THEORETICAL STATISTICAL DISPLACEMENT ANALYSIS
OF COMPOSITE BUILDING STRUCTURES IN TIME
OF MODELLED EARTHQUAKE ACTION

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SUMMARY

The constituent parts of the force system replacing earthquake action change according to instationary stochastic process, which can be described by the product of a periodical deterministic function and a nonstationary, stepwise stochastic process which can be represented in each halfperiod by one of to each other correlated Gaussian variables. With the aid of the solution of the differential equation system due to the linear elastic dynamic effect with damping the mean values and covariance functions of displacements of the structure can be determined. With various weighting of the components differing from each other in frequency we can follow the real earthquake spectra. Simple numerical example is added.

INTRODUCTION

In the actual civil engineering practice most buildings to be exposed to horizontal dynamic loads (replacing effect of earthquake) have vertical load-bearing structures of non-symmetrical floor plan. Structurally it means that the vertical load-bearing structures of the building are frameworks, columns, independent or connected bearing walls or combinations thereof. Deterministic analysis of such structures can be simplified by using the linear viscoelastic structural model suggested by the first author, which consists of rigid horizontal floor planes and of bar-connections substituting the vertical load-bearing elements. (See Ref.1)

NUMERICAL METHOD IN CASE OF DETERMINISTIC LOADS

The behaviour of the previously described modelled structure can be characterised by the matrix differential equation of motion (Ref.2) (notations see at the end of the paper)

$$\underline{\underline{M}}\,\ddot{\underline{\underline{f}}} + \underline{\underline{C}}\,\dot{\underline{\underline{f}}} + \underline{\underline{K}}\,\underline{\underline{f}} = \underline{\underline{p}}(t) \qquad (1)$$

Here $\underline{\underline{f}}$ represents a hypervector of 3 times n dimensions (n is number of floor planes) containing displacements in direction of floor plane axes and rotations in the plane of rigid floor platforms, $\underline{\underline{p}}(t)$ contains the (in work expressions to $\underline{\underline{f}}$ corresponding) time-dependent generalized forces.

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Time-dependence of coefficient matrices is neglected. The numerical method of solution (Ref.3) is based on the assumption $\underline{\underline{C}} = \alpha_c \underline{\underline{K}}$ on a linear transformation in the form $\underline{\underline{f}} = \underline{\underline{Z}}\ \underline{q}$ and upon a multiplication of (1) by Z* from the left side respectively. It is possible to perform these after determination of eigenvalues and eigenvectors of the problem without damping. ($\underline{\underline{Z}}$ contains these eigenvectors in a reduced form.) The obtained 3 times n unconnected differential equations for the transformed generalized displacements can be solved and the last step of the algorithm needs a re-transformation to the originally unknown functions.

INVESTIGATION OF THE STRUCTURE UNDER EFFECT OF EARTHQUAKE-LIKE STOCHASTIC LOADS

Random excitation of a structure due to earthquake can be described mathematically by the aid of stochastic processes. (Ref. 5, 6) The solution of the problem was sought first for stationary parts of earthquake motion by the spectral method. The use of this method was extended for special instationary cases by use of envelope functions (Ref. 7, 8, 9).
In the following we shall deal only with the solution of one differential equation of the previously described transformed system in case of stochastic loads:

$$\ddot{q} + d\ \dot{q} + \omega_o^2\ q = G(t) \qquad (2)$$

The experiences of matrix solution for static random loads given in (Ref.10) should be used in multidimensional case, but we don't touch upon this side of the problem. According to the nature of loading the reduced function G(t) can be built up as sum of products of a deterministic sinusoid function $g_1(t)$ and a stepwise random function $g_2(t)$ (See Fig.1).

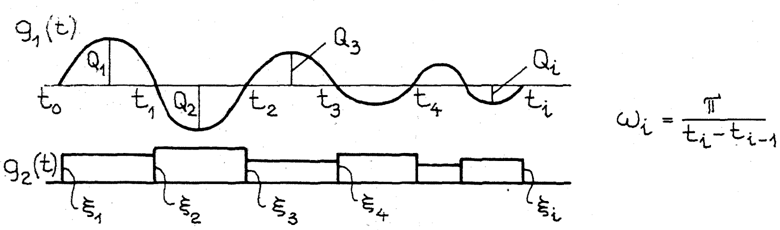

Fig.1.

If Q_i are chosen as mean values, then ξ_i are Gaussian variables with mean value 1,0 and covariance matrix $\underline{\underline{B}}_{\xi\xi}$, which generally is of banded nature. The principle of linear superposition is taken as valid and for one load component history as represented on Fig.1. mean values and covariance functions of q(t) can be evaluated as follows (q_o and $\dot{q}_o$ are taken as of each other and of ξ_i values independent normal variables)

$$E[q(t)] = b(t)\ E[q_o] + h(t)\ E[\dot{q}_o] + \sum_{i=1}^{m+1} a_i(t)\ E[\xi_i]$$

here $t_m < t \le t_{m+1}$ (formulas of b(t), h(t) and $a_i(t)$ are given in Appendix)

$$B_q(t_I,t_{II}) = b(t_I)\cdot b(t_{II})\ \sigma^2[q_o] + h(t_I)\cdot h(t_{II})\ \sigma^2[\dot{q}_o] + \underline{\underline{a}}^*(t_I)\ \underline{\underline{B}}_{\xi\xi}\ \underline{\underline{a}}(t_{II})$$

here $t_{m_1} < t_I \le t_{m_1+1}$ and $t_{m_2} < t_{II} \le t_{m_2+1}$ order of vector $\underline{\underline{a}}$ and of quadratic matrix $\underline{\underline{B}}_{\xi\xi}$ is max $[m_1+1, m_2+1]$.

As various harmonic functions have different rates of occurence we can use the method of weighted realizations (Ref.11 , 12) to obtain final values of $E[q]$, $B_q(t_I,t_{II})$. We shall use the assumptions that series values of ξ_i belonging to different harmonics are independent that the domain of frequency field of excitation is finite and width of it is rather small respectively.

NUMERICAL EXAMPLE

A structure considered as particle with mass 10 t on a subgrade characterised with spring coefficient 31047 t/s^2 and damping coefficient 111,44 t/s (Fig.2) shall be investigated numerically, if $f_o = 0$, $\dot{f}_o = 0$ and 3 equal halfperiod long excitation (ω_i=20 s^{-1}) is given, as on Fig.1. The amplitudes are given by $E[p_{max}]$ = 1000 MN, $E[\xi_i]$ = 1.0 and $\sigma^2[\xi_i]$ = 0.0025 (i = 1,2,3). Correlation of ξ_i (i = 1,2,3) is neglected.

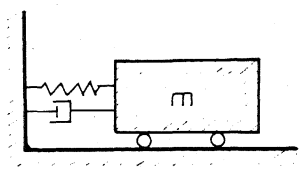

Fig.2.

Only one degree of freedom motion is considered, the resulting displacement as Gaussian stochastic process is drawn on Fig.3. The structure can be examined in view of permissible displacement limit.

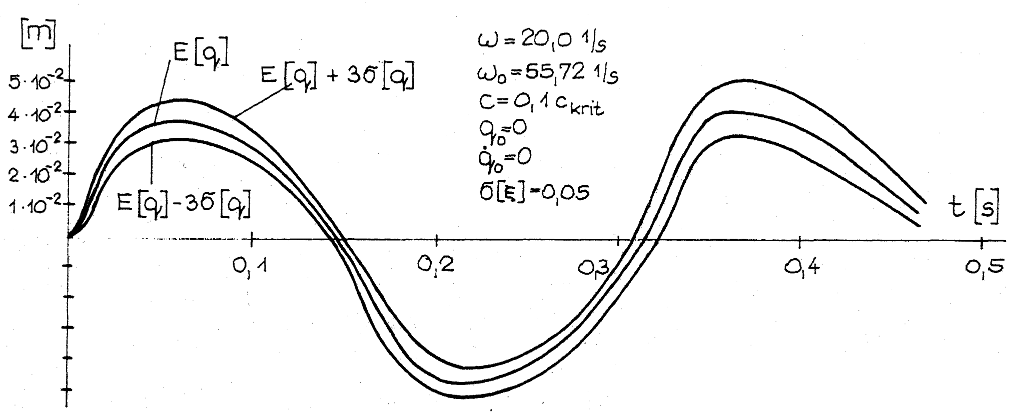

Fig.3.

NOTATION

The following symbols are used in this paper:

$\underline{\underline{a}}(t)$	auxiliary vector-function
$\overline{b}(t)$	auxiliary function
$B_q(t_I, t_{II})$	covariance function of q
$\underline{\underline{B}}_{\xi\xi}$	covariance matrix of random variables $\xi_1 \ldots \xi_i \ldots$
c	damping coefficient
$\underline{\underline{C}}$	damping matrix of the structure
$\overline{d}$	damping coefficient after reduction
$E[\,.\,]_{..}$	expected value of random expression in the brackets
$\underline{\underline{f}}, \underline{\underline{\dot{f}}}, \underline{\underline{\ddot{f}}}$	displacement hypervector, first, second time derivatives respectively
g(t), G(t)	given time dependent function
h(t)	auxiliary function
i,j,k	integer numbers (indices)
$\underline{\underline{K}}$	stiffness matrix of the structure
$\underline{\underline{M}}$	generalized mass matrix of the structure
$\overline{\max}[\;]$	maximum value of constants in the brackets
n	number of floor planes of the structure
$\underline{p}(t)$	hypervector of time dependent generalized forces
$\overline{\overline{q}}(t)$	generalized displacement after transformation
q_o	initial value of q
$\dot{q}_o$	initial value of first time derivative of q
Q_i	absolute value of reduced amplitudos (generally mean value)
t	time parameter
t_o	starting time of loading
$\underline{\underline{Z}}$	transformation matrix built up of reduced natural mode shapes of the structure
α_c	constant
ξ_i	correlated sequence of normally distributed random numbers
$\sigma[\;]$	standard deviation of random expression in the brackets
Σ	summation convention
ω_o	associated natural frequency
ω_i	changing frequency of force system of excitation in period i ($i \neq 0$)

APPENDIX

Computation of $\underline{a}(t)$, b(t), h(t)

If $t = t_k$, $m = m_k$ and $t_{m_k} < t_k \leq t_{m_k+1}$ respectively for $i = 1 \ldots m$

$$a_i(t) = (-1)^{i-1} Q_i \left\{ \omega_i \frac{e^{-\frac{d}{2}(t-t_{i-1})}}{(\omega_i^2-\omega_o^2)^2 + d^2\omega_i^2} \left(\frac{\frac{d^2}{2} + (\omega_i^2-\omega_o^2)\sin\left(\sqrt{\omega_o^2 - \frac{d^2}{4}}(t-t_{i-1})\right)}{\sqrt{\omega_o^2 - \frac{d^2}{4}}} + \right.\right.$$

$$\left.\left. + d\cos\left(\sqrt{\omega_o^2 - \frac{d^2}{4}}\,(t-t_{i-1})\right)\right) - \omega_i \cdot \frac{d\cos(\omega_i(t_i-t_{i-1}))}{(\omega_i^2-\omega_o^2)^2 + d^2\omega_i^2} + \frac{(\omega_i^2-\omega_o^2)\sin(\omega_i(t_i-t_{i-1}))}{(\omega_i^2-\omega_o^2)^2 + d^2\omega_i^2} \right\}$$

$$a_{m+1}(t) = (-1)^m Q_{m+1}\left\{\omega_{m+1}\frac{e^{-\frac{d}{2}(t-t_m)}}{(\omega_{m+1}^2-\omega_o^2)^2+d^2\omega_{m+1}^2}\left(\frac{\frac{d^2}{2}+(\omega_{m+1}^2-\omega_o^2)\sin\left(\sqrt{\omega_o^2-\frac{d^2}{4}}(t-t_m)\right)}{\sqrt{\omega_o^2-\frac{d^2}{4}}}+\right.\right.$$

$$\left.\left.+\, d\cos\left(\sqrt{\omega_o^2-\frac{d^2}{4}}\,(t-t_m)\right)\right)-\omega_{m+1}\frac{d\cos(\omega_{m+1}(t-t_m))}{(\omega_{m+1}^2-\omega_o^2)^2+d^2\omega_{m+1}^2}+\frac{(\omega_{m+1}^2-\omega_o^2)\sin(\omega_{m+1}(t-t_m))}{(\omega_{m+1}^2-\omega_o^2)^2+d^2\omega_{m+1}^2}\right\}$$

$$a_{m+2} = a_{m+3} = \ldots = 0$$

$$b(t) = e^{-\frac{d}{2}t}\left\{\cos\left(\sqrt{\omega_o^2-\frac{d^2}{4}}\,t\right)+\frac{d}{2}\,\frac{\sin\left(\sqrt{\omega_o^2-\frac{d^2}{4}}\,t\right)}{\sqrt{\omega_o^2-\frac{d^2}{4}}}\right\}$$

$$h(t) = \frac{\sin\left(\sqrt{\omega_o^2-\frac{d^2}{4}}\,t\right)}{\sqrt{\omega_o^2-\frac{d^2}{4}}}$$

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