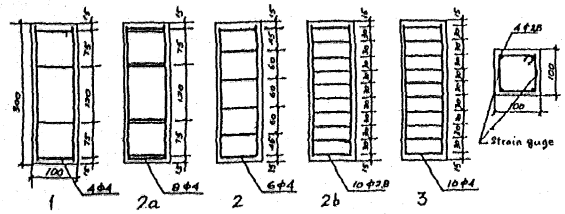

Fig. 2. Skeleton Construction of Stirrup.

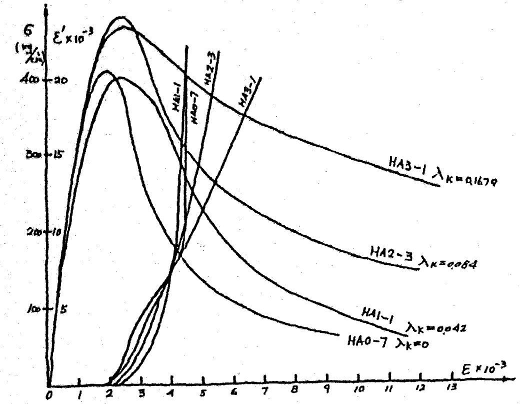

Fig. 3 Complete Stress - Strain Curve of Concrete (R=500 kg/cm²) Confined with Stirrup.

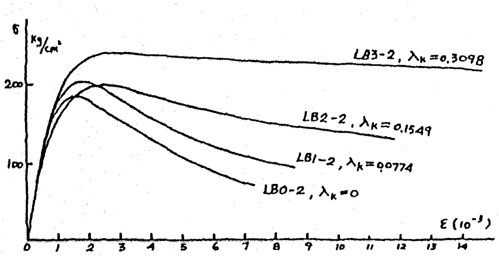

Fig. 4. Envelopes of Complete Stress-Strain Curve of Concrete (R=300 kg/cm²) Confined with Stirrup under Cyclic Loading B.

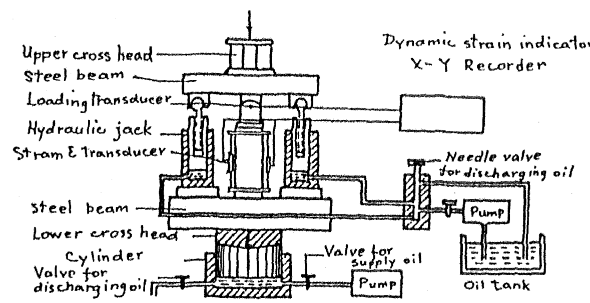

Fig. 1 Equipment to measure Stress-Strain Curve.

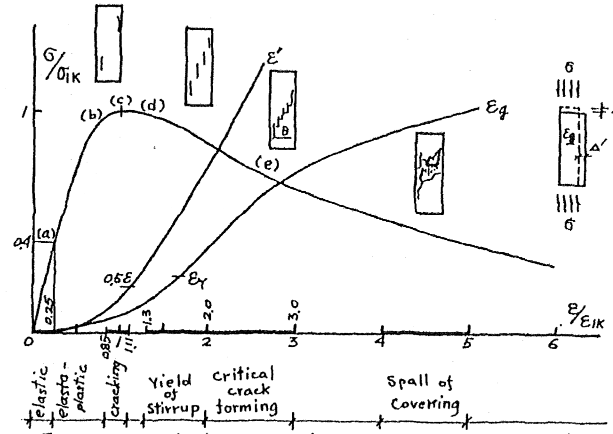

Fig. 5. Typical Diagram to show Rupture Development and Increment of Strain.

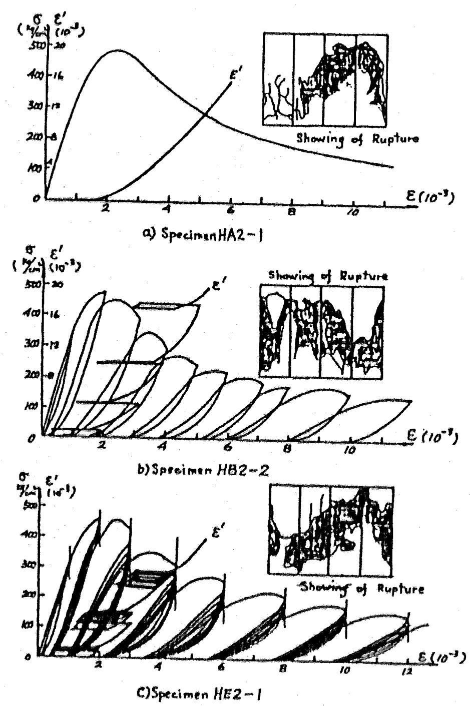

Fig. 6. Complete Stress - Strain Curves of Confined Concrete (R = 500 kg/cm²) under Cyclic Loading B. E.

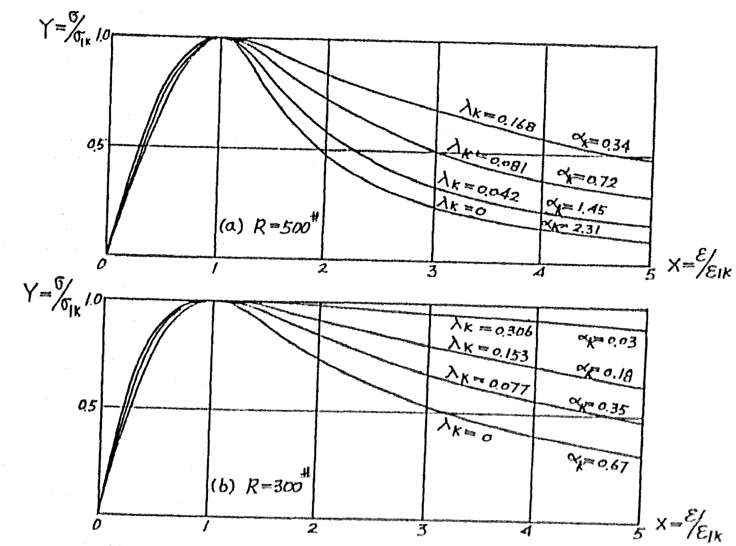

Fig. 7. Complete Stress - Strain Curves of Concrete Confined with Stirrup.

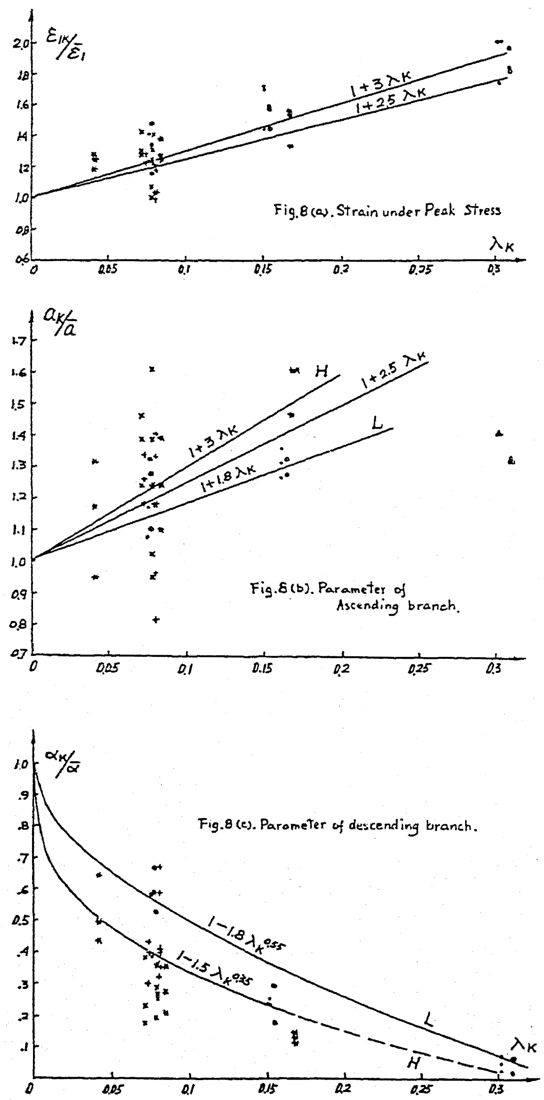

Fig.8. The Relationship between λK and Strain under Peak Stress, Parameters of Ascending and Descending Branch.

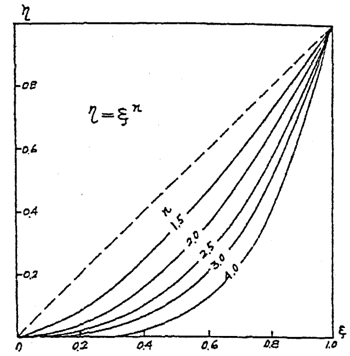

Fig.9. Equation of unloading Curve.

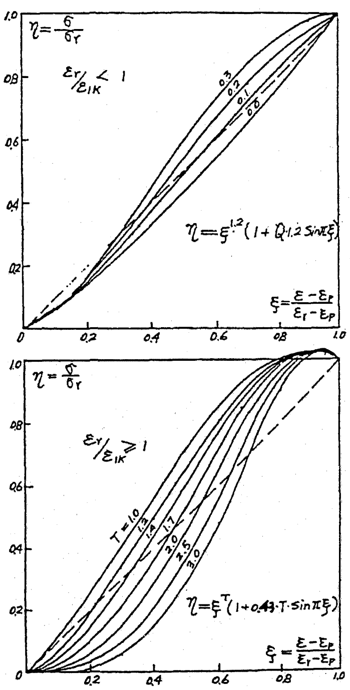

Fig.10. Equation of Reloading Curve.