SUBSTRUCTURING AND RIGID DIAPHRAGM SYSTEM SOLUTIONS IN 3-D BUILDING ANALYSIS

J. C. Jeing (I)

SUMMARY

The building is idealized as a series of independent plane substructures interconnected by horizontal rigid diaphragms, common-degree-of-connectivity nodes, or both. Any large frame or large structural system can be divided into smaller, simpler substructures. Shear walls with openings can be modeled independently in finer detail without increasing the size of the problem overall. The slab with openings can be modeled as a flexible horizontal plane. The substructuring technique reduces the complexity of modeling problems, improves computational efficiency, and provides flexibility in analyzing design modifications. The computer program SABS applies the procedure to elastic and dynamic analysis of three-dimensional building systems.

INTRODUCTION

Several computer programs currently available for three-dimensional static and dynamic analysis of building systems idealize the structure as an assemblage of plane frames linked by rigid floor diaphragms, but they do not enforce compatibility among displacements occurring at joints that are common to more than one frame. Moreover, many modeling problems occur when these programs are applied to certain types of buildings, such as those containing shear walls with openings, flexible diaphragms or mezzanines, discontinuous diaphragms, different levels of foundations, complex framing systems, etc.

The purpose of the research described here was to develop the capability to define two types of connected degrees of freedom at any joint of a frame. These degrees of freedom can either be connected to a connectivity node for a common degree of freedom or be slaved to any position on a rigid diaphragm.

The advantages of defining these two types of connected degrees of freedom in an analysis procedure are briefly as follows.

- Each individual frame may be any type of complex plane structure.
- Greater freedom is created by specifying joints that can be linked to connectivity nodes for the purpose of defining common degrees of freedom or by defining nodes to be slaved at locations on floor diaphragms.
- Any two or more frames can be selectively linked by connectivity nodes and rigid diaphragms.
- Any large frame or large structural system can be divided into smaller, simpler substructures. This capability not only can

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(I) Associate, URS/John A. Blume & Associates, Engineers, San Francisco, California, USA

reduce the problem of size but also can significantly reduce the cost of obtaining the solution for a large, complex structure.

- With the introduction of connectivity nodes, it is relatively simple to add substructures to an existing model in order to reflect design modifications. Renumbering and remodeling the entire structure are not required. Moreover, treating a complex structure as a composition of several simpler parts significantly reduces the complexity of preparing and checking input data.

- Utilizing these two types of connected degrees of freedom permits particular portions of a structure to be treated as substructures that can be modeled in finer detail, by the use of superelements, without increasing the size of the problem overall. For example, shear walls with openings, or particular joints, can be modeled independently as groups of superelements attached to their own frames.

The computer program SABS (Structural Analysis of Building Systems) combines features of two earlier programs, SAP IV (Ref. 1) and TABS (Ref 2). The structural types of SAP IV and the modeling techniques of TABS are used to produce an efficient program for analysis of certain types of buildings that cannot be modeled in the TABS program and that would have too large a numbering system, as well as bandwidth problems, if modeled in the SAP IV program. SABS makes input preparation easier and improves computational efficiency.

A typical example of a structure to which the technique described here and the computer program SABS are applicable is shown in Fig. 1.

STRUCTURE IDEALIZATION

The idealization that was selected for the analysis of a framed building is essentially identical to the TABS (Ref. 2) idealization. The positioning and connectivity of individual frames and horizontal rigid diaphragms are modified, however, in such a way as to increase the number of available modeling options. The structural idealization can be summarized as follows.

The building must be separated into a series of discrete plane frames that are connected either by connectivity nodes (common displacements) or by horizontal rigid diaphragms (slaved displacements). Each frame must be either in the vertical plane or in the horizontal plane (a flexible diaphragm, for example). Vertical-plane frames may be arbitrarily oriented, as indicated in Fig. 2. Each frame is modeled in H,V,R (horizontal, vertical, and rotational) local coordinates, while the diaphragm is modeled in X,Y,Θ global coordinates.

A typical frame elevation is shown in Fig. 3. The connection between frames can be accomplished through either slaved displacements or common displacements. Horizontal rigid diaphragms (slaved displacements) may be located at any horizontal plane. As shown in the figure, there may be two or more independent diaphragms at any level. It is not necessary to connect all frames to a given diaphragm; moreover, one frame can be connected to two or more diaphragms at any level.

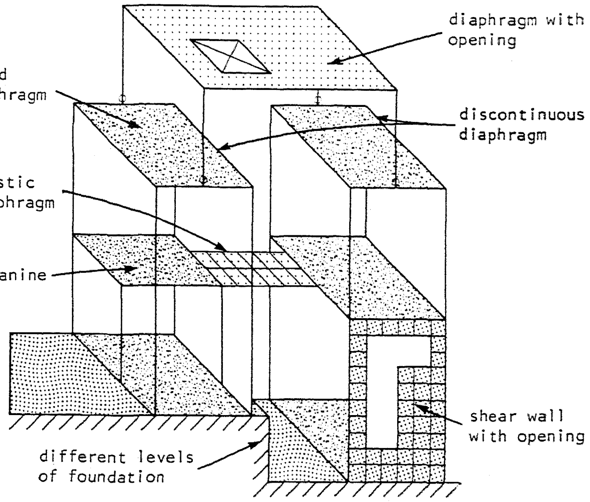


Fig. 1 Example Building for SABS Analysis

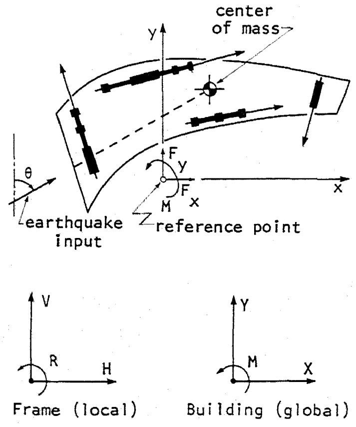


Fig. 2 Plan View of a Typical Building (from Ref. 2)

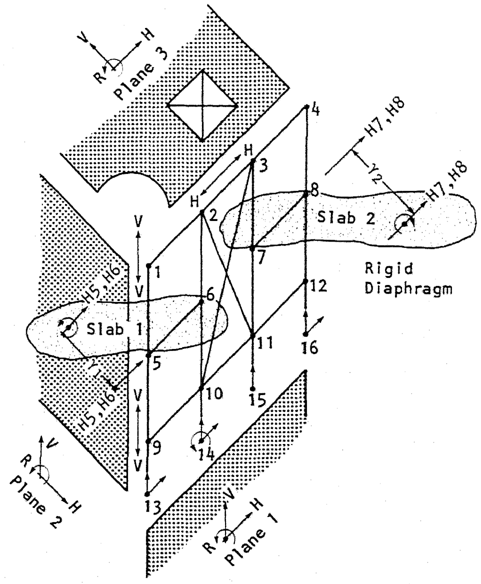


Fig. 3 Typical Frame Elevation

The connectivity nodes for common displacements at joints that are connected to two or more frames must occur in the same plane for any vertical, horizontal, or rotational connection (Fig. 3). Each joint has three common degrees of freedom to be connected to in-plane substructures -- translational (H and V) and rotational (R); but only the translational degrees of freedom can be connected to out-of-plane substructures. The displacements of connectivity nodes are the displacements at the joints of any frame that is connected to the connectivity nodes.

The horizontal displacements at the joints of frames that are connected to diaphragms are kinematically related to the diaphragms. The relationship depends on the orientation of the frame and its relation to the center of the diaphragm (Fig. 3).

Any large frame or large structural system can be divided into several smaller or simpler substructures that are linked by connectivity nodes. For example, a frame such as the one shown in Fig. 4a can be substructured into four smaller frames, as shown in Fig. 4b.

For dynamic analysis, the mass of the structure may be lumped selectively at the joints of the frames specified by the connectivity nodes, or it can be lumped entirely at the centers of diaphragms. The period and mode-shape calculations are based on the condensed external degrees of freedom of the connectivity nodes and diaphragms. The connectivity nodes need not be actually connected to two or more substructures; any joint of a frame that is transformed from an internal degree of freedom into an external degree of freedom is specified by a connectivity node and is taken into account in period and mode-shape calculations.

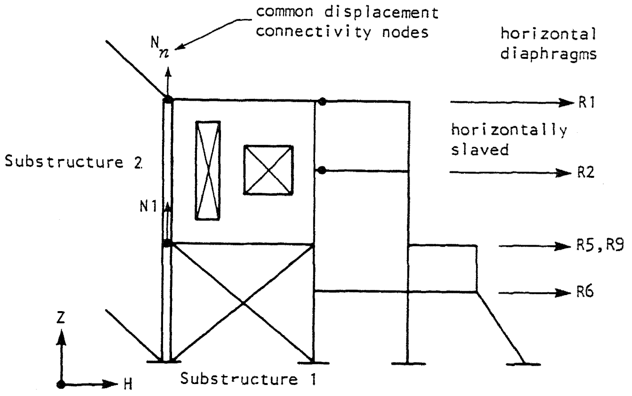
For static analysis, forces may be applied at the joints of frames or centers of diaphragms and at connectivity nodes. Fixed-end forces can be applied at any member of a beam or column element.

Static and dynamic analyses can be run at the same time. Numbers of static load cases and dynamic load cases are calculated independently and are combined dynamically according to the SABS user's specifications, shown in the accompanying table.

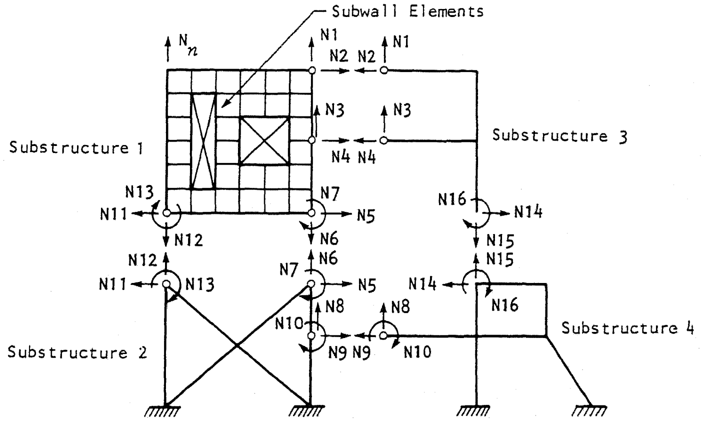
COMPUTATIONAL PROCEDURE

The substructuring technique is applicable to structures that can be separated into a series of plane frames that are tied together by horizontal rigid diaphragms, connectivity nodes, or both. Individual frames are modeled in the same way that two-dimensional plane frames are modeled in the SAP IV program (Ref. 1) except for specification of nodes that are connected to the diaphragms or the connectivity nodes. Each frame has internal degrees of freedom and connected (external) degrees of freedom (i.e., common degrees of freedom and slaved degrees of freedom).

The computational procedure for each frame consists first of obtaining a stiffness matrix and load vector for the connected degrees of freedom only. This is accomplished by static condensation to remove the internal degrees of freedom. The condensed matrices can then be combined to form a stiffness matrix and load vector for the entire structure, expressed in terms of the



a. Elevation of Frame

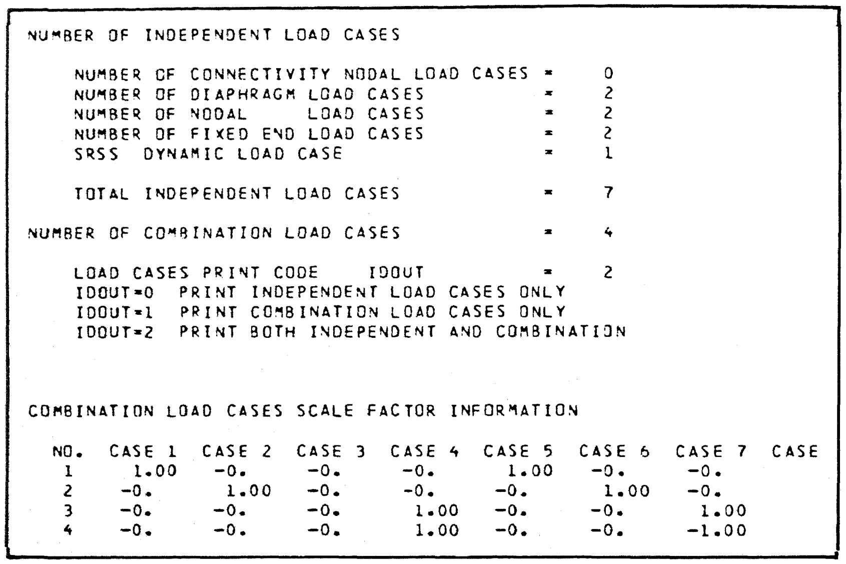


b. Substructural Parts and Connectivity Nodes

Fig. 4 Example of Substructuring

common joint displacements of the connectivity nodes and the rigid body displacements of the horizontal rigid diaphragms. The stiffness matrix for a representative frame is shown in Fig. 5. The relationship between the horizontal displacements of the diaphragm and the frame is shown in Fig. 6. Fig. 7 shows the stiffness matrix for the entire building.

LOAD CASE CONTROL INFORMATION



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NUMBER OF INDEPENDENT LOAD CASES

    NUMBER OF CONNECTIVITY NODAL LOAD CASES =    0
    NUMBER OF DIAPHRAGM LOAD CASES          =    2
    NUMBER OF NODAL      LOAD CASES         =    2
    NUMBER OF FIXED END LOAD CASES          =    2
    SRSS  DYNAMIC LOAD CASE                 =    1

    TOTAL INDEPENDENT LOAD CASES            =    7

NUMBER OF COMBINATION LOAD CASES            =    4

    LOAD CASES PRINT CODE    IDOUT          =    2
    IDOUT=0  PRINT INDEPENDENT LOAD CASES ONLY
    IDOUT=1  PRINT COMBINATION LOAD CASES ONLY
    IDOUT=2  PRINT BOTH INDEPENDENT AND COMBINATION



COMBINATION LOAD CASES SCALE FACTOR INFORMATION

  NO.  CASE 1  CASE 2  CASE 3  CASE 4  CASE 5  CASE 6  CASE 7  CASE
   1     1.00  -0.     -0.     -0.       1.00  -0.     -0.
   2    -0.      1.00  -0.     -0.     -0.       1.00  -0.
   3    -0.    -0.     -0.       1.00  -0.     -0.       1.00
   4    -0.    -0.     -0.       1.00  -0.     -0.     -1.00
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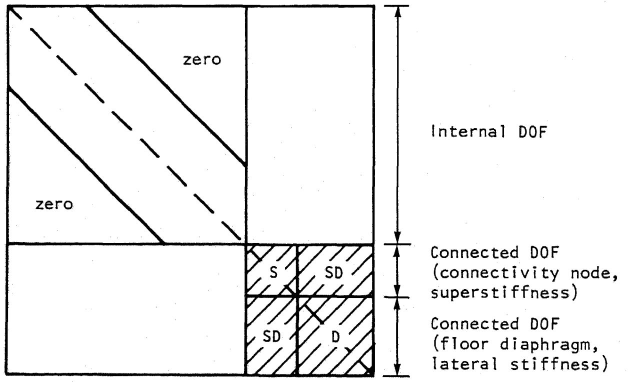


Note:

Cross-hatched portion (lower right) becomes condensed substructure stiffness for assembly into building stiffness.

Fig. 5 Frame Stiffness Matrix

The computational procedure then requires solving for the lateral displacements of the horizontal diaphragms and the common displacements of the connectivity nodes of the entire building (Fig. 7). Once that has been done, it is possible to compute the connected displacements for each frame (Fig. 6). The internal displacements of each frame can then be obtained by back substitution (Fig. 5). In this way, the joint displacements of the frame are computed successively, and individual member forces may be computed at the same time in standard fashion.

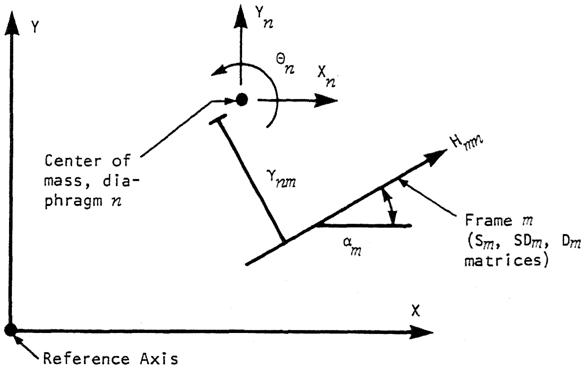
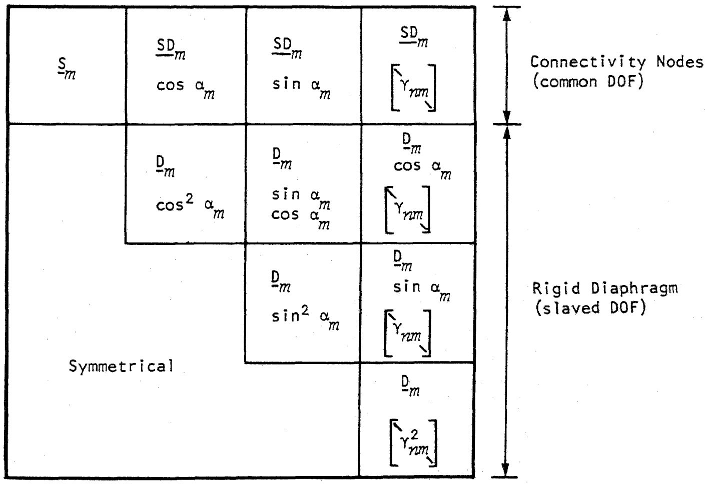


Fig. 6 Diaphragm and Frame Horizontal Displacements (after Ref. 2)



$\underline{S}_m$	$\underline{SD}_m$ cos α_m	$\underline{SD}_m$ sin α_m	$\underline{SD}_m$ $\left[\diagdown Y_{nm} \diagdown \right]$	Connectivity Nodes (common DOF)
	$\underline{D}_m$ cos^2 α_m	$\underline{D}_m$ sin α_m cos α_m	$\underline{D}_m$ cos α_m $\left[\diagdown Y_{nm} \diagdown \right]$	Rigid Diaphragm (slaved DOF)
		$\underline{D}_m$ sin^2 α_m	$\underline{D}_m$ sin α_m $\left[\diagdown Y_{nm} \diagdown \right]$	
Symmetrical			$\underline{D}_m$ $\left[\diagdown Y^2_{nm} \diagdown \right]$	

Fig. 7 Complete Building Stiffness Matrix

EXAMPLES OF SABS ANALYSIS

The following are practical examples of structures that have been analyzed by the SABS program.

The example shown in Fig. 8 is a 33-story high-rise hotel consisting of three connected structures, labeled A, B, and C in the figure. Originally, there was only a 16-story hotel building (A) and 14-story garage (B), with an expansion joint between the two structures. A 19-story hotel structure (C) was built above the original structures a few years later. The SABS program first analyzed the three parts of the hotel independently and then combined them for analysis as one structure by the use of connectivity nodes.

Fig. 9 shows a V-shaped 23-story hotel. The structure has a large opening around an elevator core at the corner between the two wings, A and B. The structural analysis assumed A and B to be rigid diaphragms linked by the flexible substructure of the area around the elevator core.

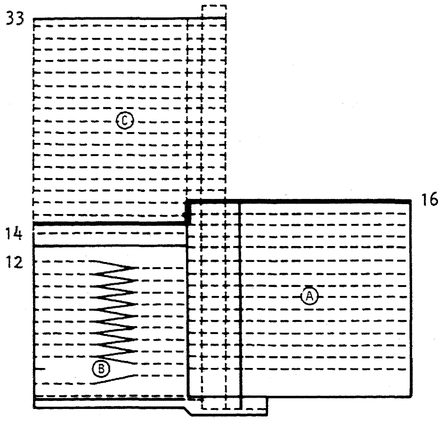


Fig. 8 Elevation of 33-Story Structure

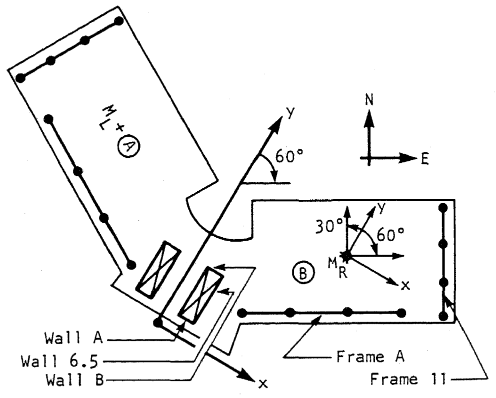


Fig. 9 Plan View of 23-Story Structure

REFERENCES

1. Wilson, E. L., and Bathe, K-J., *SAP IV*, Report EERC 73-11, Earthquake Engineering Research Center, University of California, Berkeley, USA, 1973.
2. Wilson, E. L., and Dovey, H. H., "Three-Dimensional Analysis of Building Systems - TABS," Report EERC 72-8, Earthquake Engineering Research Center, University of California, Berkeley, USA, 1972.