SEISMIC ANALYSIS OF BUILDING FRAMES WITH SEMIRIGID CONNECTIONS

C. K. Chen (I)

SUMMARY

This paper presents a simplified method by which the story drift and the fundamental period of vibration of multistory steel frames with flexible beam-column connections can be estimated quickly and accurately, either by hand or with a small desk-top computer. Two multistory frames (3 stories and 10 stories) with assumed joint rotational stiffnesses ranging from 10^3 to 10^8 kip-in./rad were used to test the accuracy of the method. The results are in good agreement with the results of computer analyses.

INTRODUCTION

In conventional analysis of steel structures, beam-column and member-end connections are usually considered to be either hinged or completely fixed. These assumptions are not entirely realistic; however, they have been adopted in practice because of their simplicity for use in analysis and design. Although a more economical design would result if the effects of semirigid connections were considered in the analysis of steel frames, these effects have been almost entirely neglected because information concerning the behavior of such connections is limited, and the analytical method is too cumbersome to be used even for moderately large frames.

The primary purpose of this paper is to present a manual procedure for estimating story deflections of unbraced steel frames with semirigid connections that can be performed quickly and, for all practical purposes, accurately. When this procedure is used with the modified Rayleigh procedure, fundamental periods of vibration can also be estimated.

To demonstrate the effectiveness of the method, it was applied to two representative multistory frames, and the results were compared with the results of computer analyses. The frames considered were a 3-story multibay frame and a 10-story multibay frame. Assumed joint rotational stiffnesses ranged from 10^3 to 10^8 kip-in./rad.

ELEMENTS OF THE METHOD

Period of Vibration

The modified Rayleigh procedure--Equation (12-3) in the 1982 *Uniform Building Code* (*UBC*, Ref. 1)--is used for manually determining the period of vibration, T. The value of T can be expressed as follows:

(I) Project Engineer, URS/John A. Blume & Associates, Engineers, San Francisco, California, USA

$$T = 2\pi \sqrt{\frac{\Sigma m_i \delta_i^2}{\Sigma F_i \delta_i}} \tag{1}$$

where the values of F_i represent any lateral force distributed approximately in accordance with the formula in the *UBC* or any other rational distribution. The deflections, δ_i, can be calculated using the applied lateral forces, F_i. The values of m_i represent mass assigned to level i.

Drifts

The deflection or drift of a frame can be determined from the deflections between successive stories. Figure 1 shows the deflected centerlines of an interior column extending between the centers of successive stories of a frame and the elastic lines of the two beams framing into the column. The following idealizing assumptions facilitate the development of the deflection, Δ, shown in Figure 1.

- Deflections caused by axial deformation of members and by shear stress are disregarded.
- Beams and columns have points of contraflexure at midspan or midstory.
- The center-to-center spacing, l, of columns is constant across the frame, and the story height, h, is constant.

The first step in calculating the story deflection of a frame is to distribute the story shear, V_i, to the columns in the story in direct proportion to their stiffnesses so that all columns will have identical deflections. The D-value distribution method developed by Muto (Ref. 2) is recommended because it is simple to apply, and the results are accurate for all practical purposes. Muto's method is outlined in Figure 2. Since the deflections will be the same for any column line of a frame, a typical interior column line can be selected as the basis for calculating story deflections.

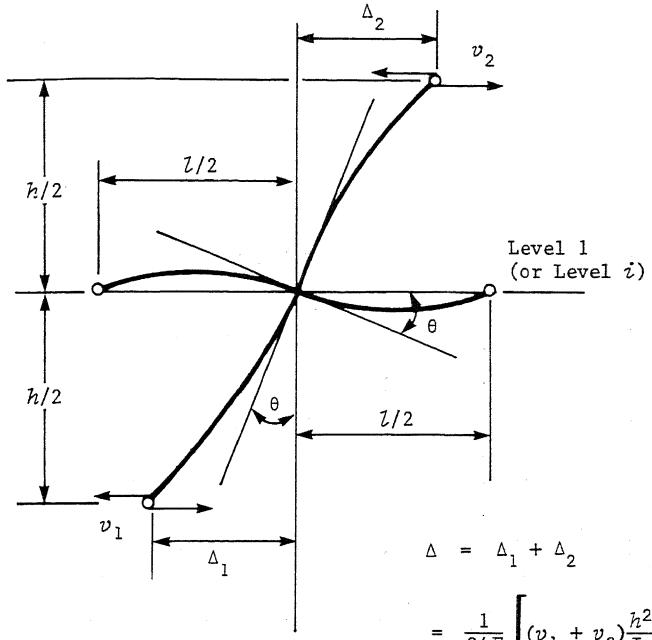
The story deflection, δ_i, at level i may be rearranged from the formula given in Figure 1 and presented as follows:

For upper stories, i.e., $i \geq 2$:

$$\delta_i = \delta_{i-1} + \frac{v_i}{12E}\left[\frac{h^2 l}{I_{bi}} + \frac{(h-d)^3}{I_{ci}}\right] \tag{2}$$

For the bottom story, i.e., $i = 1$:

$$\delta_1 = \frac{v_1 \left(\frac{h}{2}\right)^3}{3EI_{c1}} + \frac{v_1}{24E}\left[\frac{h^2 l}{I_{b1}} + \frac{(h-d)^3}{I_{c1}}\right] \tag{3}$$

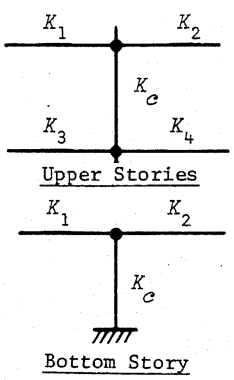


$$\Delta = \Delta_1 + \Delta_2$$

$$= \frac{1}{24E}\left[(v_1 + v_2)\frac{h^2 l}{I_{b1}} + (h - d)^3\left(\frac{v_1}{I_{c1}} + \frac{v_2}{I_{c2}}\right)\right]$$

Note: See text for notation.

FIGURE 1 DEFLECTED SHAPE OF AN INTERIOR COLUMN BETWEEN TWO SUCCESSIVE STORIES



$$\bar{K} = \frac{\Sigma K_i}{2K_c}$$

$$D = \frac{\bar{K}}{2 + \bar{K}} \cdot K_c$$

$$\bar{K} = \frac{\Sigma K_i}{K_c}$$

$$D = \frac{0.5 + \bar{K}}{2 + \bar{K}} \cdot K_c$$

K_i = relative stiffness of beam

K_c = relative stiffness of column

$\frac{D}{\Sigma D}$ = shear distribution factor

FIGURE 2 MUTO'S *D*-VALUE METHOD FOR SHEAR DISTRIBUTION

where:

δ_i = deflection at story level i
v_i = shear of a column below story level i
I_{bi} = moment of inertia of a beam at story level i
I_{ci} = moment of inertia of a column below story level i
h = story height
l = bay width
d = depth of beam

Effects of Joint Flexibility

The moment of inertia of the beam under consideration (I_{bi}) must be modified to account for the effects of joint flexibility. This flexibility can be accounted for by reducing the beam rigidity according to the following formula:

$$\left(\frac{I_b}{L_b}\right)_{\text{red}} = \left[\frac{1}{1 + \frac{6EI_b}{K_\theta L_b}}\right] \frac{I_b}{L_b} \tag{4}$$

where:

I_b = moment of inertia of beam (in.^{4})
L_b = length of beam (in.)
E = Young's modulus of elasticity (kip/in.^{2})
K_θ = joint rotational spring stiffness (kip-in./rad)

Although this formula was suggested by Driscoll (Ref. 3) for determining the effective length of columns with semirigid connections, a later study (Ref. 4) has shown that it may be used to model unbraced frames with semirigid connections.

STEPS IN THE METHOD

The period of vibration and story deflections can be calculated manually in the following steps:

1. Assume an initial value of T, and calculate the base shear, V, in accordance with the *UBC* formula.
2. Estimate story forces, F_x, in accordance with the *UBC* formula.
3. Select an interior column line and estimate story deflections, δ_i, using column shears v_i from Equations (2) and (3).
4. Calculate T using the modified Rayleigh procedure as shown in Equation (1).

5. Compare the calculated value of T with the initial value of T and repeat steps 1 through 4 using the calculated value of T, if necessary.

Although the period obtained from step 4 is theoretically correct, step 5 is required for obtaining accurate values of F_i and δ_i for seismic design.

VERIFICATION OF THE METHOD

Computer Analysis

The purpose of the computer analysis performed in this study was twofold: to verify Equation (4) and to test the accuracy of the results predicted by the manual method. Two computer programs were used: DRAIN-2D (Ref. 5) and SAP IV (Ref. 6). DRAIN-2D can be used to model semirigid joints as deformable elements. SAP IV has no capability to handle the semirigid joint problem directly; however, if beam rigidities are modified in accordance with Equation (4), SAP IV can be used to model semirigid elements. The results obtained by the two computer codes were virtually identical. Equation (4) is therefore a valid way of accounting for the effects of joint flexibility. SAP IV was used to test the adequacy of the proposed method.

3-Story Frame

The results of the analysis of the 3-story frame by both methods are shown in Table 1 along with the frame's configuration and structural properties. The 3-story, 2-bay frame is 180 in. high and 188 in. wide and was selected from the frames studied in Reference 4 to represent a typical rack used for merchandise storage. The structure is made of cold-formed steel, and the beam end connections (shelf connectors) are of the clip-in type. Although the value of K_θ was experimentally determined to be 10^3 kip-in./rad, three other cases--5 x 10^3, 10^4, and 10^6 kip-in./rad--were also considered. The value of 10^6 kip-in./rad represents perfect connection rigidity.

As shown in Table 1, the results obtained by the proposed manual method are in good agreement with those of the computer method. In general, deflections are overestimated as the connections become very flexible and underestimated as the connections become more rigid. The predicted periods of vibration are remarkably accurate in comparison with those obtained by the computer method.

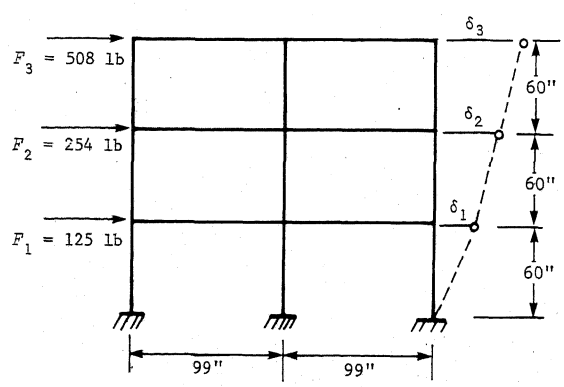
10-Story Frame

The 10-story frame is typical of those used in commercial office buildings. Both exterior and interior spans are 20 ft, and the story heights are 12 ft. The structure is made of W14x68 exterior columns and W14x95 interior columns; the girder sections are the same across all spans and all levels. Two joint rotational stiffnesses were assigned to this frame: 10^6 and 10^8 kip-in./rad. The limiting case of perfect connection rigidity is represented by $K_\theta = 10^8$ kip-in./rad, and flexible connection is represented by $K_\theta = 10^6$ kip-in./rad. The results of the manual analysis of the 10-story frame shown in Table 2 are in excellent agreement with the results of the computer analysis.

TABLE 1

RESULTS FOR THE 3-STORY FRAME

Joint Rotational Stiffness, K_θ (kip-in./rad)	Deflection (in.)			Period, T (sec)
	δ_3	δ_2	δ_1	
10^3				
Manual	2.00	1.38	0.50	1.66
Computer	1.80	1.20	0.45	1.56
5×10^3				
Manual	0.87	0.63	0.27	1.11
Computer	0.90	0.65	0.28	1.13
10^4				
Manual	0.74	0.54	0.24	1.02
Computer	0.77	0.56	0.25	1.04
10^6				
Manual	0.59	0.44	0.21	0.92
Computer	0.64	0.47	0.22	0.95



Column:

$A = 0.69$ in.^{2}

$I = 1.15$ in.^{4}

Beam (unreduced):

$A = 1.29$ in.^{2}

$I = 3.27$ in.^{4}

Joint:

K_θ varies

Story Mass:

$m_1 = m_2 = m_3$

$= 16.2$ lb-sec^2/in.

Lateral Force:

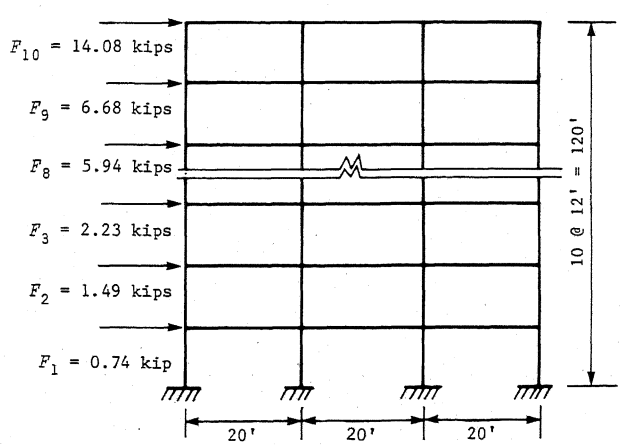
F_i is based on $T = 2.0$ sec

$V = \Sigma F_i = 887$ lb

TABLE 2

RESULTS FOR THE 10-STORY FRAME

Joint Rotational Stiffness, K_θ (kip-in./rad)	Deflection (in.)										Period, T (sec)
	δ_{10}	δ_9	δ_8	δ_7	δ_6	δ_5	δ_4	δ_3	δ_2	δ_1	
10^6											
Manual	4.11	3.94	3.68	3.35	2.96	2.51	2.01	1.48	0.92	0.34	2.29
Computer	4.31	4.13	3.85	3.49	3.06	2.58	2.04	1.46	0.87	0.31	2.33
10^8											
Manual	3.27	3.14	2.94	2.69	2.25	1.91	1.54	1.14	0.71	0.27	2.03
Computer	3.35	3.21	2.99	2.72	2.39	2.02	1.61	1.16	0.71	0.27	2.06



Column:

W14x68 (exterior)

W14x95 (interior)

Beam:

W18x40

Joint:

K_θ varies

Story Mass:

$m_1 = m_2 = \ldots = m_{10}$

$= 0.262$ kip-sec^2/in.

Lateral Force:

F_i is based on $T = 2.0$ sec

$V = \Sigma F_i = 47.5$ kips

CONCLUSIONS

The results of analyses performed using the simplified method developed for determination of story drift and fundamental period of vibration of multistory steel frames with rigid and flexible beam-column connections show good agreement with the results of computer analyses. The proposed procedure is suitable for manual calculation or for use with a small desk-top computer. The study also demonstrates that joint rotations contribute significantly both to deflections and to the period of vibration. They should, therefore, be considered in the analysis and design of buildings with flexible connections.

REFERENCES

1. International Conference of Building Officials, *Uniform Building Code*, Whittier, California, 1982.
2. Muto, K., "Seismic Analysis of Reinforced Concrete Buildings," *Proceedings, First World Conference on Earthquake Engineering*, San Francisco, California, 1956.
3. Driscoll, G. C., "Effective Length of Columns with Semi-rigid Connections," *Engineering Journal*, American Institute of Steel Construction, Vol. 13, No. 4, 1976.
4. Chen, C. K., R. E. Scholl, and J. A. Blume, *Seismic Study of Industrial Steel Storage Racks*, URS/John A. Blume & Associates, Engineers, San Francisco, California, June 1980.
5. Kanaan, A., and G. H. Powell, *DRAIN-2D: A General Purpose Computer Program for Dynamic Analysis of Inelastic Plane Structures*, EERC 73-6 and EERC 73-22, Earthquake Engineering Research Center, University of California, Berkeley, April 1973 (revised September 1973 and August 1975).
6. Bathe, K.-J., E. L. Wilson, and F. E. Peterson, *SAP IV: A Structural Analysis Program for Static and Dynamic Response of Linear Systems*, EERC 73-11, Earthquake Engineering Research Center, University of California, Berkeley, June 1973 (revised April 19, 1974).